DYNAMICAL CONSTRAINTS VERSUS HAMILTONIAN SYMMETRY

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In the paper we consider the relationship between supersymmetric transformation of Hamiltonian and its dynamical constraints for some models of supersymmetric field theory. It is found that requirement of invariance of canonical Hamiltonian under global supersymmetry generates dynamical constraints of the model. For our examples, it is an alternative way to obtain the constraints (besides the Dirac-Bergmann algorithm). On the other hand, we have showed the invariance of these models on the surface of dynamical constraints.

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1. Introduction

Supersymmetric Lagrangians of field theory are singular. The occurrence of constraints results, among others, from the existence of fermionic fields in representations of supersymmetry. It is possible to quantize the Hamiltonians of supersymmetric models with the use of Dirac-Bergmann method (1):

— when passing to phase space, we get the primary constraints,

— secondary constraints will be calculated from Poisson brackets of primary constraints and of primary Hamiltonian,

— we reckon Dirac brackets in relation to the first and second class constraints and pass to quantum Dirac brackets.

Another method of quantization of models with constraints is the Fradkin-Vilkovisky method of functional quantization (2).

Both methods of quantization require knowledge of the primary and secondary constraints. The examination of several supersymmetric models revealed the existence of a relation between the properties of Hamiltonian symmetry and the form of constraints. We start with a canonical Hamiltonian $H = H(\varphi_i, \phi_j, \pi_k)$ with density $\mathscr{H} = \dot{\phi}_i\pi_i - \mathscr{L}$ (where

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$\mathscr{L}$ — Lagrangian density of a certain action invariant under global supersymmetry). At first, we treat all fields, velocities of fields and momenta as independent variables. Then, we investigate supersymmetric variation of Hamiltonian density (su-sy transformations of fields and velocities are known). The requirement of invariance of Hamiltonian under supersymmetry implies condition $\delta\mathscr{H} \approx$ 3s-div and the same relations between fields, velocities of fields, momenta and variations of momenta. We have calculated them step by step with an assumption that relations obtained in each stage are preserved under su-sy transformation. It appears that these relations coincide with the dynamical constraints of the model, our assumption being now a preserving of dynamical surface with respect to transformations of supersymmetry. In Chapters 2 and 3, two models are dealt with, namely: a chiral model and a supersymmetric extension of free electromagnetic model. The dynamical constraints of these models (obtained in the first stage of Dirac-Bergmann quantization) are generated by the requirement of Hamiltonian invariance under transformation of global supersymmetry.

Chapter 4 contains the analysis of this relation for the s-QED model. This model is invariant under su-sy transformations on-shell. Hence, the canonical Hamiltonian has a non-zero variation on the surface of dynamical constraints. Generation of dynamical constraints takes place here on the assumption that the variation of canonical Hamiltonian density is independent of the field velocities and accelerations. Hamiltonian of chiral model in superfield formulation is examined in Chapter 5.

2. *Supersymmetric Hamiltonian of chiral model*

We shall begin with a simple example, namely, with a massive chiral Lagrangian with interaction (3). The chiral superfield contains scalar fields A and F, pseudoscalar ones B and G and a Majorana spinor ψ_α. The Lagrangian density is the following:

$$\mathscr{L} = -\tfrac{1}{2}\,\partial_\mu A\partial^\mu A-\tfrac{1}{2}\,\partial_\mu B\partial^\mu B-\frac{i}{2}\,\bar{\psi}\bar{\partial}\psi+\tfrac{1}{2}F^2+\tfrac{1}{2}G^2+m\left(AF+BG-\frac{i}{2}\,\bar{\psi}\psi\right)$$

$$+gF(A^2-B^2)+2gGAB-ig\bar{\psi}(A-\gamma_5 B)\psi. \tag{2.1}$$

The action obtained from this density is invariant under transformation of global supersymmetry, i.e. variation of density (2.1) is a total divergence. Being acquainted with the transformations of component fields (see Appendix A) we can calculate this variation:

