POLARIZED γ-QUANTA PRODUCTION IN $\tau \to \nu+\pi+\gamma$ DECAY

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The Stokes parameters of the photons produced in the τ-lepton decay, $\tau \to \nu+\pi+\gamma$, have been found. The study of γ-quanta polarization parameters in this decay allows us to obtain the essential information about the vector and axial formfactors of $\gamma W\pi$ vertex in a broad interval of the time like momentum transfer which are inaccessible in the radiative pion decay. We have calculated the contribution in the Stokes photon parameters which is caused by the anomalous magnetic moment of a heavy lepton.

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1. The investigation of τ-lepton hadronic decays (very numerous due to the large τ mass) allows one to tackle a great number of different problems of weak interaction physics [1]. It is enough to mention the checking of the vector weak current conservation hypothesis (strangeness being conserved) in $\tau^- \to \nu+\pi^-+\pi^0$ and $\tau^- \to \nu+\pi+\omega(\varphi)$ decays [2] and the locality of the τ-lepton weak interaction with hadrons [3]. The study of the hadronic decays may be interesting for the search of the second-type weak currents [4–6]. The polarized τ-lepton decays may help to better the ν_τ mass estimate (which is rather rough now). The $\tau \to \nu+\pi+\varrho$ decay is of a great importance for checking of the different predictions of current algebra and especially for the A_1-meson properties studies [7–9].

This paper is a study of the radiative decay of a τ-lepton, $\tau \to \nu+\pi+\gamma$. This decay may be interesting first of all for the study of the weak $\gamma W^*\pi$-vertex (W^*-virtual W-boson) formfactors in the time-like momentum transfer region. This region turns out to be large enough, $m_\pi^2 \leqslant t \leqslant m_\tau^2 = 3.16$ GeV^2. The same formfactors define also the radiative pion decay, $\pi \to e+\nu+\gamma$, the momentum transfer interval being very small, $t \leqslant m_\pi^2$.

The τ-lepton radiative decays may be used in principle for the estimate of the anomalous magnetic moment of a τ-lepton. Until this moment will have been measured, we may consid-

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er the τ-lepton anomalous moment to be small, since the experimental data about the $e^+ + e^- \to \tau^+ + \tau^-$ reaction at large energies [10] are in agreement with the QED predictions.

It will be shown further, that the study of the polarization properties of the γ-quantum produced in the $\tau \to \nu + \pi + \gamma$ decay is extremely important for the reconstruction of the weak formfactors of the $\gamma W^* \pi$-vertex.

2. The matrix element of the $\tau \to \nu + \pi + \gamma$ process, which corresponds to the diagrams in Fig. 1, takes the form:

$$\mathcal{M} = \mathcal{M}^{(0)} + \mathcal{M}^{(S)},$$

$$\mathcal{M}^{(0)} = f_\pi G \frac{m_\tau}{\sqrt{2}} \bar{u}(p_2) \left(-e \cdot P + \frac{\hat{k}\hat{e}}{2k \cdot p_1} \right) (1-\gamma_5) u(p_1), \quad P = \frac{p_1}{k \cdot p_1} - \frac{q}{k \cdot q},$$

$$\mathcal{M}^{(S)} = -f_\pi G \frac{1}{\sqrt{2}} \bar{u}(p_2) \left[-i\gamma_\alpha (1+\gamma_5) \varepsilon_{\alpha\mu\varrho\sigma} e_\mu k_\varrho q_\sigma V(t) \right.$$

$$\left. + \gamma_\alpha (1+\gamma_5)(e_\alpha k \cdot q - k_\alpha e \cdot q) a(t) \right] u(p_1),$$

where G — the Fermi constant of weak interaction, f_π — the constant of $\pi \to \mu + \nu$ decay, m_τ — τ-lepton mass, e_α — the γ-quantum polarization 4-vector (the definitions of 4-momentaa re given in Fig. 1).

The structure radiation term $\mathcal{M}^{(S)}$ is characterized by the two electroweak form factors: the vector $V(t)$ and the axial vector $a(t)$, $t = (p_1 - p_2)^2 = (k+q)^2$. Due to the isovector part conservation hypothesis of the weak vector current the formfactor $V(t)$ must coincide with the electromagnetic formfactor of the $\pi^0 \gamma\gamma^*$-vertex which may be measured in the $e^+e^- \to \pi^0 + \gamma$ reaction for the time-like momentum transfer. But it is only $|V(t)|^2$ that may be found in this reaction, whereas in the time-like momentum region the formfactors have to be complex: $\mathrm{Im}\, V(t) \neq 0$, $t \geq 4m_\pi^2$, $\mathrm{Im}\, a(t) \neq 0$, $t \geq 9m_\pi^2$.

