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CORRECTIONS TO STATIC PROPERTIES OF HADRONS*

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Quantum gluon corrections to the static properties of hadrons are calculated within the framework of quark bag model using noncovariant perturbation theory. As an example baryon magnetic moments are considered. Comparison of quantum and classical approaches is given.

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Introduction

It is well known at present that static properties of hadrons measured experimentally differ from predictions of additive quark model. These deviations appear in the phenomenological constituent quark model as well as in the bag model. Thus, for instance, the value of g_A for transition $n \to p$ predicted by the bag model is equal to 1.09 while experimentally $g_A \approx 1.25$; the difference between experimental values of magnetic moments and additive quark model predictions is of the order of 0.2–0.3 n.m., that is 10–20%, even if anomalous magnetic moments of quarks μ_u, μ_d, μ_s are considered as free phenomenological parameters. This difference is probably due to: 1) quark interactions described in α_s-approximation, where α_s — constant of strong interaction; 2) presence of π-meson clouds.

It is certainly interesting to find out whether it is possible to explain this difference within the framework of the models [1, 2] mentioned above. This question was discussed in many papers [1–8]. The problem of baryon magnetic moments was considered also within the framework of sum rules and other models [9, 10].

In this paper we shall describe calculations of α_s-corrections in the bag model. The results obtained by different authors are contradictory. We will give here the complete calculation of the corrections to baryon magnetic moments using quantized gluon field.

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1. Gluon corrections to magnetic moments of hadrons. Classical method

In this chapter we consider first calculation of magnetic moments treating gluon field classically as it was done in Ref. [6]. For simplicity anomalous magnetic moments and masses of quarks are assumed to be zero. This approximation does not change the essence of the problem and allows one to make description more transparent. We will use the MIT bag model, however, the results, as will be evident further, have general validity. The method of calculation which we call classical is the following one: consider solution of Dirac equation for a quark in a homogeneous magnetic field

$$(\hat{p}-e\hat{A})\psi = 0, \tag{1}$$

satisfying confining boundary condition

$$i\hat{n}\psi = \psi. \tag{2}$$

In Eq. (1) $\vec{A} = \frac{1}{2}\vec{H}\times\vec{r}$, $A_0 = 0$ and $\vec{H}$ is a small external field. Let us write the solution in the form $\psi = \psi_0+\delta\psi$, where ψ_0 — solution of free Dirac equation with the boundary condition (2), corresponding to the ground $1s_{1/2}$ state of quark, and $\delta\psi$ — the correction to the wave function linear in $\vec{H}$. Note, that $\delta\psi$ also satisfies boundary condition (2). Next, consider the solution for the gluon field $F^a_{\mu\nu}$, created by color current of quarks $j^a_\mu = \bar{\psi}\gamma_\mu\lambda^a\psi$. It can be found by solving Maxwell equations with the confining condition

$$n_\mu F^a_{\mu\nu} = 0$$

at the boundary [2, 11].

In these solutions we also keep terms linear in $\vec{H}$. Once $F^a_{\mu\nu}$ is known we can find the energy of the gluon field

$$W = \int \frac{(\vec{E}^a)^2-(\vec{B}^a)^2}{2}\,dV, \tag{3}$$

which gives the gluon correction to the mass of a hadron in the presence of magnetic field $\vec{H}$. $\vec{E}^a$ and $\vec{B}^a$ in Eq. (3) are gluoelectric and gluomagnetic fields, respectively. The derivative $-\partial W/\partial\vec{H}$ taken in the limit $\vec{H}\to 0$ defines the QCD correction of the order of $\alpha_s = g^2/4\pi$ to the magnetic moment of a hadron. It has the form [6]

$$\delta\vec{\mu} = -\tfrac{3}{8}\,\alpha_s A \sum_{i\neq k}\lambda^a_i\lambda^a_k Q_i\vec{\sigma}_k, \qquad A = 0.0406. \tag{4}$$

Indices i and k correspond to different quarks, Q_i — quark charge, A — numerical coefficient depending on quark wave function; $\vec{\sigma}_k$ and λ^a_k are understood as matrix elements of corresponding operators. Let us note that the operator structure of the correction $\delta\vec{\mu}$ is essentially different from that of the "basic" magnetic moment

$$\vec{\mu} = \mu_0 \sum_i Q_i\vec{\sigma}_i$$

made of Dirac magnetic moments of quarks.