$$\delta\mathscr{L} = -\frac{i}{2}\,\bar{\zeta}\gamma^0\gamma^\mu\partial_0(\partial_\mu A+\gamma_5\partial_\mu B)\psi+\frac{i}{2}\,\bar{\zeta}\gamma^0\partial_0(F-\gamma_5 G)\psi$$

$$+im\bar{\zeta}\gamma^0\partial_0(A-\gamma_5 B)\psi+ig\bar{\zeta}\gamma^0\partial_0\psi(A^2-B^2)-2ig\bar{\zeta}\gamma^0\gamma_5\partial_0\psi AB+\text{3s-div}. \tag{2.2}$$

Now, the transformation of a canonical Hamiltonian density is considered:

$$\mathscr{H} = (\partial_0\varphi_i)\pi_i-\mathscr{L}, \tag{2.3}$$

where

$$\varphi_i = (A, B, \psi_\alpha, F, G).$$

We shall get the variation of Hamiltonian density in the form:

$$\delta\mathscr{H} = (\partial_0 F)\delta\pi_F + (\partial_0 G)\delta\pi_G + (\partial_0 A)\delta\pi_A + (\partial_0 B)\delta\pi_B + (\partial_0\psi)\delta\pi$$
$$+ i\bar{\zeta}(\pi_F - \gamma_5\pi_G)\partial_0\bar{\partial}\psi + i\bar{\zeta}(\pi_A + \gamma_5\pi_B)\partial_0\psi + (\partial_0\partial_\mu A)\zeta\gamma^\mu\pi$$
$$-(\partial_0\partial_\mu B)\zeta\gamma_5\gamma^\mu\pi + (\partial_0 F)\zeta\pi + (\partial_0 G)\zeta\gamma_5\pi - \delta\mathscr{L}. \tag{2.4}$$

After integration with respect to the spatial variables, the Hamiltonian density (2.3) gives the Hamiltonian $H = \int d^3x\mathscr{H}(x)$. It can be inferred from the form of density variation (2.4) that the transformation of supersymmetry gives a non-vanishing variation of Hamiltonian. Assumption $\delta H = 0$ implies appearance of the constraints on the conjugate momenta and the fields. We shall get an explicit expressions by assuming $\delta\mathscr{H}$ = 3s-div.

Now, we shall rewrite the expression for variation of density as follows:

$$\delta\mathscr{H} = (\partial_0\varphi_i)X_i + (\partial_0^2\varphi_j)Y_j + 3\text{s-div}, \tag{2.5}$$

where

$$Y_\psi = -i\zeta\gamma^0\pi_F - i\zeta\gamma_5\gamma^0\pi_G$$

$$Y_A = -\pi\gamma^0\zeta + \frac{i}{2}\bar{\psi}(\gamma^0)^2\zeta$$

$$X_\psi = \delta\pi + i\bar{\zeta}\hat{\bar{\partial}}(\pi_F - \gamma_5\pi_G) - i\bar{\zeta}(\pi_A + \gamma_5\pi_B) - \frac{i}{2}\bar{\zeta}\gamma^0\bar{\partial}(A + \gamma_5 B) + \frac{i}{2}\bar{\zeta}\gamma^0(F - \gamma_5 G)$$
$$+ im\bar{\zeta}\gamma^0(A - \gamma_5 B) + ig\bar{\zeta}\gamma^0(A^2 - B^2) - 2ig\bar{\zeta}\gamma^0\gamma_5 AB$$

$$X_A = \delta\pi_A - \zeta\hat{\bar{\partial}}\pi - \frac{i}{2}\zeta\gamma^0\hat{\bar{\partial}}\psi - im\zeta\gamma^0\psi - 2ig\zeta\gamma^0\psi A + 2ig\zeta\gamma^0\gamma_5\psi B$$