3. Let us consider now the polarization characteristics of γ-quantum, produced in the nonpolarized τ-lepton decay. We write down the square of the matrix elements as $\overline{|\mathcal{M}|^2} = \frac{1}{4} G^2 f_\pi^2 X_{\mu\nu} e_\mu e_\nu^*$, where the line means the summation over the polarization of the

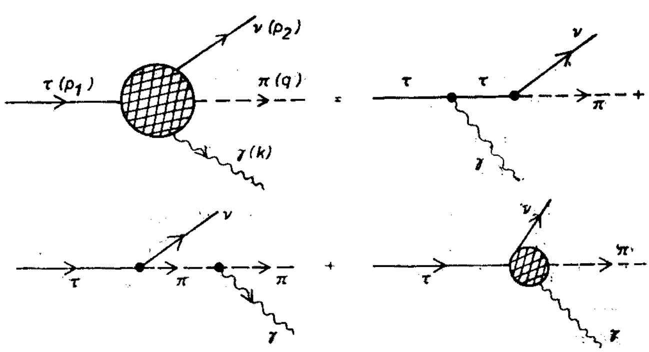

Fig. 1. The Feynman diagrams for the $\tau \to \nu + \pi + \gamma$ decay

τ-lepton and neutrino. Using only the electromagnetic current conservation for the $X_{\mu\nu}$-tensor structure we may write:

$$X_{\mu\nu} = g_{\mu\nu}A_1 + P_\mu P_\nu A_2 + (P_\mu E_\nu + P_\nu E_\mu)A_3$$

$$+ i\varepsilon_{\mu\nu\alpha\beta}k_\alpha p_{1\beta}A_4 + i\varepsilon_{\mu\nu\alpha\beta}k_\alpha q_\beta A_5, \qquad E_\mu = \varepsilon_{\mu\alpha\beta\gamma}k_\alpha q_\beta p_{1\gamma},$$

where A_i is the real structure functions depending on the two invariant variables, t and $s = (p_2+q)^2$.

The structure functions A_3–A_5 characterize the P-odd effects in the $\tau \to \nu+\pi+\gamma$ decay with the production of the polarized γ-quanta. The structure function A_3 is T-odd also, therefore $A_3 \neq 0$ is true only for the complex formfactors $V(t)$ and $a(t)$, i.e. in the region $t \geqslant 4m_\pi^2$. Therefore, in the CP-invariant theory $A_3 = 0$ for the $\pi \to \mu+\nu+\gamma$ decay where $t < m_\pi^2$.

It is convenient to characterize the polarization states of the produced γ-quantum by the Stokes parameters ξ_i [11] for which the following formulas may be obtained in terms of the structure functions A_i:

$$X\xi_1 = e^{(1)} \cdot Pe^{(2)} \cdot EA_3, \qquad 2X\xi_3 = |e^{(1)} \cdot P|^2 A_2,$$

$$X\xi_2 = \varepsilon_{\mu\nu\alpha\beta}e_\mu^{(1)}e_\nu^{(2)}k_\alpha p_{1\beta}A_4 + \varepsilon_{\mu\nu\alpha\beta}e_\mu^{(1)}e_\nu^{(2)}k_\alpha q_\beta A_5, \tag{1}$$

$$X = -A_1 - \tfrac{1}{4}\left(\frac{m_\tau^2}{k\cdot p_1^2} + \frac{m_\pi^2}{k\cdot q^2} - 2\frac{p_1\cdot q}{2k\cdot p_1 k\cdot q}\right)A_2,$$

where $e_\mu^{(1)}$ and $e_\mu^{(2)}$ are the orthogonal 4-vectors, $e^{(1,2)}\cdot k = 0$, and moreover the 3-vector $\vec{e}^{(1)}$ is on the decay plane, and the 3-vector $\vec{e}^{(2)}$ is orthogonal to this plane. It is seen from (1) that the P- and T invariance violation in the $\tau \to \nu+\pi+\gamma$ decay is defined by the Stokes parameter ξ_1. This fact has first been mentioned [12] for the $\pi \to \mu+\nu+\gamma$ decay.

