QCD SUM RULES AND WEAK TRANSITIONS IN KAON PHYSICS*

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We analyze and discuss the applications of QCD sum rules to the derivation of bounds on the matrix elements of four-quark operators appearing in the $\Delta S = 1$ sector of the weak Hamiltonian. We present the application to the calculation of ε'/ε and $K \to 2\pi$ amplitudes. We also analyze the dependence on the renormalization point μ and the momentum continuation in amplitudes.

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1. Introduction

The main subject of these lectures is the application of QCD sum rule techniques to the derivation of bounds on the matrix elements of different weak transition operators. Although the method used is rather general, we shall restrict our discussion mostly to kaon physics; the applications to other fields will be briefly discussed.

The idea of using sum rules to obtain bounds on matrix elements stems from the fact that the present techniques used in calculations, such as quark models, saturation by single vacuum state, etc., suffer definitively from uncertainties. Therefore, although we believe to have rather sufficient knowledge of the operators, our lack of knowledge of true hadronic wave functions (confinement problem) limits our ability to give exact answers. On the other hand, sum rule techniques enable one to *relate* the unknown matrix element $\mathcal{M}$ to another quantity ψ, usually a two-point function, which might be calculated more reliably than $\mathcal{M}$ itself.

In weak transitions, we are mostly concerned with operators which are either bilinear in quark fields (currents) or are a product of currents (four-quark operators). The former

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appear, for example, in semileptonic decays of mesons, such as $K \to \pi e\nu$, $D \to K e\nu$, $B \to D e\nu$, and the latter are responsible for $K \to 2\pi$ decays, $K^0 - \overline{K^0}$ mixing, etc.

Besides deriving the bounds on the matrix elements relevant to ε'/ε, $K \to 2\pi$, $K^0 - \overline{K^0}$ mixing, we analyze and discuss the dependence of the matrix elements on the renormalization point μ and the ambiguity in momentum continuation arising from the use of PCAC and the soft-pion limit.

The plan of the paper is as follows. In Sec. 2 we confine ourselves to a brief review of the QCD-corrected electroweak Hamiltonian for the $\Delta S = 1$ sector, but present a detailed analysis of the problem of μ dependence. In Sec. 3 we analyze the problem of continuation of the PCAC amplitudes to the physical ones in the framework of chiral perturbation theory. We also present the amplitudes for $K \to 2\pi$ decays in terms of electroweak theory. The QCD sum rule approach is developed in Sec. 4, where the bounds are obtained in terms of the two-point function $\psi(q^2)$. Section 5 contains the calculation of $\psi(q^2)$ and the results obtained. In Sec. 6 we discuss the reliability of the whole approach and give comments on the prospects of future improvements.

2. *Effective weak Hamiltonian and QCD corrections*

The theory [1, 2] underlying our discussion is the effective weak Hamiltonian obtained in the standard $SU(2) \times U(1)$ model using the short-distance expansion for the product of weak currents. The effective Hamiltonian is of the form [3–6]

$$H_{\mathrm{w}}^{\Delta S=1} = \sqrt{2}\, G_{\mathrm{F}} \sin\theta_1 \cos\theta_1 \cos\theta_3 \sum_{n=1}^{6} c_n O_n, \tag{2.1}$$

where O_n are local four-quark operators and c_n are Wilson coefficients "renormalized" by QCD. The angles θ_i are the generalized Cabibbo angles in the six-flavor model [1].

The local operators O_n have the form

$$\begin{aligned}
O_1 &= (\bar{s}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}) - (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu u_{\mathrm{L}}),\\
O_2 &= (\bar{s}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}) + (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu u_{\mathrm{L}}) + 2(\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{d}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}) + 2(\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{s}_{\mathrm{L}}\gamma^\mu s_{\mathrm{L}}),\\
O_3 &= (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu u_{\mathrm{L}}) + (\bar{s}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}) + 2(\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{d}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}) - 3(\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{s}_{\mathrm{L}}\gamma^\mu s_{\mathrm{L}}),\\
O_4 &= (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu u_{\mathrm{L}}) - (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{d}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}) + (\bar{s}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}),\\
O_5 &= (\bar{s}_{\mathrm{L}}\gamma_\mu \lambda^a d_{\mathrm{L}})[\bar{u}_{\mathrm{R}}\gamma^\mu \lambda^a u_{\mathrm{R}} + \bar{d}_{\mathrm{R}}\gamma^\mu \lambda^a d_{\mathrm{R}} + \bar{s}_{\mathrm{R}}\gamma^\mu \lambda^a s_{\mathrm{R}}],\\
O_6 &= (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})[\bar{u}_{\mathrm{R}}\gamma^\mu u_{\mathrm{R}} + \bar{d}_{\mathrm{R}}\gamma^\mu d_{\mathrm{R}} + \bar{s}_{\mathrm{R}}\gamma^\mu s_{\mathrm{R}}].
\end{aligned} \tag{2.2}$$

The set of operators (2.2) has definite isospin properties. The operator O_4 is a pure $\Delta I = 3/2$ operator, whereas the other operators in (2.2) are $\Delta I = 1/2$. The first four operators are present in the effective Hamiltonian before inclusion of QCD corrections; the operators O_5 and O_6 (penguins) appear after inclusion of QCD corrections when flavor symmetry is broken [4].

The asymptotically free values of the coefficients c_n are given by

$$c_1^{\mathrm{F}} = 1, \quad c_2^{\mathrm{F}} = \tfrac{1}{5}, \quad c_3^{\mathrm{F}} = \tfrac{2}{15}, \quad c_4^{\mathrm{F}} = \tfrac{2}{3}, \quad c_5 = c_6 = 0. \tag{2.3}$$

In the flavor-symmetry limit, the Hamiltonian may be written in a simple way as

$$H_{\mathrm{w}}^{\Delta S=1} = \sqrt{2}\, G_{\mathrm{F}} c_1 c_3 s_1 [c_- O_- + c_+ O_+], \tag{2.4}$$

where

$$O_\pm = (\bar{s}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu d_{\mathrm{L}}) \pm (\bar{s}_{\mathrm{L}}\gamma_\mu d_{\mathrm{L}})(\bar{u}_{\mathrm{L}}\gamma^\mu u_{\mathrm{L}}), \tag{2.5}$$

with

$$c_-^{\mathrm{F}} = c_1^{\mathrm{F}} = 1,$$

$$c_+^{\mathrm{F}} = 5c_2^{\mathrm{F}} = \tfrac{15}{2}\, c_3^{\mathrm{F}} = \tfrac{3}{2}\, c_4^{\mathrm{F}} = 1. \tag{2.6}$$

The relation (2.6) is also valid for the QCD-corrected Hamiltonian, provided flavor symmetry is not broken. Flavour-symmetry breaking slightly changes c_1 and c_2 because of the mixing of the $\Delta I = 1/2$ operators.