$$X_F = \delta\pi_F + \zeta\pi + \frac{i}{2}\bar{\psi}\gamma^0\zeta. \tag{2.6}$$

To make $\delta\mathscr{H}$ a total spatial divergence, all the coefficients (2.6) must vanish. It turns out that this requirement generates constraints of chiral model:

$$Y_A = Y_B = 0, \qquad \pi^\alpha = \frac{i}{2}(\bar{\psi}\gamma^0)^\alpha \quad (\pi^\alpha \text{ — conjugate momentum to } \psi_\alpha),$$

$$Y_\psi = 0, \qquad \pi_F = \pi_G = 0,$$

$$X_\psi = 0, \qquad \pi_A = -\partial^0 A, \quad \pi_B = -\partial^0 B,$$

$$F+mA+g(A^2-B^2) = 0,$$

$$G+mB+2gAB = 0,$$

$$X_A = 0 = X_B, \qquad (\partial\!\!\!/+m)\psi+2g(A-\gamma_5 B)\psi = 0,$$

$$X_F = 0, \qquad \delta\pi_F = 0,$$

$$X_G = 0, \qquad \delta\pi_G = 0.$$

We see that the constraints generated by the requirement of Hamiltonian invariance under global supersymmetry coincide with the dynamical constraints.

3. *Supersymmetric Hamiltonian of extended electromagnetic model*

A procedure similar to the one adopted in the previous chapter is followed here. The Lagrangian density has the form:

$$\mathscr{L}_{\text{em}} = -\tfrac{1}{4}f_{\mu\nu}f^{\mu\nu}+\frac{i}{2}\bar{\lambda}\partial\!\!\!/\lambda+\tfrac{1}{2}D^2, \tag{3.1}$$

where

$$f_{\mu\nu} = \partial_\mu V_\nu-\partial_\nu V_\mu.$$

It is interesting to notice that this model does not represent a full supersymmetric extension of electromagnetic field. To make the fields $(V_\mu, \lambda_\alpha, D)$ form su-sy multiplet it is necessary to apply the Lorentz gauge condition which, together with the equation of motion of field λ, reckons this submultiplet as a separate representation of supersymmetry. It reduces the number of independent components of field V_μ which satisfy the condition:

$$\partial^\mu V_\mu = 0. \tag{3.2}$$

Using transformations of the fields of the multiplet (see Appendix A), the variation of Lagrangian density is obtained in the form:

$$\delta\mathscr{L}_{\text{em}} = -\frac{i}{2}\bar{\zeta}\gamma^0\gamma_5\partial_0\lambda D-\tfrac{1}{2}\bar{\zeta}\sigma^{kl}\gamma^0\partial_0\lambda\partial_k V_l-\frac{i}{2}\bar{\zeta}\gamma_k\partial_0\lambda f^{0k}+\text{3s-div}. \tag{3.3}$$

Let us write the canonical Hamiltonian density of the extended electromagnetic field:

$$\mathscr{H} = (\partial_0\varphi_i)\pi_i-\mathscr{L}_{\text{em}} \quad \text{where} \quad \varphi_i = (V_\mu, \lambda_\alpha, D). \tag{3.4}$$

Transformation of this density looks as follows:

$$\begin{aligned}\delta\mathscr{H} = {} & (\partial_0 D)\delta\pi_D+(\partial_0 V_\mu)\delta\pi^\mu+(\partial_0\lambda)\delta\pi-i\bar{\zeta}(\partial_0\partial\!\!\!/\gamma_5\lambda)\pi_D \\ & +i\bar{\zeta}\gamma_\mu(\partial_0\lambda)\pi^\mu-i(\partial_0\partial_\mu V_\nu)\bar{\zeta}\sigma^{\mu\nu}\pi-i(\partial_0 D)\zeta\gamma_5\pi-\delta\mathscr{L}_{\text{em}}.\end{aligned} \tag{3.5}$$