If it is only a γ-quantum that is detected in the $\tau \to \nu+\pi+\gamma$ decay, the corresponding tensor $x_{\mu\nu}$ is defined by the two structures only:

$$X_{\mu\nu} \Rightarrow x_{\mu\nu} = g_{\mu\nu}a_1(s) + i\varepsilon_{\mu\nu\alpha\beta}k_\alpha p_{1\beta}a_2(s),$$

$$a_1(s) = \int\left[A_1 + \tfrac{1}{4}\left(\frac{m_\tau^2}{k\cdot p_1^2} + \frac{m_\pi^2}{k\cdot q^2} - 2\frac{p_1\cdot q}{k\cdot p_1 k\cdot q}\right)A_2\right]dt,$$

$$a_2(s) = \int\left(A_4 + \frac{k\cdot q}{k\cdot p_1}A_5\right)dt.$$

It is seen that the P-odd structure function A_3 does not contribute to the integral characteristics; as a result either the nonpolarized or circularly polarized γ-quanta may be produced in the $\tau \to \nu+\pi+\gamma$ decay. All these statements are true for a general case irrespective of the model used for the formfactors $V(t)$ and $a(t)$.

Now we may write the following expressions for the invariant structure functions in terms of the electroweak formfactors:

$$A_1(x_\gamma, x_\pi) = -8m_\tau^2[1+(1-x_\nu-\Delta)\,\mathrm{Re}\,(a+V)]\,\frac{1-x_\pi+\Delta}{x_\gamma}$$
$$+m_\tau^6|V|^2[4(1-x_\nu-\Delta)^2-4x_\pi x_\gamma(1-x_\nu-\Delta)-x_\nu(1-x_\nu-\Delta)^2+4x_\gamma^2\Delta]$$
$$-m_\tau^6|a|^2x_\nu(1-x_\nu-\Delta)+2\,\mathrm{Re}\,aV^*(1-x_\nu-\Delta)\,[x_\gamma(1-x_\gamma-\Delta)-x_\pi(1-x_\pi+\Delta)],$$
$$A_2(x_\gamma, x_\pi) = 8m_\tau^4(1-\Delta)+4m_\tau^6x_\gamma(1-x_\nu-\Delta)\,\mathrm{Re}\,a+m_\tau^8x_\gamma^2(1-x_\nu-\Delta)^2(|a|^2-|V|^2),$$
$$A_3(x_\gamma, x_\pi) = -8m_\tau^2\,\mathrm{Im}\,V+4x_\gamma(1-x_\nu-\Delta)\,\mathrm{Im}\,aV^*,$$
$$A_4(x_\gamma, x_\pi) = \frac{16}{x_\gamma}\left(\frac{-1+\Delta}{x_\gamma}-\frac{2\Delta}{1-x_\nu-\Delta}\right)-8m_\tau^2\,\mathrm{Re}\,V\left(\frac{x_\pi}{x_\gamma}-\frac{2\Delta}{1-x_\nu-\Delta}\right)-8m_\tau^2\,\mathrm{Re}\,a$$
$$\times\left(1-\frac{1-x_\pi+\Delta}{x_\gamma}\right)-2m_\tau^4|a|^2[(1-x_\nu)^2-\Delta^2]$$
$$+4\,\mathrm{Re}\,aV^*x_\gamma(1-x_\gamma-\Delta)\left(\frac{x_\pi}{x_\gamma}-\frac{2\Delta}{1-x_\nu-\Delta}\right),$$
$$A_5(x_\gamma, x_\pi) = \frac{8}{x_\gamma}\left(\frac{2}{x_\gamma}-\frac{x_\pi}{1-x_\nu-\Delta}\right)+8m_\tau^2\,\mathrm{Re}\,V\left(\frac{1+x_\pi-\Delta}{x_\gamma}-\frac{x_\pi}{1-x_\nu-\Delta}\right)$$
$$+8m_\tau^2\,\mathrm{Re}\,a\left(1+\frac{x_\pi-1-\Delta}{x_\gamma}\right)+2m_\tau^4|V|^2[x_\gamma(1-x_\gamma-\Delta)-x_\pi(1-x_\pi-\Delta)]$$
$$-2m_\tau^4|a|^2x_\pi(1-x_\nu-\Delta)-4m_\tau^4\,\mathrm{Re}\,aV^*[x_\pi^2+x_\gamma^2+x_\pi x_\gamma-(x_\gamma+x_\pi)(1+\Delta)],$$

where $x_\gamma = 2E_\gamma/m_\tau$, $x_\pi = 2E_\pi/m_\tau$, $x_\nu = 2E_\nu/m_\tau$, $x_\gamma+x_\pi+x_\nu = 2$, E_γ, E_π and E_ν are energies of a γ-quantum, pion and neutrino, respectively.