In Ref. [6] the Dirac equation has been solved explicitly. Here we will use perturbation theory which allows us to obtain the corrections to the energy linear in $\vec{H}$. As is known, $\delta\psi$ is orthogonal to the wave function of the ground state ψ_0. Now let us take into account that solutions of Dirac equation with positive and negative frequencies satisfying boundary condition (2) form a complete set of functions.

To find the expansion of $\delta\psi$ it is not necessary to know $\delta\psi$ explicitly; to do that we may use the Dirac equation written in the form

$$(E_0\gamma_0 - i\vec{\nabla}\vec{\gamma})\delta\psi = -\frac{e}{2}[\vec{H}\times\vec{r}]\vec{\gamma}\psi_0 - \delta E\gamma_0\psi_0. \tag{5}$$

Here E_0 — the energy of $1s_{1/2}$ state of quark. The structure of the solution up to terms linear in $\vec{H}$ could be easily found by multiplying the equation (5) by $\bar{\psi}_\alpha$ and integrating over bag volume; here ψ_α — solution of Dirac equation for $\vec{H} = 0$, α denotes the set of corresponding quantum numbers. The integral of left hand side of the equation is equal to

$$E_0\int \psi_\alpha^\dagger\delta\psi dV - i\int \bar{\psi}_\alpha\vec{\nabla}\vec{\gamma}\delta\psi dV. \tag{6}$$

The integral with $\vec{\nabla}$ could be rewritten as

$$-i\int \vec{\nabla}(\bar{\psi}_\alpha\vec{\gamma}\delta\psi)dV + i\int (\vec{\nabla}\bar{\psi}_\alpha)\vec{\gamma}\delta\psi dV. \tag{7}$$

The first integral in expression (7) is reduced to the surface integral

$$\int \vec{n}\bar{\psi}_\alpha\vec{\gamma}\delta\psi dS$$

which vanishes because ψ_α and $\delta\psi$ satisfy boundary condition (2). The second integral in expression (7) can be transformed using the Dirac equation and expressed in the form

$$-E_\alpha\int \psi_\alpha^\dagger\delta\psi dV.$$

After integration of r.h.s. of Eq. (5) multiplied by $\bar{\psi}_\alpha$ we obtain the following equation

$$(E_0 - E_\alpha)\int \psi_\alpha^\dagger\delta\psi dV = -\frac{e}{2}\vec{H}\int \vec{r}\times\bar{\psi}_\alpha\vec{\gamma}\psi_0 dV - \delta E\int \psi_\alpha^\dagger\psi_0 dV. \tag{8}$$

When $\alpha = 1s_{1/2}$ we get the well known result for the linear in $\vec{H}$ correction for quark energy

$$\delta E = -\vec{\mu}_0\vec{H}$$

where $\vec{\mu}_0$ — Dirac magnetic moment of quark:

$$\vec{\mu}_0 = \frac{e}{2}\int \vec{r}\times\bar{\psi}_0\vec{\gamma}\psi_0 dV.$$

For $\alpha \neq 1s_{1/2}$ Eq. (8) is reduced to the equation

$$\int \psi_\alpha^\dagger \delta\psi dV = \frac{-\frac{e}{2}\vec{H}}{E_0 - E_\alpha} \int \vec{r} \times \bar{\psi}_\alpha \vec{\gamma} \psi_0 dV, \tag{9}$$

which allows one to find expansion for $\delta\psi$. It is rather obvious that this expansion includes wave functions corresponding to $ns_{1/2}$ (excluding $n = 1$) and $nd_{3/2}$ states. The integrals in r.h.s. of Eq. (9) represent coefficients of expansion of $\delta\psi$. It is seen that expansion of $\delta\psi$ includes also solutions with negative energies. In that case $E_\alpha = -|E_\alpha|$.