Let us write it in the form:

$$\delta\mathscr{H} = (\partial_0 D)X_D+(\partial_0 V_\mu)X^\mu+(\partial_0\lambda)X_\lambda+(\partial_0^2 V_k)Y^k+(\partial_0^2\lambda)Y_\lambda+\text{3s-div}, \tag{3.6}$$

where

$$Y_\lambda = i\pi_D\bar{\zeta}\gamma^0\gamma_5$$

$$Y^k = i\pi\sigma^{0k}\zeta - i\bar{\zeta}\gamma^k\lambda$$

$$X_\lambda = \delta\pi - i\pi^\mu\bar{\zeta}\gamma_\mu - i\partial_k\pi_D\bar{\zeta}\gamma^k\gamma_5 - \frac{i}{2}D\bar{\zeta}\gamma^0\gamma_5 - \tfrac{1}{2}\partial_k V_l\bar{\zeta}\sigma^{kl}\gamma^0 + \frac{i}{2}f^{k0}\bar{\zeta}\gamma_k$$

$$X^\mu = \delta\pi^\mu - i\partial_k\pi\sigma^{k\mu}\zeta - \tfrac{1}{2}\bar{\zeta}\sigma^{lk}\gamma^0\partial_k\lambda\delta^{l\mu} - \frac{i}{2}\bar{\zeta}\hat{\partial}\lambda\delta^{0\mu}$$

$$X_D = \delta\pi_D + \pi\gamma_5\zeta + \frac{i}{2}\bar{\zeta}\gamma^0\gamma_5\hat{\lambda}. \tag{3.7}$$

Now, we make all the coefficients (3.7) equal to zero. In such a way the variation of Hamiltonian density becomes a total spatial divergence and we get a canonical Hamiltonian invariant under supersymmetry transformation. This requirement generates the constraints:

$Y_\lambda = 0,$	$\pi_D = 0,$	
$Y_k = 0,$	$\pi^\alpha = -\frac{i}{2}(\bar{\lambda}\gamma^0)^\alpha,$	(π^α — conjugate momentum λ_α)
$X_\lambda = 0,$	$\pi^0 = 0,$	(π^μ — conjugate momentum to V_μ)
	$\pi^k = -f^{0k},$	
$X_k = 0,$	$D = 0,$	equations of motion of fields D, λ
	$\hat{\partial}\lambda = 0,$	
$X_0 = 0,$	$\delta\pi^0 = 0,$	
$X_D = 0,$	$\delta\pi_D = 0.$	

It is easy to see from the form of Lagrangian density that the obtained constraints and the form of conjugate momenta are identical to the dynamical constraints and the momenta of the model. It is interesting to observe that the condition of invariance under supersymmetry does not reckon the constraint $\dot{\pi}^0 = 0$. This condition will be obtained after complementation of the transformations by the gauge transformation of field V_μ. In fact, Lagrangian density is invariant under gauge transformation of photon field:

$$\tilde{\delta}V_\mu = \partial_\mu\varphi. \tag{3.8}$$

The variation of Hamiltonian density undergoes alternation:

$$(\delta+\tilde{\delta})\mathscr{H} = \partial\mathscr{H} + (\partial_0\partial_\mu\varphi)\pi^\mu + (\partial_0 V_\mu)\tilde{\delta}\pi^\mu = \delta\mathscr{H}$$
$$+(\partial_0\varphi)X_\varphi + (\partial_0^2\varphi)Y_\varphi + (\partial_0 V_\mu)\tilde{\delta}\pi^\mu. \tag{3.9}$$

Additional coefficients have the following form:

$$X_\varphi = \partial_k \pi^k, \qquad \partial_k \pi^k = 0,$$
$$Y_\varphi = -\pi^0, \qquad \pi^0 = 0. \tag{3.10}$$

It was shown in the supersymmetric extension of free electromagnetic model that the application of the condition of invariance under transformation of supersymmetry and the gauge transformation generates all the conjugate canonical momenta and all the dynamical constraints of the model respectively.

