NONPERTURBATIVE EFFECTS IN ELECTRON POSITRON ANNIHILATION INTO MESONS

By K. Forinash*

Department of Physics, East Tennessee State University, Johnson City, TN 37601, USA

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A cross section for electron positron annihilation into three pions is calculated which includes contributions made by exact solutions of nonlinear equations.

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In the past few years Burt [1] has developed a quantum field theory that has met with some success in describing persistently self-interacting spin-zero meson systems. By including contributions made by exact solutions of nonlinear equations, problems of nonrenormalizability are avoided for field equations with self interacting terms. Using this method a solitary-wave exchange potential has been found [2] which gives phase shifts in excellent agreement with experiment [3]. A mass spectrum that matches the known neutral pseudoscalar mesons has also been found [4]. In this paper Burt's theory of persistently self-interacting mesons is applied to the calculation of electron positron annihilation. A cross section for this process, which includes the effects of self-interacting mesons, is found using persistently self-interacting meson propagators.

The equation describing self-interacting mesons is the nonlinear Klein-Gordon equation,

$$\partial^{\mu}\partial_{\mu}\phi+m^{2}\phi+\lambda\phi^{3} = 0. \tag{1}$$

Exact solutions to this equation are found by Burt [5] to be

$$\phi^{(\pm)}(x) = \frac{A_k^{(\pm)}\exp(\mp ik\cdot x)}{(Dk\omega V)^{1/2}}\left[1-\frac{A^{(\pm)^3}\exp(\mp 3ik\cdot x)}{10m^2}\right]^{-1/3}. \tag{2}$$

In this expression V is the volume of the system, m is the pion mass and $A^{(\pm)}$ are creation and annihilation operators of the Klein-Gordon equation which satisfy the commutation relations

$$[A_{\bar{k}}^{(+)}, A_{\bar{k}'}^{(-)}] = \delta_{n_{\bar{k}}n_{\bar{k}'}}. \tag{3}$$

* Present address: Natural Sciences Division, Indiana University Southeast, 4201 Grant Line Road, New Albany, Indiana 47150, USA.

k is the four momentum so that

$$k^2 = k_0^2 - \bar{k}^2 = \omega_k^2 - \bar{k}^2 = m^2. \tag{4}$$

These solutions are generalizations of solitary waves and can be expanded in a series of Gegenbauer polynomials [6]. In momentum space the persistently interacting propagator constructed from these solutions [7] can be written as

$$P(k, (3n+1)^2m^2) = i \sum_{n=0}^{\infty} \frac{(C_n^{1/3})^2(3n+1)^{3n+1}(3n+1)!b^{3n}}{(\bar{k}^2+(3n+1)^2m^2)^{3n}D^{3n+1}} \Delta_F(k, (3n+1)^2m^2). \tag{5}$$

Here

$$\Delta_F(k, (3n+1)^2m^2) = \frac{1}{(\bar{k}^2-(3n+1)^2m^2-i\varepsilon)} \tag{6}$$

is the Feynman propagator for the linear Klein-Gordon field with the mass replaced by $(3n+1)m^2$. $C_n^{1/3}$ represents the Gegenbauer polynomial of order 1/3. The expression b contains the coupling constant λ and is

$$b = \frac{\lambda}{V \cdot 10m^2}, \tag{7}$$

where the volume of the system has been absorbed by redefining the fields and coupling constant as

$$\phi' = V^{1/2}\phi \quad \text{and} \quad \lambda' = \lambda V^{3/2}. \tag{8}$$

The constant D is an arbitrary function of momentum with the simplest form [1]

$$D = a_N\left(1+\gamma\frac{\bar{k}^2}{(3n+1)^2m^2+1}\right) \text{ with } \gamma = \left(\frac{m^2}{\lambda^1}\right)^{4/3} m^2. \tag{9}$$

The series expansion for the propagator is asymptotic and for practical calculations must be truncated. Once a choice of truncation is made, the constant a_N appearing in equation (9) can be determined using the requirement that the probability for intermediate states is unity. If no perturbative contributions to meson propagation are included, a_N can be shown [1] to satisfy

$$1 = \sum_{n=0}^{N} (C_n^{1/3})^2(0.1)^{2n}\gamma^{-3n/2}(3n+1)!(3n+1)^{3n-2}a_N^{-3n-1}. \tag{10}$$

Here N is the value of the summation index n where the asymptotic series for the propagator is truncated.

