ELECTROMAGNETIC MASS DIFFERENCES OF MESONS IN A QCD POTENTIAL MODEL

By D. Y. Kim, S. N. Sinha*

Department of Physics and Astronomy, University of Regina, Regina, Canada S4S 0A2

and S. I. H. Naqvi**

TRIUMF, Vancouver, Canada V6T 2A3

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Considering the gluonic contribution to mass difference in the non-relativistic QCD potential model of Schnitzer, we calculate the electromagnetic mass differences of strange, charm, b-quark and t-quark mesons. The numerical results are found to be in agreement with the available experimental values and with the theoretical values of others. We predict $T^+(\bar{d}t)-T^0(\bar{u}t) \simeq T^{*+}-T^{*0} \simeq 3.6$ MeV.

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The electromagnetic (e-m) mass differences of mesons are important [1] in determining the production ratios of various mesons and of their decay mechanism. There has been extensive discussion [1–4] on the estimation of mass splitting of D^+ and D^0. It has been found that the simple non-relativistic models could be used to estimate the e-m mass splitting of heavy mesons. Such a model has been used by Lane and Weinberg [2]. They estimate the D^+-D^0 mass difference to be 6.7 MeV as compared to the experimental value of (4.7 ± 0.3) MeV. In their calculation, however, they ignored the gluonic and magnetic contribution to the mass difference. Considering gluonic [5] and magnetic term explicitly, Chan [3] calculated the D^+-D^0 and other hadron mass differences and found the D^+-D^0 mass difference to be 5.2 MeV, which is a closer value to the experiment. The gluonic contribution term is significant and cannot be ignored. The D^+-D^0 mass difference has also been calculated by others [4]. The B^+-B^0 mass difference has been discussed by Chan, Eichten, Singh et al., and Kim and Sinha [6–8]. Recently the evidence [9] of t-quark hadron has been found at the CERN pp collider and the mass of t-quark is set at 40 ± 10 GeV. Recently, there has been renewed interest in the e-m mass difference

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* Present address: Theoretical Physics Institute, University of Alberta.

** Present address: University of Regina.

of t-quark isodoublets ($T^0 \equiv \bar{u}t$, $T^+ \equiv \bar{d}t$ and T^{*0}, T^{*+}). This will entail calculations of the production ratios of these mesons [7, 8].

One of the main objectives of this work is to evaluate the e-m mass difference of pseudoscalar and vector mesons in a QCD potential model of Schnitzer [10]. The characteristic of this model is the explicit use of the effective coupling parameter which is a function of mass, in contrast to a fixed coupling constant in a usual phenomenological model [7, 8]. The model also provides the hyperfine splitting of mesons and the other properties of hadrons.

In the present note, we utilize the technique and analysis of Lane and Weinberg [2] a la Schnitzer to estimate the e-m mass differences of strange, charm, b-quark and t-quark mesons.

In his model, Schnitzer [10] considered a Fermi-Breit type of Hamiltonian, which contains a short range Coulomb type $1/r$ vector exchange term with a long range scalar confining force. Taking the effective coupling constant for gluon to be mass dependent and assuming the long range vector exchange to be absent, one can express S-wave hyperfine splitting for mesons as