4. *The properties of symmetry of Hamiltonian of s-QED model versus dynamical constraints*

Supersymmetric electrodynamics is a combination of the models examined earlier, namely, the massive chiral superfield and the extension of electromagnetic field. The term describing the interaction is added to the free massive chiral Lagrangian and to the free Lagrangian of electromagnetic field. It contains a part describing the interaction of the matter-current j^μ with the photon field V_μ as well as the terms improving the properties of the complete Lagrangian under supersymmetry transformation. The action is invariant on the surface containing the equations of motion of auxiliary fields F_i, G_i and the fermionic fields $\psi_{i\alpha}$. The following fields are built in the model:

a) matter — two chiral multiplets $(A_i, B_i, \psi_{i\alpha}, F_i, G_i)$, $i = 1, 2$,

b) electromagnetic field-multiplet $(V_\mu, \lambda_\alpha, D)$. Lagrangian density of s-QED has the form (4):

$$\mathscr{L} = -\tfrac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{i}{2} \lambda \bar{\partial} \lambda + \tfrac{1}{2} D^2$$
$$+\tfrac{1}{2} \sum_{i=1}^{2} [D_\mu A_i D^\mu A_i + D_\mu B_i D^\mu B_i + i\bar{\psi}_i \bar{\partial} \psi_i + F_i^2 + G_i^2 + m(2A_i F_i + 2B_i G_i + \bar{\psi}_i \psi^i)]$$
$$+\bar{\lambda}(A_1 + \gamma_5 B_1)\psi_2 - \bar{\lambda}(A_2 + \gamma_5 B_2)\psi_1 + (A_1 B_2 - B_1 A_2) D + i\bar{\psi}_1 \gamma_\mu \psi_2 V^\mu, \tag{4.1}$$

where

$$D_\mu A_1 = \partial_\mu A_1 + V_\mu A_2 \qquad D_\mu A_2 = \partial_\mu A_2 - V_\mu A_1.$$

Supersymmetric transformation of Lagrangian density (for transformations of component fields see Appendix A) looks as follows:

$$\delta\mathscr{L} = \tfrac{1}{2} \zeta \gamma^0 \gamma^\mu \partial_0 (\partial_\mu A_i + \gamma_5 \partial_\mu B_i)\psi_i + im\bar{\zeta}\gamma^0 \partial_0 (A_i - \gamma_5 B_i)\psi_i$$
$$+ \frac{i}{2} \bar{\zeta}\gamma^0 \partial_0 (F_i - \gamma_5 G_i)\psi_i - \frac{i}{2} \bar{\zeta}\gamma^0 \gamma_5 \partial_0 \lambda D - \tfrac{1}{2} \zeta \sigma^{kl} \gamma^0 \partial_0 \lambda \partial_k V_l$$
$$- \frac{i}{2} \zeta \gamma_\nu \partial_0 \lambda f^{0\nu} + i\zeta \gamma^0 \gamma_5 \partial_0 \lambda C + i\zeta \sigma^{0\nu} \partial_0 \chi V_\nu + 3\text{s} - \text{div} \tag{4.2}$$

+linear combination of equations of motion of fields F_i, G_i, ψ_i,

where

$$C = A_1B_2 - B_1A_2, \quad \chi = (A_1 + \gamma_5 B_1)\psi_2 - (A_2 + \gamma_5 B_2)\psi_1.$$

Hamiltonian density of the s-QED model is:

$$\mathscr{H} = (\partial_0 \varphi_i)\pi_i - \mathscr{L}_{\text{mat}} - \mathscr{L}_{\text{em}} - \mathscr{L}_{\text{int}}. \tag{4.3}$$

The variation of canonical Hamiltonian density can be formulated in the following form:

$$\delta\mathscr{H} = (\partial_0\varphi_i)X_i + (\partial_0^2\varphi_j)Y_j + iV_\mu\bar{\zeta}\gamma^\mu[(A_1 - \gamma_5 B_1)\hat{R}\psi_2 - (A_2 - \gamma_5 B_2)\hat{R}\psi_1] \tag{4.4}$$

$$+ \text{linear combination of equations of motion of } F_i,\ G_i,$$

where $\hat{R}\psi_i = R\psi_i - i\gamma^0\partial_0\psi_i$ and R represents the operator of equation of motion (see Appendix A).