μ dependence

The coefficients c_i in (2.1) are calculated in perturbative QCD and summed up in the leading log approximation using renormalization-group techniques. Generally, these coefficients are functions which depend on m_{W}, the masses of *heavy quarks* m_{t}, m_{b}, m_{c}, the QCD parameter Λ_{QCD}, and the *renormalization point* μ. This μ dependence of the Wilson coefficients has to be cancelled by the μ dependence of the operators O_i when the latter are taken between hadronic states. The *explicit* μ dependence of the matrix elements of the operators O_i might be calculated only in QCD. However, the calculation of matrix elements in QCD is typically nonperturbative and at present one does not know how to solve this problem. Therefore, one relies on some phenomenological QCD-like models, such as the bag model, but the explicit μ dependence is lost.

To be more explicit, let us assume that some operator O_i is multiplicatively renormalizable. The μ dependence of its Wilson coefficients $c_i(\mu)$ could be easily factorized as

$$c_i(\mu) = \hat{c}_i[\alpha_s(\mu^2)]^{d_i}, \tag{2.7}$$

where $\hat{c}_i$ does not depend on μ, α_s is given by

$$\alpha_s(\mu^2) = \frac{4\pi}{9\ln\left(\dfrac{\mu^2}{\Lambda^2}\right)}, \tag{2.8}$$

and d_i is proportional to the anomalous dimension of the operator O_i. Now it is clear that the product

$$c_i(\mu)\,\langle \mathrm{f}|O_i|\mathrm{i}\rangle\,(\mu) \tag{2.9}$$

is μ independent since the operator O_i does not mix with other operators. It follows that the μ dependence of $\langle f|O_i|i\rangle$ could also be factorized as

$$\langle f|O_i|i\rangle = [\alpha_s(\mu^2)]^{-d_i}\langle f||O_i||i\rangle, \tag{2.10}$$

where $\langle f||O_i||i\rangle$ is a μ-independent quantity.

Now, the problem is as follows. In all phenomenological QCD-like models one calculates the *left-hand side* of (2.10), i.e., $\langle f|O_i|i\rangle$, and *not* $\langle f||O_i||i\rangle$, and it is not clear *at which* μ the calculation is performed. This is known as the μ-dependence problem. Our discussion is quite general since the weak Hamiltonian could always be written as the sum of multiplicatively renormalizable operators [7]:

$$H_{\mathrm{w}}^{\Delta S=1} = \sqrt{2}\, G_{\mathrm{F}} s_1 c_1 c_3 \sum_{i=1}^{5} \tilde{c}_i \tilde{Q}_i, \tag{2.11}$$

where $\tilde{c}_i$ and $\tilde{Q}_i$ could be read off in Ref. [7].

The Buras-Słominski diagonal operator basis in (2.11) is particularly useful if one wants to calculate the μ dependence in QCD explicitly, because it simply factorizes out as in (2.10). On the other hand, Buras and Słominski have shown [7] that the relations between the operators obtained in the valence-quark approximation are at variance with the μ dependence predicted by QCD. These relations could be in agreement for at most one value of μ.

3. *Effective chiral Lagrangian, current algebra and quark models*

A simple method used to relate different matrix elements of a given operator is chiral perturbation theory [8]. This method involves the use of the pseudo-Goldstone boson sector of the theory in the form of an effective Lagrangian.

The unitary chiral matrix field U is defined by

$$U = \exp\left(\frac{2i}{f}\Phi\right), \tag{3.1}$$

where Φ is the octet matrix of pseudo-Goldstone fields:

$$\Phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{bmatrix} \tag{3.2}$$

and $f = f_\pi = 132$ MeV. Then strong interactions are described by the effective chiral Lagrangian density, correct to quadratic order in meson masses and momenta:

$$\mathscr{L} = \frac{f^2}{8}\,\mathrm{tr}\,(\partial_\mu U \partial^\mu U^\dagger) + v\,\mathrm{tr}\,(MU + (MU)^\dagger), \tag{3.3}$$

where M is the quark matrix, $M_{ii} = (m_u, m_d, m_s)$, $M_{i\neq j} = 0$, and

$$v = \frac{f^2 m_{\pi^+}^2}{4(m_u+m_d)} = \frac{f^2 m_{K^+}^2}{4(m_s+m_u)} = \frac{f^2 m_{K^0}^2}{4(m_s+m_d)}. \tag{3.4}$$

The Lagrangian (3.3) is consistent with $SU(3)_L \times SU(3)_R$, parity, charge conjugation and chiral symmetry breaking by the bare quark masses in strong interactions.

In the framework of the effective chiral Lagrangian field theory, the operators O_i, which have definite transformation properties under chiral rotations in respect to $SU(3)_L \times SU(3)_R$, have the following realization:

$$O_i = a_i(\partial_\mu U \partial^\mu U^\dagger)^2_3, \quad i = 1, 2, 5, 6 \tag{3.5}$$

and

$$O_n = a_n T^{ij}_{kl}(U\partial_\mu U^\dagger)^k_i (U\partial^\mu U^\dagger)^l_j, \quad n = 3, 4$$

$$T^{13}_{12} = T^{31}_{12} = T^{13}_{21} = T^{31}_{21} = \tfrac{1}{2}, \quad T^{23}_{22} = T^{32}_{22} = 1, \quad T^{33}_{32} = T^{33}_{23} = -\tfrac{3}{2} \quad (O_3),$$

$$T^{13}_{12} = T^{31}_{12} = T^{13}_{21} = T^{31}_{21} = \tfrac{1}{2}, \quad T^{23}_{22} = T^{32}_{22} = -\tfrac{1}{2} \quad (O_4),$$

with the coefficients a_i *not fixed* by the chiral symmetry requirements alone.

