ON THE PHYSICAL MEANING OF THE GALILEAN SPACE-TIME COORDINATES

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An operational definition of the Galilean space-time coordinates is discussed. It is shown that the fifth coordinate has a deep physical meaning.

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1. *Introduction*

The space-time coordinates, like any physical quantity, should possess their own operational definitions. This evident fact has found its realization only in the case of relativistic physics, where such operational definitions were given by Einstein in 1905 [1]. In the case of non-relativistic or Galilean physics it is usually assumed that the space coordinates are provided by sticks and that there is no need to synchronize the clocks. A little thought shows, however, that sticks cannot provide any closed operational definition of space coordinates because in order to read the indications of the ends of the sticks we always need light signals, and they are not present in the Galilean world. Similarly, as we shall show below, it is not true that the non-relativistic clocks do not need synchronization.

The aim of the present paper is to discuss the operational meaning of the Galilean space-time coordinates. We shall closely follow the Einsteinian way of defining relativistic space-time coordinates and therefore we start our considerations by recalling briefly its main points.

According to Einstein [1], a coordinate frame is defined as an arrangement of synchronized clocks which represent the elementary events. The clocks are mutually synchronized by the exchange of light signals executed by an arbitrary observer equipped with a clock and instruments which emit and detect light pulses. If T_1 is the reading of the observer's clock when the instrument emits a light pulse towards a given tested clock and T_2 is the reading of the same observer's clock when the light pulse after being reflected at the posi-

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tion of the tested clock returns to the instrument, then at the moment of reflection of the light pulse the tested clock should show the time T which satisfies the synchronization condition

$$T - T_1 = T_2 - T. \tag{1.1}$$

It is, of course, assumed that the light pulse received at the tested clock is immediately reflected in the direction from which it came.

In addition to the time T, the diatance to the tested clock is defined as being equal to

$$X = \frac{c(T_2 - T_1)}{2}, \tag{1.2}$$

where c is an universal constant equal to the velocity of light in vacuum. The numbers T and X together with the angles of the direction of the emitted light pulse provide the space-time coordinates for all elementary events. Since the readings T_1 and T_2 depend on the observer, the space-time coordinates provided by (1.1) and (1.2) are specific to the given observer. A different observer will use other coordinates, which according to the Einstein relativity principle are related by the Poincaré transformations.

In the case when the Einstein relativity principle is replaced by the corresponding Galilean one, the procedure needs some modification. The main reason for such a modification comes from the lack of universal light signals in the Galilean world. Any signal used to synchronize clocks must move with a velocity which depends on the reference system and the problem of the operational meaning of the space-time coordinates used in non-relativistic physics needs clarification. A satisfactory solution of this problem is necessary for the closeness of the Galilean physics.

2. *General properties of coordinate system*

Before going into details of the Galilean space-time coordinates, we shall define the general notion of space-time manifolds. Generalizing the Einsteinian way of defining the space-time coordinates we shall say that the notion of space-time is specified provided we know:

1) the class of observers,
2) the class of elementary events,
3) the class of signals used to communicate between the observers and elementary events,
4) the interaction of the signals used with the elementary events.

In the case of Einstein, the class of observers coincides with the class of elementary events and these consist of clock with light emitters, detectors and reflecting mirrors. As communication signals between the events and observers the relativistic physics uses light pulses with their universal velocity of travelling and with the laws of geometric optics to describe the interaction of signals with the elementary events.

It is clear that this Einsteinian realization of the general requirements in items 1 to 4 determines the classical character of the notion of space-time manifolds. In our discussion of Galilean space-time coordinates we shall still stay within this classical framework. But we shall also see a way of passing to the quantum notion of space-time, which in the world of elementary particles is more adequate than the commonly used classical notion.

3. *The Galilean space-time coordinates*

It is clear that in the case of the Galilean world we cannot use the Einsteinian operational way of defining the space-time coordinates. As we have already mentioned, this is connected with the difference in the classes of signals used in the Einstein and Galilean cases. As a consequence we must modify the synchronization condition (1.1) The only possible generalization of this condition which has a clear intuitive meaning is of the form

$$T-T_1 = A(T_2-T), \tag{3.1}$$

where A is some yet unknown constant.

The Einsteinian condition (1.1) is a particular case of our condition (3.1) corresponding to the choice $A = 1$. Physically, condition (3.1) means that the time of the signal's travel from the observer to the given elementary event is in general not equal to the time of the return travel. As a weaker synchronization condition for all clocks we adopt the proportionality of these two times.

