SYMMETRY BREAKING IN GRAVITY INDUCED SUPERPOTENTIAL

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The portion of the Nanopoulos-Srednicki superpotential which yields desired symmetry breaking is generalized to include terms proportional to Tr A^4 and (Tr $A^2)^2$. We show that the usual symmetry breaking scheme stays unaltered by addition of such terms. In particular we can generate a symmetry breaking SU(5) → SU(3)×SU(2)×U(1), if we introduce realizable restrictions on parameters associated with the terms.

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In a recent paper Choudhury [1] extended gravity induced superpotential introduced by Nanopoulos and Srednicki [2] and checked the validity of the symmetry breaking schemes SU(5) → SU(4)×U(1) and SU(5) → SU(3)×SU(2)×U(1). As was pointed out there, the addition of a term of the form Tr A^4, where A is the adjoint $\underline{24}$-representation of SU(5) used for the symmetry breaking purpose, does not automatically guarantee the intended breaking and has to be checked individually. With a term proportional to Tr A^4, assumed to be generated by the influence of gravity, this scheme works perfectly. But unfortunately this is not the only term of fourth power in $A^\alpha{}_\beta$ the potential can have. In addition to Tr A^4, we can also introduce a term proportional to (Tr $A^2)^2$. In this short note we want to show that even in this generalization, the symmetry schemes remain intact.

The portion of the superpotential which is relevant to the symmetry breaking and contains the most general superposition of terms upto the fourth power of the adjoint $\underline{24}$-representation of SU(5), $A^\alpha{}_\beta$, is given by the SU(5) invariant expression

$$v = (\tfrac{1}{2})\lambda_0 \operatorname{Tr} A^2 + (\tfrac{1}{3})\lambda_1 \operatorname{Tr} A^3 + (\tfrac{1}{4})\lambda_2 \operatorname{Tr} A^4 + (\tfrac{1}{4})\lambda_3 (\operatorname{Tr} A^2)^2, \tag{1}$$

where the last term is the new additional one.

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As was indicated by Witten [3], the conditions which have to be satisfied to get the vacuum expectation values of the fields by minimizing v with respect to them are

$$v_a(\Phi) = \frac{\partial v(\Phi)}{\partial \Phi_a} = 0, \tag{2}$$

and

$$K_\alpha = \overline{\Phi}_a T^a_{\alpha b} \Phi^b = 0, \tag{3}$$

where $T^a_{\alpha b}$ are the generators of the gauge group.

The extremum of v obtained from Eq. (2), along with the restriction that $A^\alpha{}_\beta$ must be traceless, determines the condition when SU(5) is going to stay intact or is going to be broken. Using Eq. (2) we get

$$(\lambda_0+\lambda_3 \operatorname{Tr} A^2)A^\beta{}_\alpha+\lambda_1(A^\beta{}_\gamma A^\gamma{}_\alpha-(\tfrac{1}{5})\delta^\beta{}_\alpha \operatorname{Tr} A^2)A^\beta{}_\alpha$$
$$+\lambda_2(A^\beta{}_\varrho A^\varrho{}_\gamma A^\gamma{}_\alpha-(\tfrac{1}{5})\delta^\beta{}_\alpha \operatorname{Tr} A^3)A^\beta{}_\alpha = 0. \tag{4}$$

To check for conditions of retention or breaking of the symmetry, we only need to look for diagonal solutions of $A^\beta{}_\alpha$. We assume

$$A^\beta{}_\alpha = c_\beta \delta^\beta{}_\alpha. \tag{5}$$

Eq. (4) then yields

$$\{(\lambda_0+\lambda_3 k_1)c_\beta+\lambda_1(c_\beta c_\alpha-k_1/5)+\lambda_2(c_\beta c_\alpha^2-k_2/5)\}\delta^\beta{}_\alpha = 0, \tag{6}$$

where

$$k_1 = \operatorname{Tr} A^2 \quad \text{and} \quad k_2 = \operatorname{Tr} A^3. \tag{7}$$

For $\alpha = \beta$, the following condition must be satisfied:

$$(\lambda_0+\lambda_3 k_1)c_\beta+\lambda_1(c_\beta^2-k_1/5)+\lambda_2(c_\beta^3-k_2/5) = 0. \tag{8}$$

Eq. (7) is quite complicated to solve in general because of the dependence of k_1 and k_2 on c_α's. But since we are only interested in finding out whether $A^\beta{}_\alpha$ possesses the forms pertinent to the symmetry breaking, we will just check whether such solutions are realizable.