The corresponding amplitudes for $K \to 2\pi$ decays are given by[1]

$$\langle \pi^+\pi^- | O_i^{1/2} | K_s^0 \rangle = i a_i \frac{2\sqrt{2}}{f^3}[k_- \cdot p + k_+ \cdot p + 2(k_+ \cdot k_-)],$$

$$\langle \pi^+\pi^0 | O_4^{3/2} | K^+ \rangle = i a_4 \frac{2\sqrt{2}}{f^3}[4(k_0 \cdot p) + 3(k_0 \cdot k_+) - (k_+ \cdot p)], \tag{3.6}$$

where p is the kaon momentum and $k_\pm$ and k_0 are momenta of charged and neutral pions, respectively. One notices that the amplitudes are *quadratic* in *momenta*. At physical values, $p^2 = m_K^2$, $k^2 = m_\pi^2$, the amplitudes are given by

$$\langle \pi^+\pi^- | O_i^{1/2} | K_s^0 \rangle = i a_i \frac{4\sqrt{2}}{f^3}(m_K^2 - m_\pi^2),$$

$$\langle \pi^+\pi^0 | O_4^{3/2} | K^+ \rangle = i a_4 \frac{12}{\sqrt{2} f^3}(m_K^2 - m_\pi^2). \tag{3.7}$$

The coefficients a_i could be obtained by calculating the matrix elements in quark models. However, thus far, in these models, it has been possible to calculate the matrix

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[1] See Ref. [9].

elements between single states. One is therefore forced to use PCAC and the soft-pion limit (SPL) in order to reduce the K → 2π amplitude to K → π transitions. The problem arises when one tries to recover the physical limit again, $k^2 = m_\pi^2$.

Continuation problem

Using the SPL one easily obtains, for example,

$$\langle \pi^+\pi^0 | O_4 | \mathrm{K}^+ \rangle = \kappa \frac{1}{f} \frac{3}{\sqrt{2}} \mathscr{M}^{3/2}_{\mathrm{K}^+\pi^+}, \tag{3.8}$$

where

$$\mathscr{M}^{3/2}_{\mathrm{K}^+\pi^+} \equiv \langle \pi^+ | O_4 | \mathrm{K}^+ \rangle = a_{3/2} \frac{4}{f^2} (k \cdot p) \tag{3.9}$$

and κ is the continuation factor

$$\kappa = \frac{m_\mathrm{K}^2 - m_\pi^2}{k \cdot p}. \tag{3.10}$$

Since, in the SPL, $k = p$, it is not clear what the value of $k \cdot p = p^2$ is ! Of course, the $(k \cdot p)$ dependence of the continuation factor κ cancels the $(k \cdot p)$ dependence of the matrix element $\mathscr{M}^{3/2}_{\mathrm{K}^+\pi^+}$. However, the problem (similar to the μ-dependence problem) is that, in quark models, one calculates $\langle \pi^+ | O_4 | \mathrm{K}^+ \rangle$ which implicitly contains the unknown $(k \cdot p)$ dependence, i.e., one calculates

$$a_{3/2} \frac{4}{f^2} (k \cdot p),$$

and not

$$a_{3/2} \frac{4}{f^2}.$$

This uncertainty, which is due to the $(k \cdot p)$ dependence, amounts roughly to a factor of 2. In the following we consistently use $(k \cdot p) = m_\mathrm{K}^2$, giving

$$\kappa = 0.923. \tag{3.11}$$

If there were no continuation, κ would be equal to 1/2.

Translated into the language of quark operators, the amplitudes (3.7) would read, using PCAC and the SPL,

$$a^{3/2}(\mathrm{K}^+ \to \pi^+\pi^0) = \kappa \frac{3}{\sqrt{2} f_\pi} (\sqrt{2} G_\mathrm{F} s_1 c_1 c_3) c_4 \langle \pi^+ | O_4 | \mathrm{K}^+ \rangle,$$

$$a^{1/2}(\mathrm{K}^0 \to \pi^+\pi^-) = \kappa \frac{1}{f_\pi} (\sqrt{2}\, G_\mathrm{F} s_1 c_1 c_3) \{ c_- \langle \pi^+ | O_- | \mathrm{K}^+ \rangle$$

$$+ c_+ \langle \pi^+ | O_+^{1/2} | \mathrm{K}^+ \rangle \} + a^{\mathrm{peng}}(\mathrm{K}^0 \to \pi^+\pi^-), \tag{3.12}$$

where $O_+^{1/2} = O_+ - \frac{2}{3} O_4$.

The amplitude a^{peng} comes from the penguin operator O_5 and is more complicated because of the presence of the anomalous commutator term [10]:

$$a^{\text{peng}}(K^0 \to \pi^+\pi^-) = \kappa \frac{1}{f_\pi}(\sqrt{2}\, G_F s_1 c_1 c_3)\, \text{Re}\, c_5$$

$$\times \{\langle \pi^+ | O_5 | K^+ \rangle + \langle \pi^+ | O_5^{(c)} | K^+ \rangle\}. \tag{3.13}$$

The second term in (3.13) arises from the commutator of the *normal-ordered* operator O_5 with an axial charge Q^5

$$[Q^5, O_5] = -[Q, O_5] + \tfrac{32}{9} \langle 0 | \bar{d} d | 0 \rangle : \bar{s}(1+\gamma_5) d :, \tag{3.14}$$

the last term being equal to $-2O_5^{(c)}$. Its matrix element is given by

$$\mathcal{M}^{(c)} = \langle \pi^+ | O_5^{(c)} | K^+ \rangle = \left(1 - \frac{1}{N_c^2}\right) \frac{f_\pi m_\pi^2}{m_u + m_d} \frac{f_K m_K^2}{m_s + m_u}. \tag{3.15}$$

The experimental values for (3.12) are given by

$$a^{1/2}(K^0 \to \pi^+\pi^-) = 27.06 \times 10^{-8}\ \text{GeV},$$

$$a^{3/2}(K^+ \to \pi^+\pi^0) = 1.83 \times 10^{-8}\ \text{GeV}, \tag{3.16}$$

which leads to the "$\Delta I = 1/2$ rule"

$$\frac{a(K_S^0 \to \pi^+\pi^-)}{a(K^+ \to \pi^+\pi^0)} = 21.6. \tag{3.17}$$

Calculation of transition amplitudes

Apart from the question of the ambiguity in the continuation factor κ, the amplitudes in (3.12) are reduced to the calculation of the matrix elements of the four-quark operators. This can, in principle, be calculated in quark models [11, 12]. However, the quark model calculations of the transition amplitudes are expected to work within a factor of 2. The same is true for the vacuum-saturation method [4].

The hope arises that lattice calculations could provide us with a more reliable estimate. Recent estimates based on lattice QCD [13] seem to support the vacuum-saturation estimate. However, the reported results are only tentative and still subject to large systematic errors. A similar approach, still in progress, uses chiral perturbation theory and lattice Monte Carlo techniques [14].