In principle, we may have two possible cases. In the first case the proportionality coefficient A does not depend on the observer chosen while in the second case it depends on the observer. In order to distinguish these two cases let us first define the distance from the observer to the event through the formula

$$X = B(T_2-T_1), \tag{3.2}$$

where again B is a constant which may or may not depend on the observer. The velocity of the signal which travels uniformly from the observer to the given tested event is equal to

$$c_1 = B\frac{1+A}{A}, \tag{3.3}$$

while the velocity of the returning signal is given by

$$c_2 = B(1+A). \tag{3.4}$$

From (3.3) and (3.4) we get

$$A = \frac{c_2}{c_1} \quad \text{and} \quad B = \frac{c_1 c_2}{c_1+c_2}. \tag{3.5}$$

In the case when the constant A does not depend on the observer the ratio $\frac{c_2}{c_1}$ therefore does not depend on the observer, either. But the only signals for which such ratio may be constant in Nature are precisely the light signals and this case thus implies $A = 1$. We see therefore that apart from the Einsteinian case the constants which participate in the construction of space-time coordinates always depend on the observer who performs the construction. This is, in particular, true for the Galilean space-time coordinates.

Using all the preceding formulas we get

$$T = \frac{c_1 T_1 + c_2 T_2}{c_1 + c_2}, \tag{3.6}$$

$$X = \frac{c_1 c_2}{c_1 + c_2}(T_2 - T_1) \tag{3.7}$$

as our operational definitions for the space and time coordinates. Clearly in the case of $c_1 = c_2$ the formulas (3.6) and (3.7) reduce to the corresponding formulas of the Einsteinian case.

In spite of the fact that (3.6) and (3.7) define the space-time coordinates in terms of exactly the same data as it happens in the Einstein case, there is a fundamental difference between our Galilean case and the Einsteinian one. The difference lies in the fact that the constants c_1 and c_2 in our case depend on the observer. More exactly, if a second observer moves uniformly with a velocity v with respect to the observer who uses the constants c_1 and c_2, then in the Galilean case the second observer must use the constants

$$c_1' = c_1 - v$$

and

$$c_2' = c_2 + v. \tag{3.8}$$

Similarly, if the first observer emits the signal at T_1 and receives it back at T_2 then the absolute character of the Galilean time implies that the second observer must emit the signal at

$$T_1' = \frac{c_1}{c_1 - v} T_1 \tag{3.9}$$

and receives it back at

$$T_2' = \frac{c_2}{c_2 + v} T_2 \tag{3.10}$$

provided that at $T = 0$ both observers were at the same position. Even if we have manipulated our experimental equipment in such a way that $c_1 = c_2$, we see from (3.8) that this condition may be satisfied only for one observer. But from (3.8) it follows that

$$c_1' + c_2' = c_1 + c_2 \tag{3.11}$$

and this invariant is a reminiscence of the invariant light velocity c of the relativistic case.

Now we are ready to discuss the essential point of difference between the Galilean and Einsteinian cases. The two cases are exactly the same in spirit but differ in completeness. The Einsteinian case needs only one fundamental constant c while the Galilean case must use two relative constants c_1 and c_2. These two constants form the invariant (3.11) but they may be different for different signals. Each Galilean observer therefore needs one additional control parameter in order to know the individual velocities c_1 and c_2. Otherwise he cannot exclude the possibility that for some events he used different c_1 and c_2 than for others. Without the additional control parameter any Galilean coordination will be non-unique and may lead to confusion in the space-time ordering of events. This is a highly undesirable situation and in order to avoid it we must admit the role of the control parameter as a tool which organizes our space-time orientation.

There is, of course, the question how to choose the needed control parameter. In order to answer this question let us observe that in addition to (3.11) we have two other invariants at our disposal

$$\alpha_1 = \frac{c_1 T_1}{c_1+c_2} \tag{3.12}$$

and

$$\alpha_2 = \frac{c_2 T_2}{c_1+c_2}. \tag{3.13}$$

In terms of these invariants the formulas (3.6) and (3.7) take the form

$$T = \alpha_1+\alpha_2 \tag{3.14}$$

$$X = c_1\alpha_2 - c_2\alpha_1.$$

From these formulas and (3.11) we see that we have already exploited all independent linear combinations formed from c_1 and c_2 with invariant coefficients. Our control parameter should therefore be at least a quadratic function of c_1 and c_2. The most convenient combination is the following

$$\theta = \tfrac{1}{2}(c_1^2\alpha_2+c_2^2\alpha_1), \tag{3.15}$$

which under the Galilean passage to the moving observer transforms according to the rule

$$\theta \to \theta' = \theta - vX + \tfrac{1}{2}v^2 T \tag{3.16}$$

familiar from the one-parameter central extension of the Galilei group.

