THE MODIFIED VERSION OF THE CENTRE-OF-MASS CORRECTION TO THE BAG MODEL*

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We propose the improvement of the recently considered version of the centre-of-mass correction to the bag model. We identify a nucleon bag with a physical nucleon confined in an external fictitious spherical well potential with an additional external fictitious pressure characterized by the parameter b. The introduction of such a pressure restores the conservation of the canonical energy-momentum tensor, which was lost in the former model. We propose several methods to determine the numerical value of b. We calculate the Roper resonance mass, as well as static electroweak parameters of a nucleon with centre-of-mass corrections taken into account.

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The MIT bag model [1] exists since more than the decade, being the useful tool for investigations of the hadron interior. During this time the model evolved substantially: the starting point was the identification of a nucleon bag with this particle at rest. The one has identified the bag with the wave packet of physical particles [2], in order to get rid of the spurious centre-of-mass motion. Next, the new method of removing the centre-of-mass effect from physical quantities was put forward [3], where one treats the hadron bag as a physical hadron bound in a certain fictitious potential. For numerical calculations, within the bag model framework, the spherical infinite well potential was used [3, 4], and the theoretical predictions for Roper resonance mass and static electroweak parameters of a nucleon were presented with a good agreement with experimental data. However, in

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such approach the energy momentum tensor $T^{\mu\nu}$ conservation at the boundary of the well is lost, as it was the case in the pre-bag model of Bogoljubov [5]. In the MIT bag model one avoids the energy-momentum flow through the surface of the bag by adding the external pressure B of such strength that it balances the internal pressure of the Dirac fields. We propose to restore the $T^{\mu\nu}$ conservation in Szymacha model [3] by introducing the fictitious external pressure b, which balances the pressure of the nucleon field on the boundary of the well. In our model the nucleon bag (i.e. three independently moving quarks in a spherical cavity of radius R_B with the external pressure B acting on it) corresponds to the physical nucleon confined in the fictitious spherical well potential (which radius is R_W), being also pressed from outside by the fictitious pressure, characterized by the parameter b. Such a scheme has a property that it gives a consistent result for a one quark bag, provided that $B = b$. In the model proposed in [3] the one quark in the bag corresponds to one quark in a spherical well, which is a contradictory result. One can argue that we have no physical particles containing one quark only, however one should have no contradiction in such a limit. We do not extend this argument for the three quarks bag, but we expect that the numerical values of both quantities will not be far from each other.

Let us consider a simple bag model of a nucleon, which energy contains the kinetic energy of massless quarks and volume energy terms only. We shall consider in this paper the nucleon and its first radial excitation, i.e. Roper resonance, so we do not include other possible terms. We have:

$$E_{\mathrm{Bag}} = 3x_0/R_B + 4\pi R_B^3 B/3, \tag{1}$$

where $x_0 = 2.043...$ is the first positive root of the equation:

$$\mathrm{tg}\, x = x/(1-x). \tag{2}$$

For the Roper resonance bag we get:

$$E^*_{\mathrm{Bag}} = (2x_0 + x_0^*)/R_B^* + 4\pi R_B^{*3} B/3, \tag{3}$$

where $x_0^* = 5.396...$ is the second positive solution of (2). We have also usual bag model equalities:

$$\partial E_{\mathrm{Bag}}/\partial R_B = 0 \tag{4a}$$

and

$$\partial E^*_{\mathrm{Bag}}/\partial R^*_B = 0, \tag{4b}$$

which guarantee the canonical energy-momentum tensor conservation on the surface of a bag. For the energy of a nucleon confined in a spherical cavity of radius R_W we have:

$$E_N = \Omega/R_W + 4\pi R_W^3 b/3, \tag{5}$$

where $\Omega = (X^2+Y^2)^{1/2}$ ($Y = M_P R_W$, M_P stands for proton mass) and X is the solution of an equation:

$$\mathrm{tg}\, X = X/(1-Y-\Omega). \tag{6}$$

For the Roper resonance we get accordingly:

$$E_R = \Omega^*/R_W^* + 4\pi R_W^{*3} b/3, \tag{7a}$$

and

$$\mathrm{tg}\, X^* = X^*/(1 - Y^* - \Omega^*), \tag{7b}$$

where $\Omega^* = (X^{*2} + Y^{*2})^{1/2}$, $Y^* = M_R R_W^*$ (M_R stands for Roper resonance mass).