As explained in the Introduction, the main problem is our lack of knowledge of the exact hadronic wave functions. Therefore, the idea of setting a rigorous bound on the matrix elements instead of evaluating them seems to be very attractive. This idea was elaborated in detail in Ref. [15] and applied to the study of the matrix elements of two-quark operators appearing in the processes $\pi^+ \to \pi^0 e^+ \nu_e$, $K_L \to \pi e \nu$, $D^+ \to \overline{K^0} e^+ \nu$ (Ref. [15]) and $B^+ \to \overline{D^0} e^+ \nu$ (Ref. [16]). The QCD sum rule techniques for the matrix elements of the four-quark operators were developed and applied to $K^0 - \overline{K^0}$ mixing in Ref. [17]. The application to the $\Delta S = 1$ sector of the weak Hamiltonian [18] consists in a straightforward generalization of the method of Ref. [17]. The method of QCD sum rules is based on analyticity, unitarity and general features of QCD [1].

4. QCD sum rules

In the QCD sum rule approach, the bound is set on the matrix element by relating it to the two-point function

$$\psi(q^2) = i \int d^4x e^{iq\cdot x} \langle 0| T(O(x)O^{\dagger}(0)) |0\rangle, \tag{4.1}$$

which gives the bound of the form

$$|F(0)| \leqslant \mathscr{A}(Q^2), \tag{4.2}$$

where $|F(0)|_{MM'} \equiv \langle M'|O|M\rangle$ can be viewed as the value of a scalar form factor $F(t)$ at $t = 0$ which is a real analytic function in the complex t plane with a cut $(m_K+m_\pi)^2 \leqslant t < \infty$.

The absorptive part of $\psi(q^2)$ is given by

$$\frac{1}{\pi} \operatorname{Im} \psi(t) = \frac{1}{2\pi} \sum_{\Gamma} \langle 0|O_i|\Gamma\rangle \langle \Gamma|O_i^{\dagger}|0\rangle (2\pi)^4 \delta(p-\textstyle\sum p_\Gamma). \tag{4.3}$$

The sum in (4.3) extends over all physical intermediate states Γ that match the quantum numbers of the operator O_i. Each state gives a positive contribution and the particular one, $|MM'\rangle$, sets the lower bound.

The two-point function $\psi(q^2)$ obeys a dispersion relation defined up to an arbitrary polynomial in q^2 of fourth order at most (this is fixed by QCD). To get rid of this arbitrariness, one usually takes the derivatives of $\psi(q^2)$; in our case, it amounts to taking the fifth derivative of the function $\psi(q^2)$

$$\mathscr{F}(Q^2) \equiv -\frac{\partial^5 \psi(q^2)}{(\partial Q^2)^5} = 5! \int_{t_0}^{\infty} dt \frac{1}{(t+Q^2)^6} \frac{1}{\pi} \operatorname{Im} \psi(t)$$

$$\geqslant \frac{15}{2\pi^2} \int_{t_0}^{\infty} dt \frac{1}{(t+Q^2)^6} \left(1-\frac{t_0}{t}\right)^{1/2} \left(1-\frac{t_1}{t}\right)^{1/2} |F(t)|^2, \tag{4.4}$$

where $Q^2 = -q^2$, $t_0 = (m+m')^2$ and $t_1 = (m-m')^2$.

Let us assume that the function $\mathscr{F}(Q^2)$ could be calculated in a reliable manner. If that is true, then one is able to set an upper bound on $F(0)$. This we prove by using the Peierls inequality in (4.4):

$$\mathscr{F}(Q^2) \geqslant (\exp \int_{t_0}^{\infty} d\mu \ln \varrho)(\exp \int_{t_0}^{\infty} d\mu \ln |F(t)|^2), \tag{4.5}$$

where $d\mu(t)$ is a normalized measure

$$d\mu(t) = \frac{1}{\pi}\left(\frac{t_0}{t-t_0}\right)^{1/2}\frac{dt}{t}$$

such that $\int_{t_0}^{\infty} d\mu(t) = 1$. The quantity

$$\varrho(t, Q^2) \equiv \frac{15}{2\pi^2}\pi t\left(\frac{t-t_0}{t_0}\right)^{1/2}\left(1-\frac{t_0}{t}\right)^{1/2}\left(1-\frac{t_1}{t}\right)^{1/2}\frac{1}{(t+Q^2)^6} \tag{4.6}$$

is positive definite for $t \geqslant t_0$.

Applying the Jensen inequality for analytic functions to (4.5) gives

$$\mathscr{F}(Q^2) \geqslant |F(0)|^2 \exp\int_{t_0}^{\infty} d\mu(t)\ln\varrho(t, Q^2), \tag{4.7}$$

and from (4.7) one immediately obtains an upper bound [18]

$$|F(0)| \leqslant t_0^2(t_0\mathscr{F}(Q^2))^{1/2}\left(\frac{2\pi}{15}\right)^{1/2}[1+(1+Q^2/t_0)^{1/2}]^6$$

$$\times[1+(1-t_1/t_0)^{1/2}]^{-1/2}. \tag{4.8}$$

Obviously, the function $\mathscr{A}(Q^2)$ in (4.2) is now given explicitly by the right-hand side of Eq. (4.8).

The result (4.8) can be easily generalized to the case when a larger number of two--particle channels $|M_i\overline{M_i'}\rangle$ are included. In that case, Eq. (4.8) becomes

$$\sum_i |F_{M_iM_i'}(0)|^2\frac{15}{2\pi t_0^{i5}}\frac{1+\left(\frac{t_0^i-t_1^i}{t_0^i}\right)^{1/2}}{\left[1+\left(\frac{t_0^i+Q^2}{t_0^i}\right)^{1/2}\right]^{12}} \leqslant \mathscr{F}(Q^2). \tag{4.9}$$

If one can relate the matrix elements $F_{M_iM_i'}(0)$ by SU(2) or SU(3) symmetry, then the desired matrix element in (4.9) simply factorizes out. Since the same channels also appear in the calculation of $\mathscr{F}(Q^2)$, they give the same factor, and the form (3.8) is recovered again.