2. *Corrections to magnetic moments in perturbation theory*

Let us calculate the correction to magnetic moment of a hadron, taking into account discrete structure of energy levels for quarks and gluons. It means that we will deal with states of quarks (antiquarks) and gluons with definite energy and quantum numbers J^P. We will calculate the energy shift in the external magnetic field due to one gluon exchange within the framework of noncovariant perturbation theory. Thus, the interaction with external field $\vec{H}$ as well as emission and absorption of gluons will be considered as small perturbations. The correction to the energy which we are interested in is given by formulae of third order perturbation theory:

$$\Delta E = \sum_n{}' \sum_m{}' \frac{V_{0m} V_{mn} V_{n0}}{(E_m - E_0)(E_n - E_0)} - V_{00} \sum_m{}' \frac{|V_{0m}|^2}{(E_m - E_0)^2}. \tag{10}$$

One of the V should be understood as operator of electromagnetic interaction (u), two others — chromodynamical interaction (v). Index "0" refers to the ground (unperturbed) state of hadron. The sign prime in Eq. (10) as usually means the absence of terms with $m, n = 0$. The correction given by Eq. (10) could be described in the language of diagrams shown in Fig. 1. The diagrams not rendered there can be obtained from diagrams of Fig. 1 by permutation of quark lines. Their contribution will be taken into account by summation over different quarks. The contribution of each diagram depends on the relative position of vertices for emission of gluons and photons. As an example we will consider baryons. It is assumed that initial and final quarks are in the ground $1s_{1/2}$ state.

The diagrams, corresponding to production of $q\bar{q}$-pairs (Fig. 2) are equivalent to diagrams b, e (Fig. 1), in which $-\tilde{E}_k$ is substituted for quark energy, where $\tilde{E}_k$ is the energy of antiquark.

The presence of the second term in Eq. (10) means that the contribution of diagram a (Fig. 1) cancels the contribution of similar diagram b (Fig. 1) if the intermediate state of quark in both diagrams coincides with the ground $1s_{1/2}$ state.

The same reason leads to cancellation of contributions of diagrams g-i (Fig. 1).

Hence we see that only excited states of intermediate quarks as well as all possible states with negative energies should be taken into account in the diagrams corresponding

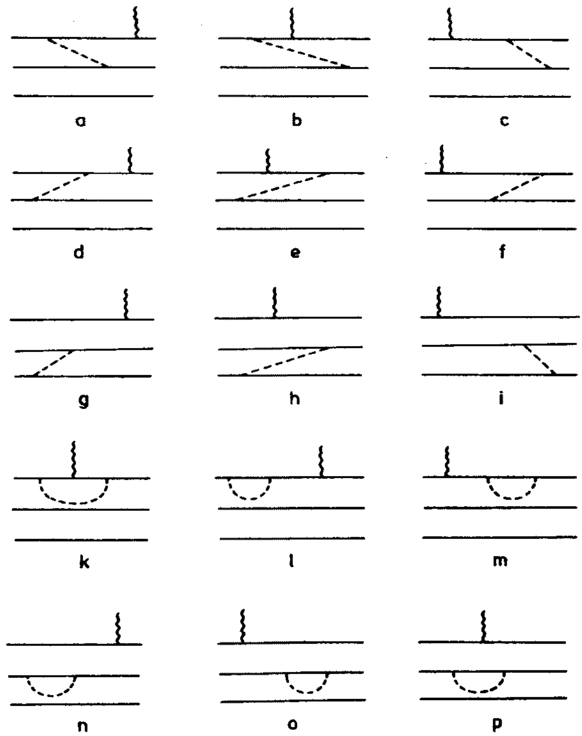

Fig. 1

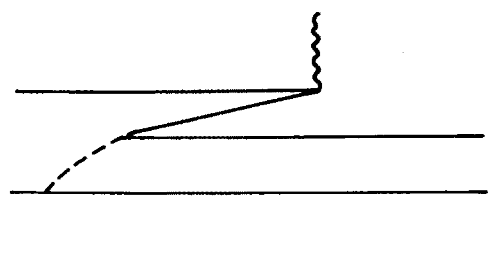

Fig. 2

to gluon exchange between different quarks. Disconnected graphs shown in Fig. 1, g-i, do not contribute. Note that formally the contribution of a single disconnected diagram is analogous to the contribution of the diagram a (Fig. 1).

All the conclusions made so far do not depend on the concrete type of interaction V, and to some extent simplify forthcoming calculations. As it was said initial and final quarks are in the ground state, therefore gluons in diagrams a-f (Fig. 1) should have $J^P = 1^+$ and any radial quantum number. It is easy to see that quarks in intermediate states in diagrams a-f (Fig. 1) can occupy $ns_{1/2}$ (excluding $n = 1$) or $nd_{3/2}$ ($n = 1, 2 \ldots$)

states, where n — the radial quantum number. When calculating contribution of negative energy states we have to sum over all possible states allowed by selection rules.