The explicit form of coefficients X_i, Y_j has been obtained from the form of transformations of component fields (they were included in Appendix A). We shall apply the condition of independence of fields velocities and accelerations to the variation of Hamiltonian density:

$$\delta\mathscr{H} \approx iV_\mu\bar{\zeta}\gamma^\mu[(A_1 - \gamma_5 B_1)\hat{R}\psi_2 - (A_2 - \gamma_5 B_2)\hat{R}\psi_1] + 3\text{s-div}. \tag{4.5}$$

The coefficients X_i, Y_j are thus equal to zero. We obtain the constraints of the model:

$$
\begin{array}{ll}
Y_{A_j} = Y_{B_j} = 0, & \pi_j^\alpha = -\dfrac{i}{2}(\bar{\psi}_j\gamma^0)^\alpha, \\[2ex]
Y_{V_k} = 0, & \pi^\alpha = -\dfrac{i}{2}(\bar{\lambda}\gamma^0)^\alpha, \\[2ex]
Y_{\psi_j} = 0, & \pi_{F_j} = \pi_{G_j} = 0, \\
Y_\lambda = 0, & \pi_D = 0, \\
X_\lambda = 0, & \pi^0 = 0, \quad \pi^k = -f^{0k}, \\
 & D + A_1B_2 - B_1A_2 = 0, \\
X_{\psi_j} = 0, & \pi_{A_j} = D^0A_j, \quad \pi_{B_j} = D^0B_j, \\
 & F_j + mA_j = G_j + mB_j = 0, \\
X_{V_k} = 0, & i\bar{\partial}\lambda + \chi = 0, \\
X_{A_j} = X_{B_j} = 0, & R\psi_j = 0, \\
X_{F_j} = 0, & \delta\pi_{F_j} = 0, \\
X_{G_j} = 0, & \delta\pi_{G_j} = 0, \\
X_D = 0, & \delta\pi_D = 0, \\
X_{V_0} = 0, & \delta\pi^0 = 0.
\end{array}
$$

The constraints obtained are identical to the dynamical constraints of supersymmetric electrodynamics resulting from Dirac-Bergmann algorithm. However, the condition of conservation of zero momentum π^0 in time was not obtained in this way.

$$\dot{\pi}^0 = \partial_k \pi^k - j^0 + V^0 \sum_{i=0}^{2} (A_i^2 + B_i^2). \tag{4.6}$$

To accomplish this aim it is necessary to complement the transformations of field V_μ by a gauge transformation:

$$\tilde{\delta} V_\mu = \partial_\mu \varphi. \tag{4.7}$$

The action of s-QED model is invariant on-shell under transformation (4.7) and the variation of Hamiltonian density looks as follows:

$$\begin{aligned} \tilde{\delta}\mathscr{H} &= \partial_\mu[\varphi(j^\mu - V^\mu \sum_{i=1}^{2} (A_i^2 + B_i^2))] + (\partial_0 \partial_\mu \varphi)\pi^\mu + (\partial_0 V_\mu)\tilde{\delta}\pi^\mu \\ &= \varphi Z_\varphi + (\partial_0 \varphi) X_\varphi + (\partial_0^2 \varphi) Y_\varphi + (\partial_0 V_\mu) X^\mu + 3\text{s-div}. \end{aligned} \tag{4.8}$$