Improvement of the bound

The bound (3.8) may be substantially improved using the techniques of Refs. [15, 18] and some reasonable phenomenological input. With some information about the slope and the convexity of the normalized form factor $F(t)/F(0)$, one obtains the improved bound

$$|F(0)| \leqslant \mathscr{A}(Q^2)\{1+[\varrho_1+\lambda_1(Q^2)]^2+\tfrac{1}{4}[\varrho_2+2\varrho_1\lambda_1(Q^2)+\lambda_2(Q^2)]^2\}^{-1/2}, \tag{4.10}$$

where ϱ_1 and ϱ_2 are the slope and the convexity of $F(t)$, respectively,

$$\varrho_1 = -4t_0 \frac{d}{dt} \frac{F(t)}{F(0)}\bigg|_{t=0},$$

$$\varrho = 16t_0^2 \frac{d^2}{dt^2} \frac{F(t)}{F(0)}\bigg|_{t=0} + 4\varrho_1, \tag{4.11}$$

and $\lambda_1(Q^2)$, $\lambda_2(Q^2)$ are given by

$$\lambda_1(Q^2) = 2+[1+(1-t_1/t_0)^{1/2}]^{-1} - 12[1+(1+Q^2/t_0)^{1/2}]^{-1},$$

$$\lambda_2(Q^2) = \lambda_1^2(Q^2)+2(1-t_1/t_0)^{1/2}[1+(1-t_1/t_0)^{1/2}]^{-2}$$

$$-24(1+Q^2/t_0)^{1/2}[1+(1+Q^2/t_0)^{1/2}]^{-2}. \tag{4.12}$$

The improvement in (4.10) requires certain assumptions about the analytic structure of $F(t)$. The form factor $F(t)$ should be

(*i*) polynomially bounded — and this really happens in QCD,

(*ii*) it should have no zeros in the complex plane. Unfortunately, we have no control of the second requirement, although it is a usual ansatz in the calculation of form factors.

In the next section we proceed to the calculation of $\psi(q^2)$.

5. *Calculation of the function $\mathscr{F}(Q^2)$*

As we have proved in Sec. 4, we are able to derive the strict bound if we know the function $\mathscr{F}(Q^2)$. The problem would be solved if we knew how to calculate the two-point function $\psi(q^2)$ reliably.

The function $\psi(q^2)$ can be calculated almost exactly in QCD at high Q^2. The corresponding diagrams are shown in Fig. 1. Diagram la gives the asymptotic QCD value

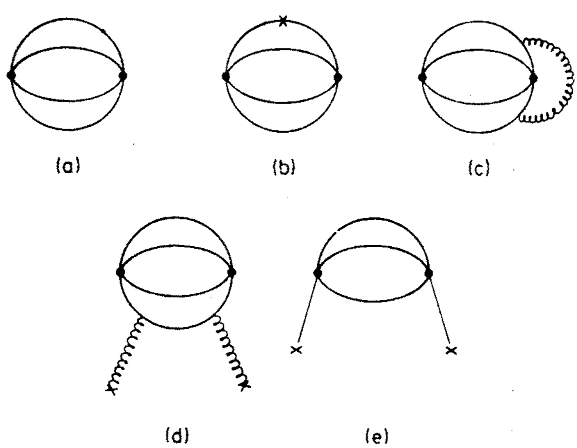

Fig. 1. QCD calculation of the two-point function $\psi(q^2)$: a) asymptotic value with chiral quarks, b) mass corrections, c) perturbative gluon corrections, d) gluon condensate corrections, e) quark condensate corrections

in the chiral limit, diagram 1b is the mass correction to the chiral limit, and diagram 1c represents perturbative QCD corrections. The "monster" diagrams 1d and 1e are non-perturbative contributions coming from gluon and quark condensates, respectively. Generally, the $\frac{1}{\pi}\,\mathrm{Im}\,\psi(Q^2)$ will be of the form

$$\frac{1}{\pi}\,\mathrm{Im}\,\psi(Q^2) = A(Q^2)^4\left(\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)}\right)^{-2d}\left[1+a\,\frac{\alpha_s(Q^2)}{\pi}+b\,\frac{\overline{m}_s^2}{Q^2}\right.$$
$$\left.+c\,\frac{\left\langle 0\left|\frac{\alpha_s}{\pi}F^2\right|0\right\rangle}{(Q^2)^2}+c'\,\frac{m_s\langle 0|\bar{s}s|0\rangle}{(Q^2)^2}+\dots\right], \qquad (5.1)$$

where $\left\langle 0\left|\frac{\alpha_s}{\pi}F^a_{\mu\nu}F^{a\mu\nu}\right|0\right\rangle$ and $m_s\langle 0|\bar{s}s|0\rangle$ are gluon and quark condensates, respectively; A, a, b, c and c' are constants, $\overline{m}_s$ is a running s-quark mass and $\alpha_s(\mu^2)$ is the running coupling constant at the renormalization point μ. The exponent d is proportional to the anomalous dimension of the operator O. As it is transparent from Eq. (5.1), one is able to calculate the μ dependence of the matrix element explicitly and factorize it out. Since the Wilson coefficient in (2.7) has the μ dependence of the form $[\alpha_s(\mu^2)]^d$, one sees that it cancels the μ dependence of Eq. (5.1) (which corresponds to the square of the matrix element), so that the final result would be μ independent.

Obviously, the result (5.1) would be valid for Q^2 large enough, so that the corrections to the asymptotic QCD value stay moderate (say, less than 20–30%). With lower value of Q^2, the corrections grow and the neglected terms (higher-dimension condensates, etc.) become important. The lowest value of Q^2 where the expression (5.1) would still be valid depends crucially on the coefficients, a, b, c and c'. These coefficients are not small at all and the lowest value of Q^2 is of the order of a few GeV^2.[2]

However, since the bound in Eq. (4.8) grows as $(Q^2)^{5/2}$, it is not very restrictive for large Q^2. Thus, in calculating the bound, we make an ansatz for the spectral function $\frac{1}{\pi}\,\mathrm{Im}\,\psi(t)$ (see Fig. 2)

$$\frac{1}{\pi}\,\mathrm{Im}\,\psi(t) = \text{sum of low-energy hadronic contributions} + \text{QCD continuum}. \qquad (5.2)$$

The last term in Eq. (5.2) is given by the asymptotic behavior of $\frac{1}{\pi}\,\mathrm{Im}\,\psi(t)$, i.e., by (5.1). In calculating the first term in Eq. (5.2), we have used the following ansatz:

$$\psi(q^2) \simeq i\int d^4x e^{iq\cdot x}\langle 0|T(J^\pi(x)J^{\pi\dagger}(0))|0\rangle\,\langle 0|T(J^K(x)J^{K\dagger}(0))|0\rangle. \qquad (5.3)$$

[2] For the operator O_4, the coefficients [19, 20] are $A=[5(16\pi^2)^3]^{-1}$, $a=-\frac{28}{15}$, $b=-20$, $c=-5(16\pi^2)/4$, $c'=10(16\pi^2)$, $d=-\frac{2}{9}$.