4. Now let us analyse the dependence of the decay probability of $\tau\to\nu+\pi+\gamma$ from the τ-polarization. For the matrix element square, after the summation over the polarization of a neutrino and γ-quantum we may write:

$$\overline{|\mathscr{M}|^2} = \tfrac{1}{4}\,G^2f_\pi^2X, \qquad X = X_0+X(s),$$
$$X(s) = i\varepsilon_{\mu\alpha\beta\gamma}s_\mu k_\alpha q_\beta p_{1\gamma}S_1+s\cdot kS_2+s\cdot qS_3,$$

where X_0 is a spin-independent part of $\overline{|\mathscr{M}|^2}$. The real invariant structure functions S_i are the second power polynomials in the weak formfactors $V(t)$ and $a(t)$. The structure function S_1 describes *P*-even and *T*-odd contributions to the spin-dependent part of the probability of the $\tau\to\nu+\pi+\gamma$ decay. The value $\mathrm{Im}\,aV^*$ does not occur in the polarized τ decay. The function $S_1(s, t)$ characterizes the dependence of the decay probability on the transverse (to the decay plane) τ-lepton polarization, while the structure functions $S_2(s, t)$ and $S_3(s, t)$

characterize the same dependence on the longitudinal components of the polarization vector $\vec{s}$. If only a γ-quantum is detected in the $\tau \to \nu+\pi+\gamma$ decay, the contribution from S_1 disappears (after the ν and π integration) though the dependence on the longitudinal polarization remains after the integration.

The structure functions S_i have the following form in terms of the produced particles energies ($\Delta = 0$)

$$S_1(x_\gamma, x_\pi) = -16m_\tau\left[\operatorname{Im} V\left(1-\frac{1}{x_\gamma}\right)+\operatorname{Im} a\left(1+\frac{x_\pi}{x_\gamma}\right)\right],$$

$$S_2(x_\gamma, x_\pi) = 16\frac{m_\tau}{x_\gamma}\left(\frac{-1+x_\gamma+2x_\pi}{x_\gamma}+\frac{1-x_\gamma-4x_\pi}{1-x_\nu}\right)$$

$$+4m_\tau^5(|V|^2+|a|^2)(1-x_\pi-x_\nu x_\gamma)+8m_\tau^3 \operatorname{Re}(V-a)(2-x_\pi)\left(-1+\frac{1}{x_\gamma}\right),$$

$$S_3(x_\gamma, x_\pi) = 16\frac{m_\tau}{x_\gamma}\left(\frac{2}{x_\gamma}-\frac{1+3x_\pi}{1-x_\nu}\right)-4m_\tau^5(|V|^2+|a|^2)(1-x_\pi)(1-x_\nu)$$

$$+8m_\tau^3 \operatorname{Re} V\left(1-x_\gamma-\frac{1-x_\pi}{x_\gamma}\right)+8m_\tau^3 \operatorname{Re} a\left(-1+x_\gamma-2\frac{1-x_\pi}{x_\gamma}\right).$$

The pion energy distribution is a result of the γ-quantum energy integration in the limits of $m_\tau-E_\pi-p_\pi \leqslant 2E_\gamma \leqslant m_\tau-E_\pi+p_\pi$ i.e. for the relativistic pion $1-x_\nu \leqslant x_\gamma \leqslant 1$. But one cannot make this integration in a general case, since the formfactors $V(t)$ and $a(t)$ depend on the energy E_γ. In order to obtain the γ-quantum energy distribution it is convenient to make use of the invariant integration technics in the ν and π c.m.s.:

$$(2\pi)^4\int\overline{|\mathcal{M}|^2}\delta(p_1-p_2-k-q)\frac{d^3q}{(2\pi)^3 2E_\pi}\frac{d^3p_2}{(2\pi)^3 2E_\nu} = \frac{1}{8\pi}\int d\cos\theta^*\overline{|\mathcal{M}|^2}\frac{E_\nu^*}{W^*},$$

$$d\cos\theta^* = \frac{dt}{m_\tau E_\gamma}\Bigg/\left(1-\frac{m_\pi^2}{s}\right), \quad \frac{m_\pi^4}{s}\leqslant t\leqslant m_\tau^2+m_\pi^2-s,$$

$$E_\nu^* = \frac{s-m_\pi^2}{2\sqrt{s}}, \quad s = W^2 = m_\tau^2-2m_\tau E_\gamma.$$

