# LETTERS TO THE EDITOR

## REGULARIZATION INDEPENDENCE OF THE BARYON NUMBERS OF CHIRAL BAGS

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When calculating baryon numbers of chiral bags, one uses the regularizing function $\exp(-|E|t)$ for $t \to 0$. We show that this can be replaced by any of a wide class of regularizing functions $f(|E|t)$ without changing the result.

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The baryon number of a chiral bag can be calculated [1] using the formula

$$B = -\tfrac{1}{2}\lim_{t\to 0}\sum_{E}\operatorname{sign}(E)e^{-t|E|}, \tag{1}$$

where the sum extends over all the single fermion states inside the bag. The question naturally arises, does the sum (1) depend on the special choice of the regularizing function $\exp(-t|E|)$. In this note we calculate $B$ from the formula

$$B = -\tfrac{1}{2}\lim_{t\to 0}\sum_{E}\operatorname{sign}(E)f(t|E|), \tag{2}$$

where $f$ is any regularizing function satisfying the conditions given below. We find that the result does not depend on the choice of the regularizing function.

The obvious assumptions about the regularizing function are that for given $E$ it should tend to one for $t \to 0$ and that it should improve the convergence of series (2). Thus we assume

$$f(0) = 1; \qquad \lim_{x\to\infty} f(x) = 0. \tag{3}$$

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We need moreover some technical assumptions to justify the steps in the calculation given below. A sufficient (but not necessary) set of assumptions is that:

(*i*) Function $f$ is three times differentiable everywhere in the interval $0 \leq x < \infty$. This is used when making Taylor expansions.

(*ii*) The interval $[0, \infty)$ can be decomposed into a finite number of intervals so that in every one of them each of the functions $f', f'', xf'', f''', xf'''$ is monotonic. This is used when replacing sums by integrals.

(*iii*) For $x \to \infty$ function $xf''(x)$ tends to zero and function $xf'''(x)$ tends to a finite limit (zero not excluded).

The calculation is an extension of the calculation given in Ref. [2] (further quoted I). It is also closely related to the proof of regularization independence of the anomalies in gauge theories as given by Fujikawa [3].

Formula (39) from I, after supplying a missing minus sign and generalizing from (1) to (2), takes the form

$$B = B_0 - \frac{8}{\pi} \int_0^{\pi/2} d\beta_0 \, \mathrm{tg}^2 \beta_0 \lim_{t \to 0} \sum_{\nu=1}^{\infty} \nu^2 \langle f(t|E|) \rangle . \tag{4}$$

Here

$$B_0 = -\theta \lim_{t \to 0} t \sum_{\nu=1}^{\infty} f'(t\nu\pi), \tag{5}$$

$\theta$ is a numerical constant known as the chiral angle

$$E = \frac{\nu}{\cos \beta_0} + \sum_{k=0}^{\infty} \frac{a_k(\beta_0, \theta)}{\nu^k}, \tag{6}$$

where $a_k$ are known numbers, and $\langle f \rangle$ means the average of $f\,\mathrm{sign}(E)$ over the eight states (differing in their coefficients $a_k$) corresponding to a given pair of indices $\beta_0$, $\nu$. In particular for any positive integer $k$

$$\langle 1 \rangle = \langle a_0^k \rangle = \langle a_1 \rangle = 0. \tag{7}$$

Changing in formula (5) the summation into an integration, which introduces no error for $t \to 0$, one finds using (3)

$$B_0 = \theta \lim_{t \to 0} t \int_0^{\infty} f'(tn\pi) dn = -\frac{\theta}{\pi} \int_0^{\infty} f'(x) dx = \frac{\theta}{\pi} \tag{8}$$

For the sum occurring in formula (4)

$$\lim_{t \to 0} \sum_{\nu=1}^{\infty} \nu^2 \langle f(t|E|) \rangle = \lim_{t \to 0} \int_1^{\infty} d\nu \nu^2 \left[ t \frac{\langle a_2 \rangle}{\nu^2} f' \left( \frac{t\nu}{\cos \beta_0} \right) + t^2 \frac{\langle a_0 a_1 \rangle}{\nu} f'' \left( \frac{t\nu}{\cos \beta_0} \right) \right], \tag{9}$$

performing the integrations and using relations (3) one finds

$$\lim_{t\to 0}\sum_{\nu=1}^{\infty}\nu^2\langle f(t|E|)\rangle = -\cos^2\beta_0\left[\frac{\langle a_2\rangle}{\cos\beta_0}-\langle a_0a_1\rangle\right]. \tag{10}$$

Substituting results (8) and (10) into formula (4), one reproduces formula (41) from I, which yields the standard result for the baryon number.

Our analysis can be generalized in many ways. It is possible to modify the technical assumptions. It is also possible to redefine the parameter tending to zero. E.g. function $\exp(-E^2t)$ is not of the type considered here, but replacing in it $t$ by $t^2$, one reduces it to the case we have discussed. More generally, our result applies to any regularizing function of the form

$$f(E,t) = \sum_{i=1}^{N} f_i(t^{c_i}|E|), \tag{11}$$

where $N < \infty$, $c_i > 0$ for $i = 1, \ldots, N$, $f(E, 0) = 1$ and each of the functions $f_i$ separately satisfies all our assumptions except $f_i(0) = 1$.

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## REFERENCES

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