With this observation taken into account the contribution of diagrams describing gluon exchange between different quarks is given by the first term in Eq. (10). Energies of quarks, antiquarks and gluons will be denoted as E, $\tilde{E}$ and ω, respectively. Indices n and m will refer now to the states of separate quarks, index "0" will correspond to the ground state. Thus, for example, v^a_{nm} means matrix element of gluon emission by quark "a" with simultaneous transition of the quark from state with the energy E_n to the state with the energy E_m. Matrix elements of electromagnetic interaction will be denoted as u^a_{nm}. We will assume also the summation over radial excitations of gluons as well as summation over color and spin variables of gluons.

Below we write down contributions from diagrams of Fig. 1.

a) $$\sum_{a\neq b}\sum_m{}' \frac{v^a_{0m}v^b_{00}u^a_{m0}}{(\omega+E_m-E_0)(E_m-E_0)}$$

d) $$\sum_{a\neq b}\sum_m{}' \frac{v^b_{00}v^a_{0m}u^a_{m0}}{\omega(E_m-E_0)}$$

b) $$\sum_{a\neq b}\sum_m{}' \frac{v^a_{0m}u^a_{m0}v^b_{00}}{(\omega+E_m-E_0)\omega}$$

c) $$\sum_{a\neq b}\sum_m{}' \frac{u^a_{0m}v^a_{m0}v^b_{00}}{(E_m-E_0)\omega}$$

f) $$\sum_{a\neq b}\sum_m{}' \frac{u^a_{0m}v^b_{00}v^a_{m0}}{(E_m-E_0)(\omega+E_m-E_0)}$$

e) $$\sum_{a\neq b}\sum_m{}' \frac{v^b_{00}u^a_{0m}v^a_{m0}}{\omega(\omega+E_m-E_0)}\,.$$

Here $v^b_{00}=v_b$ — matrix element of gluon emission without change of quantum numbers of quark state. Formulae defining contribution of diagrams with antiquarks can be obtained by substitution $E_m\to-\tilde{E}_m$, $v_{mn}\to\tilde{v}_{mn}$, $u_{mn}\to\tilde{u}_{mn}$. Matrix elements $\tilde{u}$ and $\tilde{v}$ are defined by integrals of the product of wave functions which are solutions of Dirac equation with negative energy, corresponding to antiquarks.

Total contribution of all diagrams considered is equal to

$$\Delta E=\sum_{a\neq b}\sum_m{}' 2(v^a_{0m}u^a_{m0}+u^a_{0m}v^a_{m0})\,\frac{1}{E_m-E_0}\cdot\frac{v_b}{\omega}$$

$$+\sum_{a\neq b}\sum_{m} 2(\tilde{v}^{a}_{0m}\tilde{u}^{a}_{m0}+\tilde{u}^{a}_{0m}\tilde{v}^{a}_{m0})\,\frac{1}{-\tilde{E}_m-E_0}\,\frac{v_b}{\omega}\,. \tag{11}$$

This expression could be rewritten as

$$\Delta E = -2\sum_{a\neq b}\frac{v_b\delta v_a^+}{\omega}\,. \tag{12}$$

δv_a means the matrix element

$$\delta v_a = \int \delta\bar{\psi}_a\hat{v}\psi_a dV+\int \bar{\psi}_a\hat{v}\delta\psi_a dV \tag{13}$$

in which

$$-\delta\psi_a = \sum_m{}' \frac{u^a_{0m}}{E_m-E_0}\,\psi^{(+)}_m+\sum_m \frac{\tilde{u}^a_{0m}}{-\tilde{E}_m-E_0}\,\psi^{(-)}_m. \tag{14}$$

$\delta\psi_a$ defines first order correction to the wave function. $\psi^{(+)}$ and $\psi^{(-)}$ are solutions of Dirac equation with positive and negative energies, respectively. It is easy to see that $\delta\psi$ coincides with the linear in $\vec{H}$ correction to the wave function found in the previous section.

