# LETTERS TO THE EDITOR

## INFLATION IN A CLOSED UNIVERSE*

By M. Demiański

Institute of Theoretical Physics, Warsaw University**

*(Received January 10, 1986)*

We investigate a class of homogeneous and isotropic cosmological models filled with radiation and constant vacuum energy. We show that in closed models inflation is possible only if the entropy is larger than $1.7 \cdot 10^{13}$.

PACS numbers: 98.80.Bp, 98.80.Dr

The inflationary model of the very early evolution of the Universe, invented by Alan Guth [1, 2], is able to explain several conundrums of the standard Big Bang scenario. For example, it explains why the temperature of the microwave background radiation is isotropic, why the observed mean density of the Universe is so close to the critical density (or why the Universe is almost Euclidean), why the matter in a large scale is uniformly distributed, and why there are so few magnetic monopoles.

Guth suggested that the GUT's phase transition is first order and that the Universe could be trapped in the symmetric state even below the transition temperature $T_c \sim 10^{14}$ GeV. The false vacuum energy very quickly starts to play dominant role. The energy density of false vacuum plays a role of cosmological constant, and the Universe expands exponentially. The initial bubble of the false vacuum can be expanded to an enormous size much larger than the presently visible Universe. Finally the latent heat is released and the Universe reheats to temperature close to $T_c$.

For the inflationary model to work one has to assume that the very early Universe was hot, its temperature should be above the transition temperature, and expanding.

I would like to consider a simple model to discuss restrictions placed by the above requirements. Let us assume that the early Universe was described by the Friedman-

* This work was supported in part by the Polish Interdisciplinary Research Project MR-I-7.

** Address: Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, Hoża 69, 00-681 Warszawa, Poland.

-Robertson-Walker metric, and was filled with relativistic particles. From the beginniug let us include the false vacuum energy but initially it will not play a dominant dynamical role. This is a simplifying assumption but nevertheless the model should quite reasonably describe the dynamics of the Universe through the GUT's spontaneous symmetry breaking transition.

The Friedman equation can be written in the form

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}(\varrho_r+\varrho_v) - \frac{k}{R^2}, \tag{1}$$

where $k = 0$, $+1$, or $-1$, $\varrho_r$ is the density of relativistic particles, and $\varrho_v$ is the constant density of false vacuum. I am using units in which speed of light, the Planck constant, and the Boltzmann constant are set to be equal to 1.

We assume that initially the matter was in thermal equilibrium and for $\varrho_r$ we take $\varrho_r = g\pi^2/30\,T^4$, where $g$ is the effective number of degrees of freedom. The expansion is adiabatic, so $RT =$ constant. In order to specify initial conditions it is useful to introduce the entropy of relativistic particles $S = \frac{4g\pi^2}{90} T^3 R^3$.

It is quite easy to solve the Friedman equation, we have

$$R^2(t) = R_0^2 \sinh \alpha t(\cosh \alpha t - \beta \sinh \alpha t), \tag{2}$$

where

$$R_0^2 = 2\left(\frac{g\pi^2}{30\varrho_v}\right)^{1/2}\left(\frac{90S}{4g\pi^2}\right)^{2/3},$$

$$\beta = \frac{3k}{16\pi G}\left(\frac{30}{\varrho_v g\pi^2}\right)^{1/2}\left(\frac{4g\pi^2}{90S}\right)^{2/3}, \tag{3}$$

and

$$\alpha = \left(\frac{8\pi G}{3}\varrho_v\right)^{1/2}.$$

When $k = 0$, or $k = -1$ for every $\varrho_v > 0$, and $S > 0$ the Universe is expanding and asymptotically $R(t) \sim R_0 e^{\alpha t}$. In a closed Universe, when $k = 1$ inflation is possible only if $\beta < 1$, or

$$\frac{3}{16\pi G}\left(\frac{30}{\varrho_v g\pi^2}\right)^{1/2}\left(\frac{4g\pi^2}{90}\right)^{2/3} < S^{2/3} \tag{4}$$

so the initial entropy cannot be too small. In SU(5) GUT theory $g = 683/8$, and the spontaneous symmetry breaking occurs at $T_c \sim 10^{14}$ GeV. In an initially closed model inflation is possible if $S > 1.7 \cdot 10^{13}$. The critical value of the initial entropy is very large. This excludes a large class of closed Universes. It is reasonable to expect that the Universe emerging from a quantum state should have rather small entropy.

It is interesting to discuss properties of solutions of Friedman equations for different values of $\beta$. When $\beta = 1$ the Universe expands from a singular state and asymptotically $R(t) \sim \frac{1}{\sqrt{2}} R_0$.

When $\beta > 1$ the Universe expands from a singular state, bounces at $R_{\max} = \frac{1}{2} R_0 |(\beta+1)^{1/2} - (\beta-1)^{1/2}|$, and recontracts.

It is worth noticing that when $\beta > 1$, there are other solutions of equation (1). They are provided by a different choice of initial conditions. There is a solution describing a Universe contracting from infinite size, bouncing, and expanding to infinity. It is given by

$$R^2(t) = \tfrac{1}{2}R_0^2(\beta+\sqrt{\beta^2-1}\cosh 2\alpha t). \qquad (5)$$

There is still another possibility. Universe can start contracting from a finite size and contracts to singularity, such situation is described by

$$R^2(t) = \tfrac{1}{2}R_0^2(\beta-\sqrt{\beta^2-1}\cosh 2\alpha t). \qquad (6)$$

We conclude that in closed Universes inflation is possible provided that the initial entropy is larger than $1.7 \cdot 10^{13}$. This condition cannot be satisfied by a generic model, and it severly restricts the class of closed Universes allowing inflation.

## REFERENCES

[1] A. H. Guth, *Phys. Rev.* **D23**, 347 (1981).
[2] A. H. Guth, *Phase Transition in the Early Universe*, eds. G. W. Gibbson, S. W. Hawking, S. T. Siklos, Cambridge University Press, Cambridge 1983.