Due to our assumption we have additional two equations:

$$\partial E_N / \partial R_W = 0 \tag{8a}$$

and

$$\partial E_R / \partial R_W^* = 0, \tag{8b}$$

which guarantee the pressure balance on the surface of the well and hence the $T^{\mu\nu}$ tensor conservation there. These equalities are new ingredients in comparison with [3], so we get different numerical predictions.

If we would like to have the mass of a Roper resonance as a prediction we have to look for one more equation, since we have seven unknown parameters in our model: B, b, R_B, R_B^*, R_W, R_W^* M_R and only six equations: $E_{Bag} = E_N$, $E_{Bag}^* = E_R$ as well as (4a, b) and (8a, b).

If we put M_R equal to its experimental value $M_R^{exp} = 1.44 \pm 0.04$ GeV [6] we get:

$$B^{1/4} = 121 \pm 7 \text{ MeV}, \qquad b = 113^{+21}_{-37} \text{ MeV},$$

which gives the 7% difference. We also have $R_B = 6.9\ \text{GeV}^{-1}$, $R_B^* = 7.7\ \text{GeV}^{-1}$ and $R_W = 5.1\ \text{GeV}^{-1}$, $R_W^* = 4.8\ \text{GeV}^{-1}$ in this case. The numerical values of predictions for electroweak parameters of a proton (p) and neutron (n) we present in Table I (they are marked by model I there) together with results which we obtain from other assumptions. The latest are as follows: first we equate the values of B and b, then we get the figures assuming that the theoretical predictions for proton magnetic moment or the sum of electromagnetic radii of proton and neutron are equal to their experimental values (i.e. $\mu_p = 2.793$ nuclear magnetons, $\langle r_p^2 \rangle_{em} + \langle r_n^2 \rangle_{em} = (3.88 \pm 0.06\ \text{GeV}^{-1})^2$). These approaches are named in Table I model II, III and IV, accordingly.

In case IV we get $\langle r_n^2 \rangle_{em} = -(0.57\ \text{GeV}^{-1})^2$, which should be compared to $\langle r_n^2 \rangle_{em}^{exp} = -(1.73 \pm 0.02\ \text{GeV}^{-1})^2$ [7]. From this table one sees that the agreement between theoretical predictions and experimental figures is quite good, except for the model IV. In all cases we cannot get the correct value for $\langle r_n^2 \rangle_{em}$, which is too small in comparison wiht its experimental value, however we get the correct sign for this quantity. In our model, as it is the case in [3], when we take the limit in which nucleon is pointlike (i.e. $R_B \to 0$, hence $R_W \to 0$), we get the Dirac values for magnetic moments of proton and neutron, which is a desired property for all composite models. There is a lack of this property in the MIT bag model without [1] centre-of-mass corrections.

We can conclude from the above considerations that the restoration of the $T^{\mu\nu}$ tensor conservation does not spoil good phenomenological predictions of [3], being more satisfactory from the theoretical point of view. Our prescription, however, introduces a new parameter b (this is a price we pay for restoration of canonical energy-momentum tensor

TABLE I

	Experiment	Model I	Model II	Model III	Model IV
$B^{1/4}$(MeV)	—	121	125	127	136
$b^{1/4}$(MeV)	—	113	125	131	156
R_B(GeV^{-1})	—	6.9	6.7	6.6	6.2
M_R (GeV)	1.44 ± 0.04	1.44 (input)	1.46	1.48	1.52
μ_p (n.m.)	2.793	2.87	2.86	2.79 (input)	2.69
μ_n (n.m.)	-1.913	-1.82	-1.77	-1.75	-1.65
g_A	1.254 ± 0.006	1.18	1.19	1.20	1.22
$\langle r_p^2 \rangle^{1/2}$ (GeV^{-1})	4.24 ± 0.06	4.10	4.09	4.06	3.94

All experimental data are taken from [6], except of the value of electromagnetic radius of proton [7].

conservation), a fictitious pressure which physical interpretation is far from being understood.

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