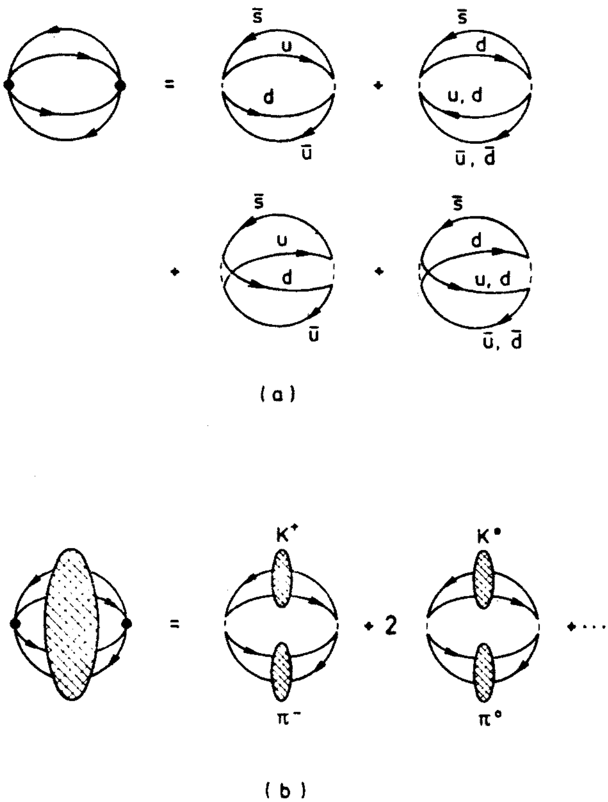

Fig. 2. Calculation of $\psi(q^2)$ for the operator O_4 using the ansatz in (5.2): a) asymptotic QCD contributions and b) leading hadronic contributions

This corresponds to summing a class of hadronic contributions which are leading in the $1/N_c$ expansion plus some nonleading contributions, but not all the subleading ones. The ansatz can be justified in the framework of the effective chiral theory where the nonfactorizable terms, not present in (5.3), are found to be negligible [20]. Unfortunately, the μ dependence can no longer be controlled in (5.3).

The ansatz (5.2) leads to the following form of $\psi(q^2)$:

$$\psi(q^2) = -i\frac{\zeta}{16}\int \frac{d^4k}{(2\pi)^4}\Pi^{\pi^-}_{\mu\nu}(k)\Pi^{\mu\nu,K^+}(k-q), \tag{5.4}$$

where ζ is a numerical factor depending on the coefficients in the sum of diagrams in Fig. 3 (for example, in the case of O_4, $\zeta = 3$).

By making use of an invariant expansion of the two-point function

$$\begin{aligned}\Pi_{\mu\nu}(k) &= i\int d^4x e^{iq\cdot x}\langle 0|T(J(x)J^\dagger(0))|0\rangle \\ &= (-\eta_{\mu\nu}k^2 + k_\mu k_\nu)\Pi_1(k^2) + k_\mu k_\nu \Pi_0(k^2)\end{aligned} \tag{5.5}$$

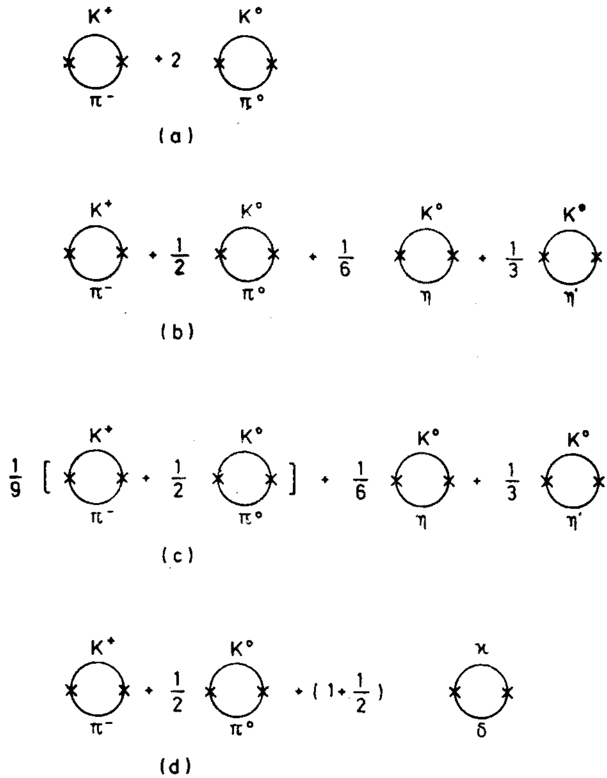

Fig. 3. Hadronic contributions to the function $\psi(q^2)$ for different operators: a) O_4, b) O_-, c) $O_+^{1/2}$, d) O_s

and using dimensional regularization, one finds [18] that

$$\mathscr{F}^{\text{low}}(Q^2) = \frac{\zeta}{256\pi^2}\int_0^\infty dt dt' \int_0^1 dx \sum_{i,j} \frac{1}{\pi}\,\text{Im}\,\Pi_i^{\pi^-}(t)\,\frac{1}{\pi}\,\text{Im}\,\Pi_j^{K^+}(t')$$

$$\times \frac{(1-x)^3x^3}{\Delta^3}\left\{1+\frac{3}{\Delta}\left[tx^2+t'(1-x)^2\right]+\frac{6}{\Delta^2}\left[A_{ij}x^2(1-x)^2\right.\right.$$

$$\left.\left.+(tx+t'(1-x))^2-2(tx+t'(1-x))(t+t')x(1-x)\right]\right\}, \tag{5.6}$$

with

$$\Delta = Q^2x(1-x)+tx+t'(1-x),$$

$$A_{11} = 8tt'+(t+t')^2,$$

$$A_{01} = A_{10} = (t-t')^2,$$

$$A_{00} = (t+t')^2. \tag{5.7}$$

A good estimate of the function $\mathcal{F}(Q^2)$ can be obtained by inserting the spectral functions of the lowest hadronic contributions, i.e., K and π poles for $\operatorname{Im}\Pi_0^{K^+}$ and $\operatorname{Im}\Pi_0^{\pi^-}$, respectively. In addition, there are $(K\pi)_{j=1}$ and $(\pi\pi)_{j=1}$ continuum contributions (dominated by the K^* and the ϱ meson) which can be described by the Breit-Wigner type of resonance.