The following integral appears in the analysis of the polarized τ-lepton decay:

$$(2\pi)^4\int s\cdot qX(s,t)\frac{d^3q}{(2\pi)^3 2E_\pi}\frac{d^3p_2}{(2\pi)^3 2E_\nu}\delta(p_1-k-q-p_2)$$

$$= s_\alpha(\mathcal{X}_1(s)k_\alpha+\mathcal{X}_2(s)p_{1\alpha}] = s\cdot k\mathcal{X}_1(s),$$

$$\mathcal{X}_1(s) = \frac{1}{8\pi}\int X(s,t)\left[-t(m_\tau^2+s)-s(m_\tau^2-s)+2m_\pi^2 m_\tau^2\right]\frac{dt}{(m_\tau^2-s)^3}.$$

One must know the weak formfactors $V(t)$ and $a(t)$ for evaluating the integral $\int\ldots dt$.

4. As it has been mentioned before, the radiative decay of the τ-lepton, $\tau \to \nu+\pi+\gamma$ may be used in principle for the calculation of an anomalous magnetic moment κ of the τ-lepton. The pole part of the matrix element depends on the value κ in the following way:

$$\mathscr{M}^{(0)} = m_\tau f_\pi \frac{G}{\sqrt{2}} \bar{u}(p_2)(1-\gamma_5)\left[-e\cdot p+\frac{\hat{k}\hat{e}}{2k\cdot p_1}+\kappa\frac{\hat{p}_2+m_\tau}{2k\cdot p_1}\frac{\hat{k}\hat{e}}{2m_\tau}\right]u(p_1).$$

After the summation over the lepton polarization we obtain the following formulas for the linear κ contribution to the structure function $A_i(s, t)$ which define the γ-quantum polarization states in the $\tau \to \nu+\pi+\gamma$ decay

$$\frac{1}{2\kappa} A_1(x_\gamma, x_\pi) = 2m_\tau^2 \frac{1-x_\pi+\Delta}{x_\gamma} - m_\tau^4 \operatorname{Re} V[-2+x_\pi+3x_\gamma+x_\pi^2-x_\gamma^2-\Delta(x_\nu+x_\gamma)]$$

$$-\operatorname{Re} a m_\tau^4(2-x_\pi+x_\nu x_\gamma+2\Delta)\frac{1-x_\nu-\Delta}{x_\gamma},$$

$$\frac{1}{\kappa} A_2(x_\gamma, x_\pi) = -m_\tau^6 x_\gamma(1-x_\nu-\Delta)\operatorname{Re}(V+a),$$

$$\frac{1}{4\kappa} A_3(x_\gamma, x_\pi) = m_\tau^2(1-x_\nu-\Delta)\operatorname{Im}(V+a),$$

$$\frac{1}{4\kappa} A_4(x_\gamma, x_\pi) = m_\tau^2 \operatorname{Re} a(1-x_\nu-\Delta)\left(1+\frac{x_\pi-2}{x_\gamma}\right)$$

$$+m_\tau^2 \operatorname{Re} V\left(2\Delta-x_\pi\frac{1-x_\nu-\Delta}{x_\gamma}\right),$$

$$\frac{1}{4\kappa} A_5(x_\gamma, x_\pi) = 2m_\tau^2 x_\pi \operatorname{Re} a - 2m_\tau^2 x_\gamma \operatorname{Re} V.$$

Let us write the expression for $X_{\mu\nu}$ due to κ^2:

$$X_{\mu\nu}^{(2)} = -2\kappa^2 p_1\cdot p_2\left(g_{\mu\nu}+\frac{i}{k\cdot p_1}\varepsilon_{\mu\nu\alpha\beta}k_\alpha p_{1\beta}\right).$$