$$\Delta E = \frac{32\pi}{9m_1 m_2} \alpha_s(M^2) \, |\psi(0)|^2 \tag{1}$$

with

$$\alpha_s(M^2) = \frac{12\pi}{(33-2n_f)} (\ln M^2/\Lambda^2)^{-1},$$

where m_1 and m_2 are the masses of quarks, $\alpha_s(M^2)$ is the effective coupling constant and $\psi(0)$ is the wave function of the particle at the origin. Taking M as mass of vector meson with $\Lambda = 500$ MeV, Schnitzer [10] calculated for $n_f = 4$ the values of α_s's as $\alpha_s(\varrho) = 1.80$, $\alpha_s(K^*) = 1.294$, $\alpha_s(D^*) = 0.545$ and $\alpha_s(F^*) = 0.524$. In our calculation, we will assume $M \equiv M_{q_1 q_2} = m_1 + m_2$ with $\Lambda = 400$ MeV. Taking the numerical values of parameters (Ref. [10]) as $m_1 = 1/2(m_u + m_d) = 313$ MeV, $m_s = 490$ MeV, $m_c = 1.6$ GeV and $m_b = 5.0$ GeV, we get for $n_f = 6$ the values of α_s as $\alpha_s(M^2_{uu}) = 1.99$, $\alpha_s(M^2_{us}) = 1.288$, $\alpha_s(M^2_{uc}) = 0.573$, $\alpha_s(M^2_{sc}) = 0.543$ and $\alpha_s(M^2_{ub}) = 0.347$. The value $\alpha_s(M^2_{ut}) = 0.194$ for $m_t = 40$ GeV (Ref. [9]). These values of α_s's are in general agreement with the values obtained from the consideration of M as a mass of vector meson with $\Lambda = 500$ MeV. The choice of $\Lambda \simeq 400$ MeV is necessary for our phenomenological calculation, which is also within the recent experimental limit of $\Lambda(310 \pm 140$ MeV) [11] and it is consistent with the values of some other calculations [11]. We calculate the hyperfine splittings of the mesons with the given experimental values of different wave functions in Ref. [12] (Table I). Considering wave function as spin-independent we have $|\psi(0)|^2_{\varrho,\pi} = 2.55-3.25 \times 10^{-3}$ (GeV)^3, $|\psi(0)|^2_{K^*,K} = 3.0-4.4 \times 10^{-3}$ (GeV)^3, $|\psi(0)|^2_{D^*,D} = 9-17 \times 10^{-3}$ (GeV)^3, $|\psi(0)|^2_{F^*,F} = 10-20 \times 10^{-3}$ (GeV)^3. The hyperfine splittings of the mesons are $(\Delta E)_{\varrho-\pi} = 577-736$ MeV (expt. 633 MeV), $(\Delta E)_{K^*-K} = 283-416$ MeV (expt. 396 MeV), $(\Delta E)_{D^*-D} = 116$–216 MeV (expt. 143 MeV) and $(\Delta E)_{F^*-F} = 78-154$ MeV (expt. [13] $139 \pm$ ± 22 MeV). The calculated values are in agreement with the values of Schnitzer [10] and with the experimental values [14] within the range of predicted values. The ranges in

TABLE I

Electro-magnetic mass differences of mesons in MeV. $(\Delta m)_{\text{hadronic}} \equiv (m_d - m_u) + (\Delta m)_{\text{gluonic}}$; Coulomb and magnetic part calculated with $\left\langle \frac{1}{r} \right\rangle_K = 108\text{–}220$ MeV with corresponding $|\psi(0)|^2$

Mass difference	$(\Delta m)_{\text{gluonic}}$	$(\Delta m)_{\text{hadronic}}$	Coulomb part	Magnetic part	$(\Delta m)_\gamma$	Total	Observed [14]
$\pi^+ - \pi^0$						4.6[a]	4.6
$K^0 - K^+$	2.35	4.86	−(0.55–0.26)	−(0.30–0.59)	−(0.85)	4.01[a]	4.01 ± 0.13
$D^+ - D^0$	0.86	3.37	0.58–1.10	0.47–0.70	1.05–1.80	4.42–5.17	4.7 ± 0.3
$B^0 - B^+$	0.29	2.80	−(0.30–0.60)	−(0.10)	−(0.40–0.70)	2.10–2.40	3.4 ± 3.6
$T^+ - T^0$	0.02	2.53	0.65–1.30	0.03	0.68–1.33	3.21–3.86	—
$D^{*+} - D^{*0}$	−0.29	2.22	0.58–1.10	−0.20	0.40–0.90	2.62–3.12	2.6 ± 1.8
$B^{*0} - B^{*+}$	−0.10	2.41	−(0.30–0.60)	0.03	−(0.27–0.57)	1.84–2.14	—
$T^{*+} - T^{*0}$	−0.01	2.5	0.65–1.30	−0.10	0.64–1.29	3.14–3.79	—

[a] input.

the values arise due to the uncertainties in the experimental values of $|\psi(0)|^2$. Note that if we take

$$|\psi(0)|^2_{K^*,K} = 4.22\times 10^{-3}\,(\text{GeV})^3,$$
$$|\psi(0)|^2_{D^*,D} = 11.15\times 10^{-3}\,(\text{GeV})^3 \tag{2}$$

then it will reproduce the experimental hyperfine splittings 396 MeV for K*-K and 143 MeV for D*-D mass differences. The values in (2) are within the given range in the experimental values of $|\psi(0)|^2$.