The explicit form of coefficients is:

$$Y_\varphi = \pi^0 = 0, \tag{4.9}$$

$$X_\varphi = \partial_k \pi^k - j^0 + V^0 \sum_{i=1}^{2} (A_i^2 + B_i^2) \approx 0,$$

$$Z_\varphi = \partial_0 [j^0 - V^0 \sum_{i=1}^{2} (A_i^2 + B_i^2)] \approx 0. \tag{4.10}$$

The condition (4.10) ensures the conservation of constraints (4.9) in the time. This was checked while examining the constraints by means of Dirac-Bergmann method. Since the constraints $\pi^0 = 0$ and $\dot{\pi}^0 = 0$ can be brought algebraically to first-class constraints, gauge condition should be introduced. There is some ambiguity here. The following condition in the s-QED can be adopted:

$$V_0 \approx 0. \tag{4.11}$$

It ensures the simultaneous generation of all dynamical constraints during supersymmetric transformation of all the fields and gauge transformation of field V_μ:

$$\delta V_\mu = i\bar{\zeta}\gamma_\mu \lambda + \partial_\mu \varphi. \tag{4.12}$$

5. The Hamiltonian of chiral multiplet in superfield formulation

In the previous chapters, the constraints of a number of supersymmetric models of field theory were examined by means of component technique. Now, we shall deal with our first example i.e. the free chiral model in superfield formulation. Apart from chiral superfield ϕ and antichiral ϕ^{+}, two new multiplets, Π and $\tilde{\Pi}$, will be introduced. They are general superfields and we shall get to know their components by applying the condition of invariance of the superfield Hamiltonian under supersymmetry transformation. The calculations were carried out in the formalism of the two-component Weyl spinors. All formulas and denotations are included in Appendix B.

The density of initial Lagrangian has the form:

$$\mathscr{L} = \tfrac{1}{8}(D^{\alpha}\bar{D}^{2}D_{\alpha}+\bar{D}_{\dot{\alpha}}D^{2}\bar{D}^{\dot{\alpha}})\phi\phi^{+}|_{\theta=\bar{\theta}=0} = -p_{\mu}Ap^{\mu}A^{*}+\tfrac{1}{2}\,\psi^{\alpha}\overleftrightarrow{\not{p}}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}+\tfrac{1}{4}\,FF^{*}. \tag{5.1}$$

Note: In formula (5.1), the measure $\frac{1}{8}(D^{\alpha}\bar{D}^{2}D_{\alpha}+\bar{D}_{\dot{\alpha}}D^{2}\bar{D}^{\dot{\alpha}}) = Z$ was used to make the Lagrangian density dependent only on first derivatives of fields. Dynamical constraints and conjugate momenta of the model can be easily calculated from the form of density:

$$\begin{aligned}
\pi_{F} &= 0, & \pi_{F^{*}} &= 0,\\
\pi_{\alpha} &= -\tfrac{1}{2}\,\sigma^{0}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}, & \pi_{\dot{\alpha}} &= -\tfrac{1}{2}\,\sigma^{0}_{\alpha\dot{\alpha}}\psi^{\alpha},\\
\pi_{A} &= -p^{0}A^{*}, & \pi_{A^{*}} &= -p^{0}A,\\
\dot{\pi}_{F} &= \tfrac{1}{4}\,F^{*} = 0, & \dot{\pi}_{F^{*}} &= \tfrac{1}{4}\,F = 0.
\end{aligned} \tag{5.2}$$

The supersymmetric transformation of Lagrangian density looks as follows:

$$\begin{aligned}
\delta\mathscr{L} &= (\varepsilon Q+\bar{\varepsilon}\bar{Q})Z\phi\phi^{+}|_{\theta=\bar{\theta}=0} = (\varepsilon D+\bar{\varepsilon}\bar{D})Z\phi\phi^{+}|_{\theta=\bar{\theta}=0}\\
&= \tfrac{1}{4}(\varepsilon^{\alpha}\not{p}_{\alpha\dot{\alpha}}D^{2}\bar{D}^{\dot{\alpha}}+\bar{\varepsilon}^{\dot{\alpha}}\bar{D}^{2}D^{\alpha}\not{p}_{\alpha\dot{\alpha}})\phi\phi^{+}|_{\theta=\bar{\theta}=0}.
\end{aligned} \tag{5.3}$$