Let us show now that correction for the energy (11) is equal to the correction which has been calculated classically. It is sufficient to notice that gluon correction to the hadron mass (including terms which do not depend on $\vec{H}$) can be calculated in the following way

$$\Delta E_g = -\sum_{a\neq b}\frac{v_a v_b^+}{\omega}\,. \tag{15}$$

It is known [2] that the correction found from Eq. (15) coincides with the correction defined as the energy of the gluon field created by quarks (see Eq. (3)). Substituting v_a, calculated with the wave function $\psi = \psi_0+\delta\psi$, into Eq. (15), we obtain in the approximation linear in $\delta\psi$

$$\Delta E_g = -\sum_{a\neq b}\frac{v_a^{(0)}v_b^{(0)+}}{\omega}-2\sum_{a\neq b}\frac{v_a^{(0)}\delta v_b^+}{\omega}\,. \tag{16}$$

The second term in this formula gives the shift of gluon correction to the mass caused by presence of magnetic field; it coincides with the correction ΔE given by Eq. (12). Note that in these formulae we took into account only the terms nondiagonal in quarks. In the next section we will return to this formula adding terms diagonal in quarks.

The correction to the energy calculated according to Eqs. (11)–(14) can be represented in terms of diagrams of the type shown in Fig. 3. Thick line in this diagram corresponds to solution of Dirac equation $\psi_0+\delta\psi$ with account of terms linear in $\vec{H}$; the other lines correspond to $\delta\psi = 0$.

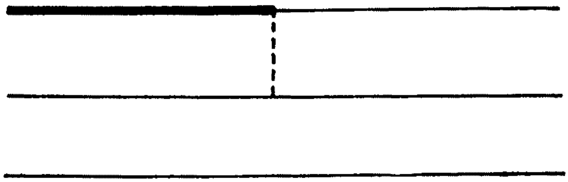

Fig. 3

Our consideration allows us to conclude that classical correction to the magnetic moment corresponds to diagrams of one-gluon exchange, shown in Fig. 1, a-f. Only excited states of quarks have to be taken into account as well as all allowed states of antiquark in the quark propagator.

Up to now we discussed corrections to the magnetic moment connected with one--gluon exchange between different quarks. This correction (see Eq. (4)) depends on quantum numbers, in particular on spin of hadron. Let us consider now contributions of diagrams k-p (Fig. 1). It is obvious from the beginning that these diagrams lead to renormalization of absolute values of magnetic moments of separate quarks

$$\vec{\mu}_q = C\vec{\mu}_0$$

where $\vec{\mu}_0$ — Dirac magnetic moment of a quark. Note that the total contribution of graphs n, o, p (Fig. 1) vanishes. Therefore corrections to magnetic moments of hadrons consisting of identical quarks do not depend neither on their quantum numbers nor on number of quarks, in a sense that taking into account these corrections results in multiplication of $\vec{\mu}_h^{(0)}$ by the same factor C, where $\vec{\mu}_h^{(0)}$ — magnetic moment of the hadron, calculated in the absence of chromodynamical interaction.

Finally the magnetic moment of hadron in α_s-approximation is equal to

$$\vec{\mu}_h = C\vec{\mu}_h^{(0)} - \tfrac{3}{8}\,\alpha_s A \sum_{i\neq k} \lambda_i^a \lambda_k^a Q_i \vec{\sigma}_k, \qquad A = 0.0406, \tag{17}$$

where $C = 1+O(\alpha_s)$; numerical value of A corresponds to massless quarks.

3. *Comparison with other calculations*

Here we will give comparison of the results obtained in previous sections with the analogous results of some other papers. We guess that this will clarify the question of validity of approximations used by different authors.

In Ref. [3] corrections to proton and neutron magnetic moment were calculated using a wave function which includes correction of first order of g-constant of chromodynamical interaction. Let us remind briefly the main points of this calculation. One-gluon exchange is considered as a two-step process: transition from the state 3q into the state 3qG, and backwards. It means that the wave function including terms proportional to g represents superposition of states $|3q\rangle$ and $|3qG\rangle$. The wave function of the state $|3qG\rangle$ antisymmetrical over quark permutations could be obtained by action of operator $\bar{\psi}\gamma_\mu \lambda^a \psi G_\mu^a$

on the state $|3q\rangle$ [3]. Taking into account only ground states of quarks and gluons it was obtained [3] that