For the b- and t-quark sector, due to the lack of knowledge of wave functions, we will assume $|\psi(0)|^2_{F^*,F} \simeq |\psi(0)|^2_{B^*,B} \simeq |\psi(0)|^2_{T^*,T} \simeq 15\times 10^{-3}\,(\text{GeV})^3$, then the hyperfine splittings are

$$(\Delta E)_{B^*-B} = 37.0\text{ MeV},\quad (\Delta E)_{T^*-T} = 2.9\text{ MeV}. \tag{3}$$

The value of B*-B = 37 MeV is of the same order [15, 7] as that of Ono (34 MeV), Singh (40 MeV), Eichten et al. (50 MeV) and Kim et al. (41 MeV). T*-T = 2.9 MeV is of the same order but lower than that of Kim et al. (5.0 MeV).

Considering gluonic, Coulomb and magnetic interaction terms explicitly, the e-m mass differences of pseudoscalar and vector mesons are [7, 8]

$$m_{p^+} - m_{p^0} = (\Delta m)_{du} + (\Delta m)_{gluon} + \alpha e_1 e_2\left[\left\langle\frac{1}{r}\right\rangle_p + \frac{2\pi}{m_1 m_2}|\psi(0)|^2_p\right],$$
$$m_{v^+} - m_{v^0} = (\Delta m)_{du} + (\Delta m)_{gluon} + \alpha e_1 e_2\left[\left\langle\frac{1}{r}\right\rangle_v - \frac{2\pi}{3m_1 m_2}|\psi(0)|^2_v\right], \tag{4}$$

where $m_1 = 1/2\,(m_d + m_u)$ and m_2 stands for quark masses of m_s, m_c, m_b and m_t. e_i represents the fractional charge of ith quark.

For numerical estimations of mass differences from relation (4) we need the values of $m_d - m_u$ and of $\left\langle\frac{1}{r}\right\rangle$. The values of quark masses, α_s and $|\psi(0)|^2$ are presented earlier. From (4) the explicit expression for K$^+$ and K^0 mass difference is

$$K^0 - K^+ = (m_d - m_u) + \frac{8\pi\alpha_s(M^2_{us})}{3m_s}|\psi(0)|^2_K\left(\frac{1}{m_u} - \frac{1}{m_d}\right)$$
$$-\frac{\alpha}{3}\left\langle\frac{1}{r}\right\rangle_K - \frac{2\pi}{3}\frac{\alpha}{m_s m_1}|\psi(0)|^2_K. \tag{5}$$

One can derive, by combining Dashen's theorem [16] and non-relativistic atomic model of K-meson, the mass relation [2]

$$\pi^+ - \pi^0 = (K^+ - K^0)_\gamma (m_K/m_\pi). \tag{6}$$

Taking $\pi^+ - \pi^0$ mass difference (4.6 MeV) as an input one gets $(K^+ - K^0)_\gamma = 0.85$ MeV.

From (5) and (6), we have

$$(\mathrm{K}^+ - \mathrm{K}^0)_\gamma = \frac{\alpha}{3}\left\langle\frac{1}{r}\right\rangle_{\mathrm{K}} + \frac{2\pi}{3}\frac{\alpha}{m_s m_1}|\psi(0)|^2_{\mathrm{K}} = 0.85 \text{ MeV}. \tag{7}$$

Substituting the numerical value of (7) and taking $\mathrm{K}^+ - \mathrm{K}^0$ mass difference (4.01 ± 0.13 MeV) as an input, we get from (5) that $m_d - m_u = 2.51 \pm 0.07$ MeV. This value of $m_d - m_u$ is consistent with the value of others [6–8]. A question of application of first order perturbation in strange quark sector might arise as $\alpha_s(M^2_{us}) \simeq 1.3$, a value larger than unity. This is the limitation of Schnitzer's model in the low mass quark sectors. However, as it estimates correct value in hyperfine splitting and a consistent value of $m_d - m_u$, we will use the value of $m_d - m_u$ (2.51 ± 0.07 MeV) from a phenomenological standpoint. On the other hand, for charm, b- and t-quark sectors no such limitation occurs as α_s is less than one [17].