The control parameter θ for each elementary event is calculated exactly from the same experimental data as the time variable T and the space distance X. It should therefore be quite clear that this variable has the same right to be called a space-time coordinate. The only difference between θ on the one hand and T and X on the other lies in their roles in our modelling of space-time orientation. The variable T orders the time evolution of events, the variable X determines the distance from the observer to the event and the variable θ says what signals were used to form the variables T and X. In this way we come to the slightly unexpected result that the Galilean space-time is not four-dimensional

(T, X + angles) as the relativistic models say but five-dimensional (T, X + angles and θ). Of course, the variable θ has been used in many Galilean theories but from now on it receives a new status and a clear operational meaning as a variable as indispensable as the remaining space-time variables.

4. *The quantum models of space-time*

Until now, we have restricted our considerations to classical physics. Now we would like to make a small step towards quantum physics. By this we mean the situation in which we replace the class of events represented previously by classical clocks with classical mirrors by the set of elementary particles. Correspondingly, we must also change the class of signals used for communication and the description of the interaction of these signals with the elementary particles. The signals by themselves should be elementary particles in order to minimize the influence of the signals on the tested elementary events. Consequently, we must admit a realistic picture of the interaction of signals with the events and this definitely excludes the classical models of space-time. The signals behave in the quantum-mechanical way and this means that it makes no sense to speak either of their velocities or of a specific time of reflection at the tested event. The complete quantum theoretical description then need a fundamental revision of the classical notion of localization and so on.

In order to retain as much as possible of the classical models of space-time we adopt an intermediate point of view which we have called a small step toward quantum physics. The merit of this step consists in retaining from the classical notion the assumption that for each elementary event there exists a number T which may still be treated as the time of reflection of the signal received by the event. Since we measure only T_1 and T_2 experimentally, we do not enter into any contradiction. Only in the case of classical physics may the number T really be independently measured and we cannot speculate in the way presented here. In the case of microparticles the number T is a number obtained from the formula (3.6) and there is no way of putting a classical clock on the microparticle. But we know that in the microworld the interaction of any elementary particle used as a communication signal with the elementary particle to which we would like to ascribe a position is such that for identically prepared tested elementary particles we can get different values of the time T_2 for the same time T_1. We have no way of knowing the mechanism of this process a priori, because that is the task of the quantum theory to be constructed on the basis of the model of space-time In order to have a logical possibility to ascribe the same value of T to identically prepared (in the sense of location in space) elementary particles we must admit that in the synchronization condition (3.1) the coefficient A for each observer is not a constant but a variable which has a range of values. The uncertainty in the value of A is the reflection of the physical uncertainty principle which makes it impossible to measure the exact value of the position of quantum objects. If A and consequently B, has no definite value then also the constants c_1 and c_2 become dispersed and cannot be treated as velocities of the signals. They are now the quantities which serve to fix the space-time coordnates. Since the quantities c_1 and c_2 may have different values for identically prepared events the control parameter may also be different for events localized at the same position. In this way the variable θ takes the role of a variable responsible for the quantum character of events. In the classical world all events lie on the plane

$$\theta-\frac{c_1c_2}{2}T+\frac{c_1-c_2}{2}X=0 \tag{4.1}$$

because the quantities c_1 and c_2 have definite values. In the quantum case the events are represented not on a plane in the five-dimensional space-time manifold but are spread over the whole five-dimensional manifold. This is a very important point for the whole quantum physics. In our approach the quantum phenomena are connected with the fluctuation in the variable θ which for the case of classical physics had only an auxiliary character. This lays the foundation for the point of view advocated in Refs [2] and [3], where it was shown that by ascribing a wave function defined over the five-dimensional Galilean space-time to any object, we may formulate in a consistent way a Galilean gauge theory and find a formulation of wave mechanics which, from the methodological point of view, is as general as the Maxwell macroscopic electrodynamics is for all electromagnetic phenomena. In particular, the new formulation of wave mechanics does not use the Planck constant as a primary constant defining the quantum character of phenomena and allows us to introduce more general quantum materials.

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