If we set $c_1 = c_2 = c_3 = c_4 = c_5 = c$, we get $k_1 = 5c^2$, $k_2 = 5c^3$, and the Eq. (8) yields

$$(\lambda_0+5\lambda_3 c^2)c = 0, \tag{9}$$

hence either $c = 0$ or $c = \pm i\sqrt{\lambda_0/(5\lambda_3)}$. For both cases the symmetry stays intact.

If we look for the symmetry breaking case SU(5) → SU(4)×U(1), we have to set $c_1 = c_2 = c_3 = c_4 = c$ and $c_5 = -4c$ in Eq. (8). In this case $k_1 = 20c^2$ and $k_2 = -60c^3$. Eq. (8) changes into

$$\lambda_0 c-3\lambda_1 c^2+(13\lambda_2+20\lambda_3)c^3 = 0. \tag{10}$$

Since we are not looking for a solution $c = 0$, we find

$$c = \frac{3\lambda_1 \pm \sqrt{9\lambda_1^2 - \lambda_0(13\lambda_2 + 20\lambda_3)}}{2(13\lambda_2 + 20\lambda_3)}. \tag{11}$$

If we set

$$9\lambda_1^2 - \lambda_0(13\lambda_2 + 20\lambda_3) = 0, \tag{12}$$

we get an unique c. Thus for a reasonable λ_i's, we can achieve such c. This indicates that the symmetry breaking SU(5) → SU(4) × U(1) can be achieved with our extended superpotential.

To look for the existence of the important symmetry breaking scheme SU(5) → SU(3) × SU(2) × U(1), we set $c_1 = c_2 = c_3 = c$ and $c_4 = c_5 = -(3/2)c$ in Eq. (8). In this case $k_1 = (15/2)\,c^2$ and $k_2 = -(15/4)\,c^3$. We obtain

$$\lambda_0 c - (\tfrac{1}{2})\lambda_1 c^2 + (7\lambda_2/4 + (\tfrac{15}{2})\lambda_3)c^3 = 0. \tag{13}$$

Since we are not looking for the solution of the type $c = 0$, we find

$$4\lambda_0 - 2\lambda_1 c + (7\lambda_2 + 30\lambda_3)c^2 = 0, \tag{14}$$

or

$$c = \frac{2\lambda_1 \pm \sqrt{4\lambda_1^2 - 16\lambda_0(7\lambda_2 + 30\lambda_3)}}{2(7\lambda_2 + 30\lambda_3)}. \tag{15}$$

If we choose

$$4\lambda_1^2 - 16\lambda_0(7\lambda_2 + 30\lambda_3) = 0 \tag{16}$$

we get real $c = \lambda_1/(7\lambda_2 + 30\lambda_3)$ provided all λ_i's are real. Therefore we see, we can achieve a realizable symmetry breaking SU(5) → SU(3) × SU(2) × U(1) if we demand that the λ_i's should satisfy the constraint Eq. (16).

We have thus shown that the additional term $(\mathrm{Tr}\, A^2)^2$ does not disturb the symmetry breaking pattern. As we have mentioned earlier that the inclusion of this term makes v of Eq. (1), the most general potential with terms proportional upto the fourth power of $A^\beta{}_\alpha$.

It would be quite interesting to study, whether potentials with arbitrary power n of the adjoint representation $A^\beta{}_\alpha$ could retain similar symmetry breaking schemes. Work is already in progress along this line.

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