The numerical value of $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K}}$ can be obtained from (7) by substituting the numerical value of $|\psi(0)|^2_{\mathrm{K^*,K}} = (3-4.4)\times 10^{-3}$ (GeV)^3. Then we have $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K^*,K}} = 160-220$ MeV, a range in numerical value rather than a fixed one due to the fluctuation in the experimental value of $|\psi(0)|^2$. For $|\psi(0)|^2_{\mathrm{K^*,K}} = 4.22\times 10^{-3}$ (GeV)^3 as in (2), the $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K^*,K}} = 171$ MeV. The value $|\psi(0)|^2_{\mathrm{K^*,K}}$ can be calculated from the theoretical relation [18] $|\psi(0)|^2 = \frac{\mu a}{2\pi}$ with $a = 0.194$ (GeV)^2. Then $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K}} = 108$ MeV. Neglecting magnetic interaction, Lane and Weinberg obtained $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K^*,K}} = 350$ MeV from (7). But if one takes the magnetic part into account then from (7) the value of $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K^*,K}}$ should be smaller than 350 MeV. We found $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K^*,K}} = 108-220$ MeV [8]. Although, we could not fix a particular value of $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K^*,K}}$, it seems to be somewhere between $108-220$ MeV, if one exploits the fact that the values of $|\psi(0)|^2_{\mathrm{K^*,K}}$ can be obtained from experimental evaluation and from the theoretical calculations [19]. The value of $\left\langle\frac{1}{r}\right\rangle_{\mathrm{K^*,K}} = 108-220$ MeV is also consistent with the value $C_{\mathrm{M}} \equiv \left\langle\frac{1}{r}\right\rangle = 205 \pm 110$ MeV, obtained by Chan [6] from a different type of analysis. The value of the matrix elements $\left\langle\frac{1}{r}\right\rangle$ for different mesons is calculated [2] by using $\left\langle\frac{1}{r}\right\rangle_{\mathrm{m}} = \left(\frac{\mu_{\mathrm{m}}}{\mu_{\mathrm{K}}}\right)^{1/3}\left\langle\frac{1}{r}\right\rangle_{\mathrm{K}}$.

At this stage we know the numerical values of all parameters and we can numerically estimate the e-m mass differences of mesons. The explicit expressions for e-m mass differences of all pseudoscalar and vector mesons can be written like relation (5) from relation (4). The mass differences are presented in Table I. In our estimations of e-m mass differences of b- and t-quark isodoublets we made an approximation in wave functions due to lack of knowledge of $|\psi(0)|^2_{B,T}$. However this approximation affects mainly the gluonic parts of these mesons, which are very small for B^0-B^+ and T^+-T^0 mesons mass differences. Hence the predictions of mass differences will not change very much and at best they change by 0.1 MeV for B^+-B^0 mesons and by 0.05 MeV for T^+-T^0 meson. Furthermore, we have not discussed, isospin-dependence of $|\psi(0)|^2$ and α_s. Isospin-dependence might be more prominent in a light meson, where the gluonic contributions are large, compared to heavy mesons. We have checked that for $m_d-m_u \simeq 2.51$ MeV the values of $|\psi(0)|^2_{K^*,K}$ and $\alpha_s(M^2_{us})$ are changed very little and the product $\alpha_s(M^2_{us})\ |\psi(0)|^2_{K^*,K}$ changes by only 0.05%. As we see by using relation: $|\psi(0)|^2 = \dfrac{\mu a}{2\pi}$ with $m_d-m_u = 2.51$ MeV, we have $|\psi(0)|^2_{us} = 5.90\times 10^{-3}$, $|\psi(0)|^2_{ds} = 5.928\times 10^{-3}$, $\alpha_s(M^2_{us}) = 1.2874$, $\alpha_s(M^2_{ds}) = 1.2816$ and $\alpha_s(M^2_{us})|\psi(0)|^2_{us} = 7.595\times 10^{-3}$, $\alpha_s(M^2_{ds})|\psi(0)|^2_{ds} = 7.598\times 10^{-3}$, in their respective units. Thus, the isospin-independent approximation of $\psi(0)$ and α_s is a fair one.