5. For the numerical estimates of various measured characteristics in the $\tau \to \nu+\pi+\gamma$ process we shall use the formfactors which are true for the vector-dominance model. Thus, approximating the vector formfactor by the ϱ-meson contribution only, and the axial-vector formfactor by the A_1-meson contribution, we may write:

$$m_\tau^2 V(t) = \vartheta \Pi_\varrho, \quad m_\tau^2 a(t) = a\Pi_A,$$

$$\Pi_\varrho^{-1} = 1-\frac{t}{m_\varrho^2} - i\frac{\Gamma_\varrho}{m_\varrho}\theta(t-4m_\pi^2)\sqrt{\frac{t-4m_\pi^2}{m_\varrho^2}},$$

$$\Pi_A^{-1} = 1 - \frac{t}{m_A^2} - i\frac{\Gamma_A}{m_A^2}\theta(t-9m_\pi^2)\sqrt{\frac{t-9m_\pi^2}{m_A^2}},$$

where ϑ and a are the formfactor values at $t = 0$, Γ_ϱ, m_ϱ, (Γ_A, m_A) — the width and the mass of $\varrho(A_1)$-meson. The value $\theta(t-4m_\pi^2)\sqrt{t-4m_\pi^2}$ guarantees both Im $V(t) \neq 0$ in the region of $t \geq 4m_\pi^2$ and the correct threshold behavior of the effective width of ϱ-meson. The ratio $\gamma = a/V$ is defined on the basis of the radiative pion decay data [13]: $\gamma = 0.44 \pm 0.12$ or $\gamma = -2.36 \pm 0.12$. Using the hypothesis of the isovector hadron weak current conservation (without the change of strangeness) [14] we may connect the constant γ to that of the radiative decay of neutral pions [15]. Assuming that $\tau(\pi^0) = (0.83 \pm 0.06)\ 10^{-15}$ s [16], we obtain:

$$\frac{m_\pi f_\pi |\vartheta|}{m_\tau^2} = 0.0261 \pm 0.0009.$$

Since the sign ϑ remains indefinite, two possible ways have to be used for estimating the vector formfactor: $m_\tau^2 V(t) = \pm 4.5\,\Pi_\varrho$ ($f_\pi = 130$ MeV). These ways may be distinguished by studying the $\tau \to \nu + \pi + \gamma$ decay in such kinematical conditions when the interference of a vector formfactor with the bremsstrahlung amplitude is essential.

Thus, the existing experimental data about the $\pi \to \mu + \nu + \gamma$ and $\pi^0 \to 2\gamma$ decay (together with the hypothesis of the vector-current conservation and vector dominance) result in the four variants of the electroweak formfactors in the analysis of the τ-lepton radiative decay:

$$m_\tau^2 V(t) = \pm 4.5\Pi_\varrho, \quad m_\tau^2 a(t) = 2(-10.6)\Pi_A.$$

As it is seen from Fig. 2 the effective mass spectrum of the $(\gamma\pi)$-system is sensitive to the value γ in the region of large $(\gamma\pi)$ masses, at $x_\nu \leq 0.7$, where the difference of inten-

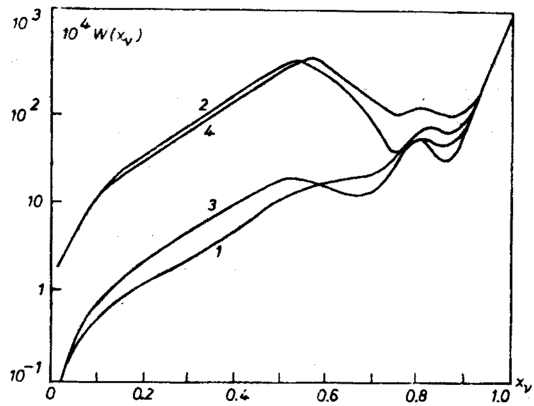

Fig. 2. The spectra of the effective masses $(\pi\gamma)$ in the $\tau \to \nu + \pi + \gamma$ decay, which are calculated for the different variants of the formfactors: (1) $\vartheta_1 = 4.5$, $a = 2.0$; (2) $\vartheta_1 = 4.5$, $a = 10.6$; (3) $\vartheta_1 = -4.5$, $a = 2.0$; (4) $\vartheta_1 = -4.5$, $a = -10.6$

sities is of two orders. The origin of this difference lies in the fact that the A_1-meson contribution prevails exactly in this kinematical region at $\gamma = -2.36$. The peak in the x_ν-distribution corresponding to the ϱ-meson production is observed for various formfactor parametrizations. It is also obvious that the ϱ-meson peak is found near the region of the intensive bremsstrahlung. Therefore in this region there must be a sensitivity to the sign of the formfactor $V(t)$ (due to the interference of the bremsstrahlung amplitude and the ϱ-production amplitude). The interference of the bremsstrahlung amplitude and the structure