We find numerically that these contributions as well as the asymptotic QCD contribution are strongly suppressed at low Q^2.

The expression (5.6) differs for the penguin contributions dominated by the operator O_5 [18]. There appear two-point functions $\Pi(t)$ with scalar and pseudoscalar currents. The expression (5.6) can be turned into the divergences of vector and axial vector currents with quark masses in the denominator. For example,

$$\frac{1}{\pi}\operatorname{Im}\Pi_{\mathrm{ps}}^{\pi}(t) = \left(\frac{f_\pi m_\pi^2}{m_u+m_d}\right)^2 \delta(t-m_\pi^2).$$

It turns out that the low Q^2 behavior for the penguin $\psi(q^2)$ is completely saturated by π and K contributions at the point Q_0^2 where the bound is optimal. In addition, the asymptotic part at Q_0^2 is completely negligible. This enables us to factorize out the vacuum-saturation result for the matrix element,

$$\mathcal{F}^{\mathrm{tot}}(Q^2) = (\mathcal{M}^{(c)})^2 I(Q^2, m_K^2, m_\pi^2), \tag{5.7}$$

with

$$I(Q^2, m_K^2, m_\pi^2) = \frac{36}{16\pi^2}\int_0^1 dx\, \frac{x^5(1-x)^5}{[Q^2x(1-x)+m_\pi^2 x+m_K^2(1-x)]^5}\,.$$

In the next section we discuss the results.

Estimate of ε'/ε

We start from the lower bound on ε'/ε derived recently by Gilman and Hagelin [22]

$$\left|\frac{\varepsilon'}{\varepsilon}\right| \geqslant 8.4(s_2c_2s_3s_\delta)\left(\frac{\operatorname{Im} c_5}{0.1}\right)\left(\frac{9\sqrt{3}}{16}\,\frac{\langle\pi^0\pi^0|4O_5|K^0\rangle}{1.4\ \mathrm{GeV}^3}\right). \tag{5.8}$$

The first parenthesis on the r.h.s. is the lower bound on the product of Kobayashi-Maskawa parameters given as a function of the B-meson lifetime τ_B, the $K^0-\overline{K^0}$ mixing parameter B and the top-quark mass m_t. The bound increases with increasing τ_B, but decreases with increasing B and m_t.

The second parenthesis is the Wilson coefficient and the third parenthesis is the matrix element of the operator O_5 which has the form (3.13).

By numerical evaluation of the integral I in (5.7) we find that

$$|\langle\pi^+|O_5|K^+\rangle| \leqslant 0.9\mathcal{M}^{(c)}, \tag{5.9}$$

with $\mathcal{M}^{(c)}$ given by (3.15).

The bound (5.9) is independent of the values of quark masses, or any other input parameter except m_π and m_K. From (5.9) the following constraints are obtained:

$$0.1\mathcal{M}^{(c)} \leqslant |\langle\pi^+|O_5+O_5^{(c)}|K^+\rangle| \leqslant \mathcal{M}^{(c)}. \tag{5.10}$$

The *lower bound* in (5.10) is obtained by subtracting two large numbers and is therefore sensitive to the approximation used. However, *both* normal and anomalous matrix elements are proportional to $\mathcal{M}^{(c)}$ and, consequently, the sensitive quark-mass dependence is factorized out. This is not the case in quark model calculations. In addition, the N_c dependence is also factorized, at least in the asymptotic limit.

Numerically, we find that with $B = 2/3$, $\tau_B = 0.9\times10^{-12}$ sec, $m_t = 40$ GeV and $\mathrm{Im}\, c_5 = 0.1$,

$$\left|\frac{\varepsilon'}{\varepsilon}\right| \geqslant 8\times10^{-4}. \tag{5.11}$$

This is an order of magnitude smaller than the Gilman-Hagelin result in Ref. [22].

This difference is due to the following:

(*i*) In their paper Gilman and Hagelin used the bag-model estimate of $\langle\pi^+|O_5|K^+\rangle$ of Ref. [11], but neglected the anomalous term. Actually, in Ref. [11] the anomalous term was estimated using the unusually large values of current quark masses obtained via bag-model calculation, and found to be rather small. We have used the running quark masses at 1 GeV^2 obtained from the QCD sum rules: $m_s = 160$ MeV, $m_d = 11$ MeV and $m_u = 5$ MeV. Our values of quark masses + anomalous term would reduce the Gilman-Hagelin estimate by a factor of ~ 2.6.

(*ii*) We have used the continuation factor $\kappa = 0.923$, which was also used in the PCAC estimate of the $K^0-\overline{K^0}$ mixing parameter B, while Gilman and Wise used $\kappa = 1.71$. This gives an additional factor of 1.85, which together with (*i*) gives a result smaller by a factor of 4.8.

Actually, with our values of quark masses and accounting for the anomalous commutator term, we have

$$\langle\pi^0\pi^0|4O_5|K^0\rangle \geqslant 0.4\langle\pi^0\pi^0|4O_5|K^0\rangle|_{\text{bag}}, \tag{5.12}$$

where

$$\langle\pi^0\pi^0|4O_5|K^0\rangle\Big|_{\text{bag}} = \begin{cases} 0.3\ \text{GeV}^3 & \text{for} \quad \kappa = 0.923 \\ 0.55\ \text{GeV}^3 & \text{for} \quad \kappa = 1.71\,. \end{cases} \tag{5.13}$$

$K \to 2\pi$ decays

The improved bounds on the matrix elements of the operators O_4 and O_- are numerically

$$|\langle\pi^+|O_4|K^+\rangle| \leqslant 1.1\times10^{-3}\ \text{GeV}^4, \tag{5.14}$$

$$|\langle\pi^+|O_-|K^+\rangle| \leqslant 0.67\times10^{-3}\ \text{GeV}^4, \tag{5.15}$$