$$|h\rangle = \left(1 - \frac{V^2}{\omega^2}\right)^{1/2} |3q\rangle - \frac{V}{\omega} |3qG\rangle, \tag{18}$$

where $V = \sum_{a=1}^{3} v_a$ is the matrix element of the transition $3q \to 3qG$. The normalization is $\langle h|h\rangle = 1$. The magnetic moment defined as matrix element $\langle h|\hat{\vec{\mu}}|h\rangle$ of the operator

$$\hat{\vec{\mu}} = \frac{e}{2} \int \vec{r} \times (\bar{\psi}\vec{\gamma}\psi) dV$$

is equal to

$$\vec{\mu}_h = \left(1 - \frac{V^2}{\omega^2}\right)\vec{\mu} + \frac{V^2}{\omega^2}\vec{\mu}'. \tag{19}$$

Here $\vec{\mu}' = \langle 3qG|\hat{\vec{\mu}}|3qG\rangle$. As it was shown in Ref. [3] for nucleon $\vec{\mu}' = \frac{2}{3}\vec{\mu}$, therefore

$$\vec{\mu}_h = \vec{\mu} - \frac{1}{3}\frac{V^2}{\omega^2}\vec{\mu}. \tag{20}$$

The last formula can be rewritten as

$$\vec{\mu}_h = \vec{\mu} - \frac{4}{3}\frac{v^2}{\omega^2}\vec{\mu}, \tag{21}$$

where v is the matrix element of emission of gluon by a single quark. It means that taking into account the correction due to transitions $3q \to 3qG$ results in multiplication of $\vec{\mu}$ by the factor $1 - \frac{4}{3}\frac{v^2}{\omega^2}$, which does not depend on quantum numbers of hadron. This result could be foreseen. According to previous sections, taking into account only ground states of quarks and gluons leads to cancellation of their contributions. Nonzero contributions arise only from graphs k-m (Fig. 1), giving rise to corrections to the magnetic moment of separate quarks. It is obvious that this correction is proportional to the magnetic moment of hadron calculated in the zero approximation.

Note, that renormalization of quark magnetic moments is not determined completely by contributions of ground states of quarks and gluons only. However, this is not essential for us now. The correction due to the difference of $\vec{\mu}'$ and $\vec{\mu}$ could have been taken into account using Eq. (16) from the previous section. Let us rewrite it in the form

$$\Delta E_g = -\frac{V^2}{\Delta\varepsilon} - 2\sum_{a \neq b} \frac{v_a \delta v_b^+}{\Delta\varepsilon}. \tag{22}$$

Here we added terms diagonal in quarks, $\sum \frac{v_a^2}{\Delta\varepsilon}$. $\Delta\varepsilon$ means difference of energies of states 3qG and 3q. In the absence of magnetic field $\Delta\varepsilon = \omega$, where ω — the energy of valence gluon. The presence of magnetic field contributes to $\Delta\varepsilon$, this additional shift being due to the difference of total magnetic moments of quarks in states 3qG (equal to $\frac{2}{3}\vec{\mu}$) and 3q (equal to $\vec{\mu}$). Hence

$$\Delta\varepsilon = \omega - \tfrac{2}{3}\vec{\mu}\vec{H} + \vec{\mu}\vec{H}.$$

Taking into account that δv is also proportional to $\vec{H}$ we obtain in the approximation linear in $\vec{H}$

$$\Delta E_g = -\frac{V^2}{\omega} + \tfrac{1}{3}\frac{V^2}{\omega^2}\vec{\mu}\vec{H} - 2\sum_{a\neq b}\frac{v_a \delta v_b^+}{\omega}. \tag{23}$$

The derivative of the second term in this formula with respect to the field $\vec{H}$ taken with minus sign coincides with the correction to the magnetic moment given by Eq. (20). Let us point out once more that this correction corresponds to the contribution of ground states of quarks and gluons in diagram n (Fig. 1). Contributions of quark excitations (particularly orbital excitations), in general lead to $\vec{\mu}' \neq \frac{2}{3}\vec{\mu}$. Eqs. (22), (23) do not include all terms diagonal in quarks, describing contributions of these excitations.