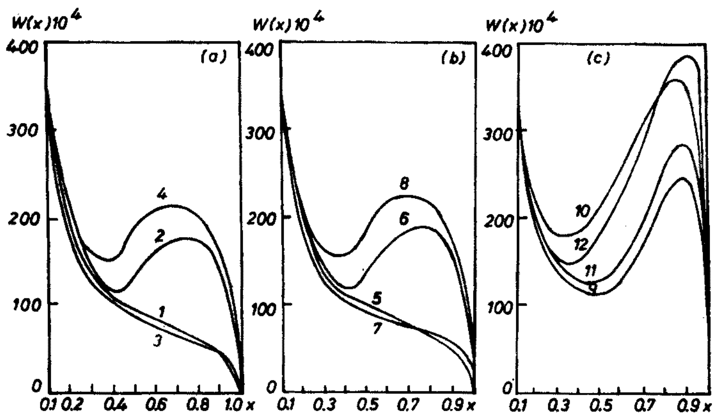

Fig. 3. The photon energy spectrum in the $\tau \to \nu+\pi+\gamma$ decay, $\Gamma_\pi w(x) = dw/dx$, $x \equiv x_\gamma$ for the different formfactors of the $\gamma W\pi$-vertex. The curves 1–4 correspond to the ρ and A_1-meson contributions with the following residues: (1) $\vartheta = 4.5$, $a = 2.0$; (2) $\vartheta = 4.5$, $a = -10.6$; (3) $\vartheta = -4.5$, $a = -2.0$; (4) $\vartheta = -4.5$, $a = 10.6$. (b) The curves 5–8 correspond to the ρ, ρ′ and A_1-meson contributions with the residues: (5) $\vartheta_1 = 5.9$, $\vartheta_2 = -1.4$, $a = 2.0$; (6) $\vartheta_1 = 5.9$, $\vartheta_2 = -1.4$, $a = -10.6$; (7) $\vartheta_1 = -5.9$, $\vartheta_2 = 1.4$, $a = 2.0$; (8) $\vartheta_1 = -5.9$, $\vartheta_2 = 1.4$, $a = 10.6$. (c) The curves 9–12 correspond to the ρ, ρ′ and A_1-meson contributions with the following residues: (9) $\vartheta_1 = 5.9$, $\vartheta_2 = -10.4$, $a = -2.0$; (10) $\vartheta_1 = 5.9$, $\vartheta_2 = -10.4$, $a = 10.6$; (11) $\vartheta_1 = -5.9$, $\vartheta_2 = 10.4$, $a = 2.0$; (12) $\vartheta_1 = -5.9$, $\vartheta_2 = 10.4$, $a = -10.6$

radiation amplitude is rather notable for $\gamma = 0.44$ and it is observable in the region of $x_\nu \leqslant 0.5$, where the decay probability for the (1) formfactor variant is twice the decay probability for the (3) formfactor variant. The relative difference between the decay probabilities of the variants (2) and (4) ($\gamma = -2.36$) is less essential (though its absolute difference is distinct).

The energy distribution of the photons produced in the $\tau \to \nu+\pi+\gamma$ decay is also sensitive to the value γ: at $\gamma = 0.44$ the monotonically decreasing spectra appear with the increase of κ, while at $\gamma = -2.36$ the photon spectra have the minimum at $x = 0.5$ and the broad maximum in the region of $x = 0.6$–0.9. In the maximum and minimum regions the photon spectra are sensitive to the sign of the vector (or axial) formfactor (Fig. 3a).

The square of the momentum transfer t may change in broad limits for the radiative decay and therefore the x and x_ν-spectra must be sensitive to a possible ϱ′-contribution [17]. Taking into account ϱ and ϱ′ mesons the vector formfactor $V(t)$ is to be described by the expression: $m_\tau^2 V(t) = \vartheta_1 \Pi_\varrho + \vartheta_2 \Pi_{\varrho'}$, $\vartheta_1 + \vartheta_2 = \vartheta$. Let us made use of the vector dominance model in order to estimate the values ϑ_1 and ϑ_2. Then, it may be written for ϑ_1:

$$\vartheta_1 m_\pi f_\pi = \frac{f(\varrho\pi\gamma)}{2\gamma_\varrho} \frac{m_\pi}{m_\varrho},$$

where $f(\varrho\pi\gamma)$ and γ_ϱ are the constants of the $\varrho \to \pi+\gamma$ and $\varrho \to e^+ + e^-$ decays. Using $\Gamma(\varrho \to \pi\gamma) = 63 \pm 7$ keV and $\Gamma(\varrho \to e^+e^-) = 6.8 \pm 0.79$ keV, we obtain $|f(\varrho\pi\gamma)| = 0.96 \pm 0.05$, $\gamma_\varrho = 2.53 \pm 0.15$. As a result we find $m_\sigma f_\pi \vartheta_1 = 0.034$ that does not coincide with the value ϑ, which has been found from the π^0 decay (using the hypothesis of the vector hadron weak current conservation). Knowing the values ϑ and ϑ_1 we may estimate the