A calculation of e-m mass differences of mesons was carried out by Barik and Jena [20] in a similar model from a different point of view, considering an argument of possible double counting of the Coulomb part. However, their numerical values are different from those presented in Table I. They used experimental mass differences of K^0-K^+ (3.99 MeV), D^+-D^0 (5.0 MeV), $K^{*0}-K^{*+}$ (4.1 MeV) and $D^{*+}-D^{*0}$ (2.6 MeV) as input and predicted the same mass differences K^0-K^+ (5.62 MeV), D^+-D^0 (4.91 MeV), $K^{*0}-K^{*+}$ (3.48 MeV) and $D^{*+}-D^{*0}$ (2.66 MeV) by choosing the free parameter $m_d-m_u = 1.9$ MeV. The deviations of input and output values in their mass differences calculation motivated us to analyze the same problem in the model of Schnitzer, using the techniques of Lane and Weinberg. It is clear from Table I that the results of mass differences are in good agreement with the observed values and consistent with the results of others [6–8], with an overall change in the numerical values due to the gluonic effect. This deviation was to be expected when comparing Schnitzer's model with that of a parametrized potential model.

In conclusion, in this report we have analyzed the e-m mass differences of strange, charm, b- and t-quark mesons in a non-relativistic potential model and have found that the numerical values are in general agreement with available experimental values. However, for $K^{*0}-K^{*+}$ meson mass difference we found 1.3 MeV much lower than the observed value. This discrepancy is discussed by DeRujula, Georgi and Glashow [5] and by others [6, 8]. Our B^0-B^+ mass difference is relatively low in value compared to the QCD-sum rule calculations of 5.8 MeV by Mathur et al. [21]. We see that their $(\Delta m)^{\text{hadronic}}_B = 5.8$ MeV is large compared to our $(\Delta m)^{\text{hadronic}}_B = 2.8$ MeV. Furthermore, Mathur et al. considered that the photon part is a relativistic one for heavy mesons like D and B. The photon contributions for D and B mesons in relativistic consideration are 0.35 MeV and 0.12 MeV respectively, a smaller value compared to non-relativistic calculations (see Table I) a fact which was also pointed out by them. However, it is of the order of the value obtained by Singh et al. [6], Chan [6] and of others [7, 8].

For this difference in claim we would also like to point out that future experiments will provide the tests for $B^+ - B^0$ mass difference predictions. Our $T^+ - T^0$ and $T^{*+} - T^{*0}$ mass differences are about 3.6 MeV, which is one MeV smaller than our earlier estimation [7]. In our earlier estimation we considered $\left\langle \frac{1}{r} \right\rangle_K = 350$ MeV, a larger value compared to that of the present calculation. However, it is in agreement with another recent calculation in a parametrized potential model [8]. Finally, in this analysis with the present model, we have considered the different values of quantities $\alpha_s(M^2)$, $|\psi(0)|^2$, and $\left\langle \frac{1}{r} \right\rangle$ for different mesons. We calculated their contributions to e-m mass differences. However it is clear from the results that these considerations will not considerably change the prediction of numerical values of mass differences and that they will remain close to those of parametrized potential model calculations. We predict $T^+ - T^0$ and $T^{*+} - T^{*0}$ mass differences are 3.6 MeV for $m_t = 40$ GeV. However we checked that for the variation of m_t, the $T^+ - T^0$ and $T^{*+} - T^{*0}$ mass differences will vary very little and for m_t from 30 to 50 GeV [9] the mass differences will change at best 0.01 MeV. In other words, the contribution to the gluonic part is getting an asymptotic value in case of a high quark mass [7].

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