In Ref. [4] the corrections to magnetic moments of nucleons arising from diagrams b, e (Fig. 1) were calculated. Only ground states of quarks were taken into account. However, as we already know, similar contributions come from diagrams a, c, d, f, g-i (Fig. 1). The last diagrams, as it was shown, cancel contributions of diagrams b, e (Fig. 1). Therefore results of Ref. [4] are erroneous, both as to the magnitude as well as to the sign of the correction. The proper sign of QCD-correction can be established rather easily in the nonrelativistic limit from the Breit potential. It is sufficient to substitute as usually $\vec{p} \to \vec{p} - e\vec{A}$ and to find the average of operator

$$\delta\vec{\mu} = -\frac{d}{d\vec{H}} U_B.$$

The sign of correction as well as its magnitude is unambiguously fixed by the form of U_B (see Ref. [13] for the magnetic moment of hydrogen and Ref. [8] where baryon magnetic moments in the potential model are considered).

In the nonrelativistic limit the QCD-correction is proportional to $(v/c)^2$. Terms of the order of $(v/c)^{2n}$, $n \geqslant 2$ do not change the sign of the correction (see Eq. (4)), so we may hope that the main effects are taken into account in the $(v/c)^2$-approximation.

Corrections to magnetic moments due to gluon exchange between different quarks were calculated also in Ref. [5]; propagators were assumed to be equal to propagators of free particles. This method of calculation assumes that the standard covariant perturbation theory is applicable. This is not obvious as far as quarks and gluons are not free. The introduction of additional parameters (effective masses of quarks and gluons) cutting off contributions of large distances does not solve all the problems connected with the formulation of Feynman rules for interactions of quarks and gluons inside hadrons. We do not see the possibility of straightforward comparison of results of Ref. [5] with ours.

3. Conclusions

The main result of our paper is the equivalence of two approaches. The first, classical, is based on classical solutions of Dirac equation for quarks, and Maxwell equations for gluons. The second, quantum-mechanical one, is based on equations of noncovariant perturbation theory; the discrete character of quark and gluon spectra is taken into account from the beginning. The quantum approach allows interpretation in terms of diagrams. Equivalence of two approaches is valid of course for the corrections due to one-gluon exchange between different quarks (Fig. 1, a-i). For this type of diagrams classical approach reduces the problem of summation over quantum numbers k, m in Eq. (10) to the solution of certain differential equations. The complete calculation of contributions of diagrams a-i (Fig. 1) was fulfilled classically in Ref. [6]. Corrections to magnetic moments of separate quarks (Fig. 1, k-p) can be calculated correctly only within the framework of quantum approach. With all corrections taken into account the magnetic moment has the form

$$\vec{\mu}_{\mathrm{h}} = \sum_i C_i \vec{\mu}_i - \tfrac{3}{8}\alpha_s \sum_{i \neq k} A_{ik} \lambda_i^a \lambda_k^a Q_i \vec{\sigma}_k,$$

where $\vec{\mu}_i$ — Dirac magnetic moment of i-th quark. Q_k and $\vec{\sigma}_k$ should be understood as matrix elements of corresponding operators. This formula is valid for hadrons consisting of quarks in $1s_{1/2}$-state. For quarks with equal masses $A_{ik} = A$, $C_i = C$. In that case taking into account the renormalization of quark magnetic moments does not change ratios of magnetic moments of different hadrons and thus does not change predictions of additive quark model. The second term, corresponding to diagrams a-i (Fig. 1) differs essentially from the "basic" magnetic moment and changes predictions of additive model. For massless quarks the bag model gives $A = 0.0406$.

It is important to emphasize that the correction to the magnetic moment calculated from diagrams of one-gluon exchange is defined by contribution of excited states of intermediate quark as well as by contribution from $q\bar{q}$-pairs. Both contributions are presumably present in calculations based on the Breit potential. Total contributions of diagrams including disconnected graphs vanish if only $1s_{1/2}$-states of quarks are taken into account. When considering correspondence between classical and quantum approaches we demonstrated one more approach (equivalent to two others) which could be called intermediate. In this approach classical solution of Dirac equation in magnetic field $\psi + \delta\psi$ was used while chromodynamical interaction was treated by considering transitions $3q \to 3qG$, where G — quantum of gluon field.

We do not give here quantitative comparison with experimental data because quantum corrections calculated in α_s-approximation coincide with classical corrections, and the complete calculation of magnetic moments requires taking into account the π-meson contributions; this was done in Ref. [14].