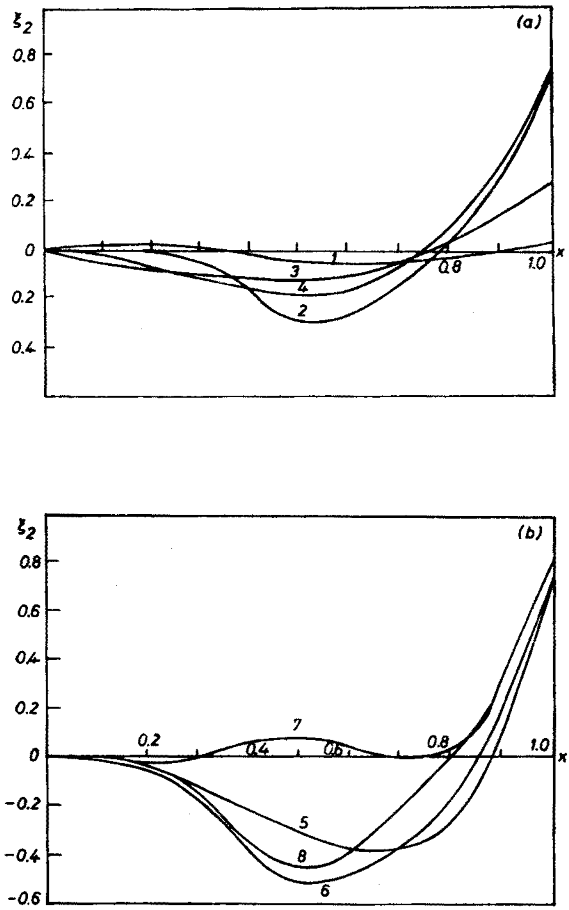

Fig. 4. The energy dependence of the circular polarization of photons (a). The curves 1–4 correspond to the ρ and A_1-meson contributions with the following residues: (1) $\vartheta = 4.5$, $a = 2.0$; (2) $\vartheta = 4.5$, $a = -10.6$; (3) $\vartheta = -4.5$, $a = -2.0$; (4) $\vartheta = -4.5$, $a = 10.6$. (b) The curves 5–8 correspond to the ρ, ρ′ and A_1-meson contributions with the following residues: (5) $\vartheta_1 = 5.9$, $\vartheta_2 = -10.4$, $a = -2.0$; (6) $\vartheta_1 = 5.9$, $\vartheta_2 = -10.4$, $a = 10.6$; (7) $\vartheta_1 = -5.9$, $\vartheta_2 = 10.4$, $a = 2.0$; (8) $\vartheta_1 = -5.9$, $\vartheta_2 = 10.4$, $a = 10.6$

value ϑ_2. And again taking into account the ϑ sign uncertainty the following expressions may be found for the vector formfactor:

$$m_\tau^2 V(t) = \pm(5.9\Pi_\varrho - 1.4\Pi_{\varrho'}), \quad m_\tau^2 V(t) = \pm(5.9\Pi_\varrho - 10.4\Pi_{\varrho'}).$$

The formfactor $V(t)$ would be more definite if we knew the width of the radiative decays $\varrho' \to \pi+\gamma$ and $\varrho' \to e^+ + e^-$. Our model gives the following products of the widths:

$$\Gamma(\varrho' \to e^+e^-)\Gamma(\varrho' \to \pi\gamma) = 4.6 \cdot 10^{-4}\ \text{MeV}^2 \quad \text{or} \quad 2.6 \cdot 10^{-2}\ \text{MeV}^2.$$

As it is seen from Fig. 3a, b the introduction of ϱ' with a small constant ($\vartheta_2 = 1.4$) does not change essentially the photon spectrum, but the variant with a large constant ($\vartheta_2 = \pm 10.4$) makes the probability of the high energy photon production rather great.

The Stokes parameters of the produced photons are also sensitive to the choice of the formfactor variants (Fig. 4).

Thus, the radiative decay $\tau \to \nu+\pi+\gamma$ may serve as an effective method not only for the measurement of the electroweak pion formfactors but also for a study of ϱ'-meson characteristics.

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