The cross section for the process shown in figure 1 is

$$d\sigma = \frac{1}{v_{\rm rel}\,2E_aE_b} \cdot \frac{|M|^2 4m^2d^3p_1d^3p_2d^3p_3}{(2\pi)^5 8E_1E_2E_3} \delta^4(p_a+p_b-(p_1+p_2+p_3)). \tag{11}$$

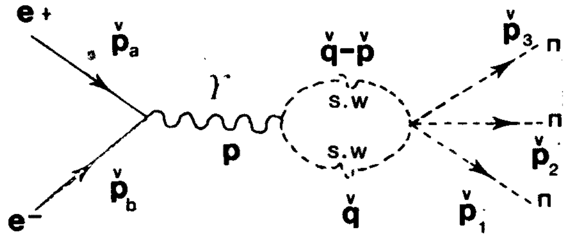

Fig. 1. Leading order diagram for $e^+e^- \to 3\pi$ with generalized solitary wave exchange

The subscripts a and b refer to the electron and positron and v_{rel} is the relative velocity between them. Quantities related to the final pions are denoted with subscripts 1, 2 and 3. The mass of the electron is m_e and the delta function assures conservation of four momentum.

Using Feynman rules [8] the matrix M is

$$M = (u^{(s_1)}(\bar{p}_a)\,(-e\gamma^{\mu})\bar{v}^{(s_2)}(\bar{p}_b))ep^{\nu}\left(\frac{\delta_{\mu\nu}}{i(p^2 - i\varepsilon)}\right)I(p). \tag{12}$$

$u(k)$ and $v(k)$ are Dirac spinors for the electron and positron with spin s_1 and s_2, γ is the standard gamma matrix [8], e is the electron charge and p is the four momentum of the internal photon. $I(p)$ represents the internal meson lines where the propagator for each meson is now a persistently interacting propagator given by equation (5) so that

$$I(p) = \frac{-1}{(2\pi)^4}\int P(q,(3n+1)^2m^2)P(p-q,(3l+1)^2m^2)d^4q. \tag{13}$$

Time and angular integrations can be evaluated analytically in the center of mass frame with momentum transfer equal to zero. The remaining momentum integral and phase space integration [9] were done numerically for several values of the parameter γ with summation indices l and n from zero to 10. Values of a_N for each value of l and n were also found numerically. A plot of the cross section for γ equal to 0.62 with center of mass energy between 1000 MeV and 1900 MeV is shown in figure 2. The location of peaks and general form of the cross section does not depend on the parameter γ. Peak values for other choices of γ are shown in Table I. Values of γ greater than five give unrealistically small cross sections.

Results of experiments done at the accelerator at Orsay indicate a possible peak in the cross section at 1650 MeV of approximately 12 nanobarns [10–12]. Data is not available for other energy regions. Although the present calculation does not include perturbative contributions, higher order diagrams or other processes which might produce three pions from electron positron annihilation, it matches the available experimental data quite well for γ equal to 0.62. For this value of γ, a peak of 12.7 nanobarns if found at 1630 MeV in the calculated cross section. Further experimental results are needed to better verify the results of this calculation.

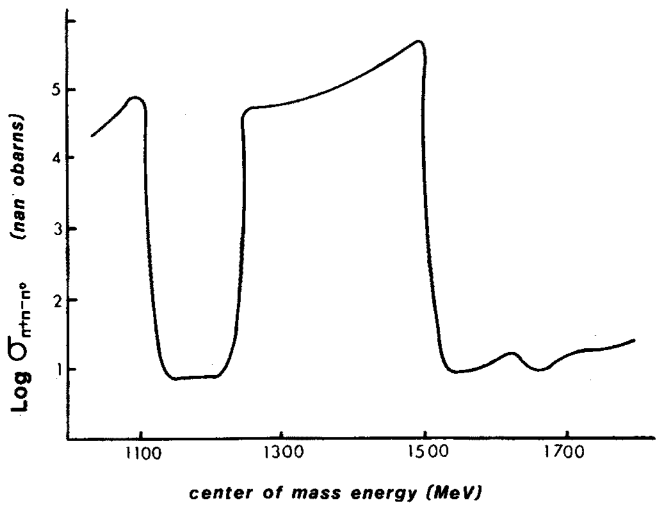

Fig. 2. Cross section for $e^+e^- \to 3\pi$ (log scale) for center of mass energies between 1000 MeV and 1900 MeV with parameter γ equal to 0.62

TABLE I

Values of the peaks in the nonperturbative contribution to the $e^+e^- \to 3\pi$ cross section in nanobarns for several values of the parameter γ

	$\gamma = 5.0$	$\gamma = 0.7$	$\gamma = 0.2$
1090 MeV	0.2173×10^0	0.9825×10^4	0.2698×10^8
1480 MeV	0.4966×10^{-2}	0.3066×10^5	0.4989×10^8
1625 MeV	0.6156×10^{-10}	0.1715×10^0	0.6456×10^5
1900 MeV	0.4097×10^{-8}	0.4249×10^1	0.6912×10^6

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