Let us note, that our main results are rather general and do not depend on the explicit form of interactions u and v. In particular, these results can be used for calculation of quadrupole moments of particles from decouplet [15], for calculation of corrections to mean square radius of charge distribution [16], corrections to axial constants of β-decay [3, 17] and so on. In each case one of the above mentioned approaches could be adequate. There are examples when only one of the approaches can be applied. These are corrections to amplitudes of decays $\varrho \to \pi\gamma$, $\Delta \to p\gamma$, etc. Classical approach is not valid in these cases, because the spin of the hadron does change, and multipole expansion is rather bad.

REFERENCES

[1] N. Isgur, G. Karl, *Phys. Rev.* **D20**, 1191 (1979); **D21**, 3175 (1980); C. Haune, N. Isgur, *Phys. Rev.* **D25**, 1944 (1982); D. A. Geffen, W. Wilson, *Phys. Rev. Lett.* **44**, 370 (1980); H. J. Lipkin, *Nucl. Phys.* **B214**, 136 (1983).

[2] T. De Grand, R. L. Jaffe, K. Johnson, J. Kiskis, *Phys. Rev.* **D12**, 2060 (1975); I. Yu. Kobzarev, B. V. Martemyanov, M. G. Schepkin, *Yad. Fiz.* **29**, 1620 (1979).

[3] I. Yu. Kobzarev, B. V. Martemyanov, M. G. Schepkin, *Yad. Fiz.* **30**, 504 (1979).

[4] A. Halprin, P. Sorba, *Phys. Lett.* **66B**, 177 (1977).

[5] W. Y. P. Hwang, *Z. Phys.* **C16**, 331 (1983).

[6] M. I. Krivoruchenko, *Yad. Fiz.* **40**, 801 (1984).

[7] G. E. Brown, M. Rho, V. Vento, *Phys. Lett.* **97B**, 423 (1980); C. E. De Tar, *Phys. Rev.* **D24**, 752, 762 (1981); P. Gonzalez, V. Vento, M. Rho, *Nucl. Phys.* A 395 (1983); S. Theberge, A. W. Thomas, CERN preprint TH. 3290 (1982); O. V. Maxwell, V. Vento, Saclay preprint DPh-T/81-76 (1981); M. M. Musakhanov, *Yad. Fiz.* **34**, 1123 (1981).

[8] M. I. Krivoruchenko, *Pisma Zh. Eksp. Teor. Fiz.* **38**, 146 (1983); R. Mignani, D. Prosperi, *Nuovo Cimento* **A75**, 221 (1983).

[9] B. L. Joffe, A. V. Smilga, *Pisma Zh. Eksp. Teor. Fiz.* **37**, 250 (1983); B. L. Joffe, A. V. Smilga, Preprint ITEP-97 (1983).

[10] H. J. Lipkin, *Phys. Lett.* **B89**, 358 (1980); H. J. Lipkin, *Phys. Rev.* **D24**, 1437 (1981); I. G. Aznauryan, A. S. Bagdasaryan, N. L. Ter-Isaakyan, *Yad. Fiz.* **39**, 108 (1984).

[11] V. N. Logunov, B. V. Martemyanov, *Yad. Fiz.* **29**, 815 (1979).

[12] L. D. Landau, E. M. Lifshitz, *Quantum Mechanics*, Nauka, Moscow 1974.

[13] H. Grotch, *Phys. Rev. Lett.* **24**, 39 (1970); R. Faustov, *Phys. Lett.* **B33**, 422 (1970).

[14] M. I. Krivoruchenko, Preprint ITEP-169, 1984.

[15] S. S. Gershtein, Yu. M. Zinoviev, *Yad. Fiz.* **33**, 1442 (1981); S. S. Gershtein, G. V. Jikia, *Yad. Fiz.* **34**, 1566 (1981).

[16] R. D. Carlitz, S. D. Ellis, R. Savit, *Phys. Lett.* **B68**, 443 (1977); B. Isgur, G. Karl, R. Koniuk, *Phys. Rev. Lett.* **41**, 1269 (1978); M. I. Krivoruchenko, *Yad. Fiz.* **41**, 777 (1985).

[17] M. G. Schepkin, in Proc. of Vth School of Physics, ITEP, Atomizdat, Moscow 1978, No 3, p. 34; M. M. Musakhanov, *Yad. Fiz.* **33**, 810 (1981).