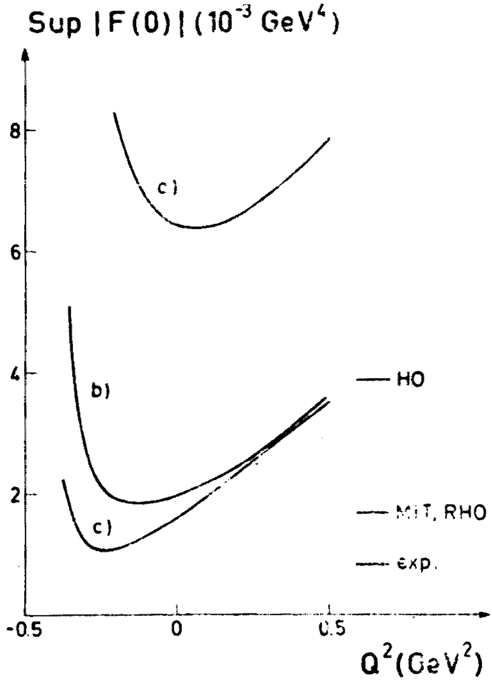

Fig. 4. Upper bounds for $|F(0)| = |\langle\pi^+|O_4|K^+\rangle|$. Curves b) and c) correspond to the improved bound in (4.10)

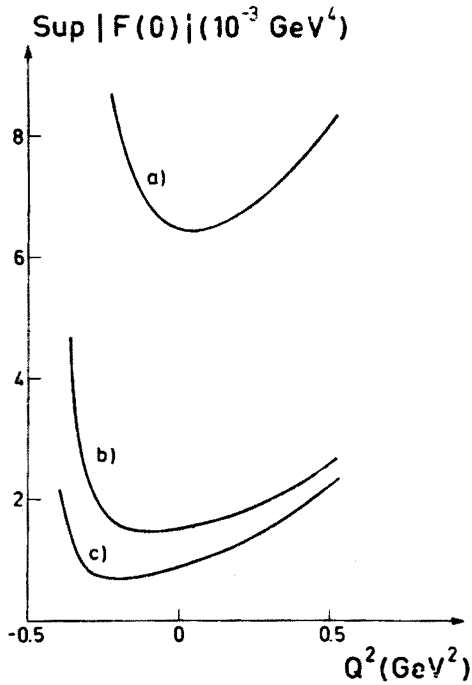

Fig. 5. Upper bounds for $|F(0)| = |\langle\pi^+|O_-|K^+\rangle|$

and the bound on the matrix element $\langle \pi^+ | O_+^{1/2} | K^+ \rangle$ is smaller by a factor of 3, which follows from the relation

$$|\langle \pi^+ | O_- | K^+ \rangle| = 3 \frac{1 - N_c^{-1}}{1 + N_c^{-1}} |\langle \pi^+ | O_+^{1/2} | K^+ \rangle|$$

taken in the large-N_c limit.

Compared with quark model calculations, we find that our upper bounds are always *below* the quark model results. The bounds derived show that quark model calculations overestimate the matrix elements of the operators with left-left helicities. The bag-model value is 1.7×10^{-3} GeV^4 for (5.14) and 0.84×10^{-3} GeV^4 for (5.15).

Having in mind that there is a large cancellation in the penguin contribution coming from the anomalous term, we find it rather difficult to account for the $\Delta I = 1/2$ rule. From our bounds it follows that O_- and $O_+^{1/2}$ can account foı at most 20% of the experimental amplitude. The rest should be ascribed to penguins.

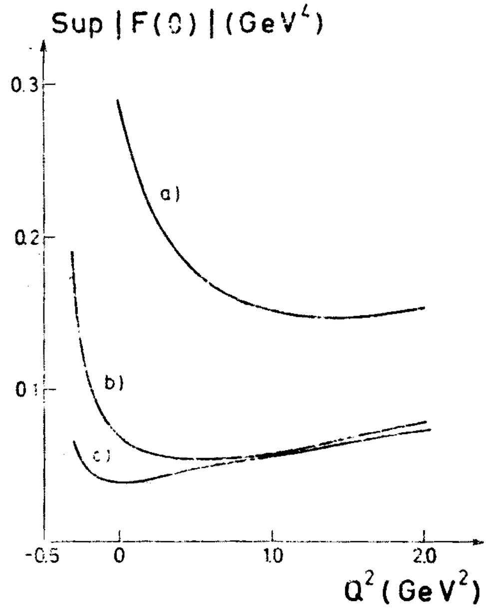

Fig. 6. Upper bounds for $|F(0)| = |\langle \pi^+ | O_5 | K^+ \rangle|$

6. *Discussion and prospects*

The derivation of the bounds on matrix elements relies on a few important assumptions. As far as the rigorous bound (depicted by curves a in Figs. 4, 5, and 6) are concerned, the underlying assumptions are as follows:

(*i*) General assumptions: analyticity, unitarity and QCD.

(*ii*) Factorization in the estimate $\mathscr{F}^{\text{low}}(Q^2)$. As we have discussed, the chiral perturbation approach [20] signals this to be a good approximation.

(*iii*) Unfortunately, the rigorous bounds are not restrictive enough to be used. Therefore, the phenomenological input is needed to improve the bound. This relies on the assumption that $F(t)$ has no zeros in the complex t plane. We have no control of this requirement.

(*iv*) In using the ansatz (5.3), we have lost the explicit μ dependence of the matrix elements. This dependence can be calculated only in QCD at higher Q^2, as in Eq. (5.1), but it does not lead to the phenomenologically useful bounds.

In addition, the theoretical calculation of the amplitudes is influenced by the ambiguity in the continuation factor.

All ambiguities discussed above can be avoided if one combines the effective-chiral-Lagrangian technique with the QCD-duality approach of Ref. [21]. The basic idea [20] is to use the amplitudes (3.7) calculated in chiral perturbation theory and to determine the unknown coefficients a_i using QCD sum rules. The duality then means that the sum rules

$$F_n = \int_0^{s_0} dt t^n \frac{1}{\pi} \operatorname{Im} \psi(t) \tag{6.1}$$

calculated in the effective chiral theory and in QCD should match in some range of s_0. The ratio of sum rules

$$M_n = \frac{F_n}{F_{n-1}} \tag{6.2}$$

can be used to determine such a range of s_0, since the unknown coefficients a_i cancel in the ratio (6.2). Then the sum rule (6.1) may be used to determine a_i. Once a_i are known, Eqs. (3.7) immediately give the desired amplitudes.

These ideas have been successfully applied to the parameter B in $K^0-\overline{K^0}$ mixing, giving $B = 1/3$ within an accuracy of 20% [20], and to the calculation of the $K^+ \to \pi^+\pi^0$ decay in the standard model [19] where the theoretical value agrees with the experimental value within a few percent. In both approaches, the μ dependence of the matrix elements is completely controlled and in the latter there is no ambiguity in momentum continuation.

The same approach can also be applied to the calculation of ε'/ε and $\Delta I = 1/2$ $K \to 2\pi$ amplitudes. The work along this line is in progress [23].

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