After passing to the components of superfields we get:

$$\delta\mathscr{L} = \tfrac{1}{4}(-2\varepsilon^{\alpha}\not{p}_{\alpha\dot{\alpha}}\psi_{\beta}\not{p}^{\beta\dot{\alpha}}A^{*}+\varepsilon^{\alpha}\not{p}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}F+2\bar{\varepsilon}^{\dot{\alpha}}\not{p}_{\alpha\dot{\alpha}}\bar{\psi}_{\dot{\beta}}\not{p}^{\alpha\dot{\beta}}A+\bar{\varepsilon}^{\dot{\alpha}}\not{p}_{\alpha\dot{\alpha}}\psi^{\alpha}F^{*}). \tag{5.4}$$

Now, let us build the density of canonical Hamiltonian:

$$\mathscr{H} = Z(\Pi\phi+\tilde{\Pi}\phi^{+}-\phi\phi^{+})|_{\theta=\bar{\theta}=0} \tag{5.5}$$

where

$$Z\Pi\phi|_{\theta=\bar{\theta}=0} = Z\Pi A-\tfrac{1}{2}\,\bar{D}_{\dot{\alpha}}D^{\alpha}\bar{D}^{\dot{\alpha}}\Pi\psi_{\alpha}-\tfrac{1}{2}\,D^{\alpha}\bar{D}^{\dot{\alpha}}\Pi\not{p}_{\alpha\dot{\alpha}}A+\tfrac{1}{2}\,\bar{D}^{\dot{\alpha}}\Pi\not{p}_{\alpha\dot{\alpha}}\psi^{\alpha}+\tfrac{1}{4}\,\bar{D}^{2}\Pi F$$

$$Z\tilde{\Pi}\phi^{+}|_{\theta=\bar{\theta}=0} = Z\tilde{\Pi}A^{*}+\tfrac{1}{2}\,D^{\alpha}\bar{D}^{\dot{\alpha}}D_{\alpha}\tilde{\Pi}\bar{\psi}_{\dot{\alpha}}-\tfrac{1}{2}\,\bar{D}^{\dot{\alpha}}D^{\alpha}\tilde{\Pi}\not{p}_{\alpha\dot{\alpha}}A^{*}+\tfrac{1}{2}\,D^{\alpha}\tilde{\Pi}\not{p}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}+\tfrac{1}{4}\,D^{2}\tilde{\Pi}F^{*}.$$

From the formula (5.5) one can conclude which components of multiplets Π and $\tilde{\Pi}$ contain conjugate momenta. They will be coefficients at the field velocities (we preserve the convention that field velocities are multiplied from the right by conjugate momenta). Hence we get:

$$\pi_A = -\tfrac{1}{2}\,\sigma^0_{\alpha\dot\alpha} D^\alpha \bar{D}^{\dot\alpha} \Pi|_{\theta=\bar\theta=0}, \qquad \pi_{A^*} = -\tfrac{1}{2}\,\sigma^0_{\alpha\dot\alpha} \bar{D}^{\dot\alpha} D^\alpha \tilde{\Pi}|_{\theta=\bar\theta=0},$$

$$\pi_\alpha = -\tfrac{1}{2}\,\sigma^0_{\alpha\dot\alpha} \bar{D}^{\dot\alpha} \Pi|_{\theta=\bar\theta=0}, \qquad \pi_{\dot\alpha} = -\tfrac{1}{2}\,\sigma^0_{\alpha\dot\alpha} D^\alpha \tilde{\Pi}|_{\theta=\bar\theta=0}. \tag{5.6}$$

The variation of Hamiltonian equals zero when its independent parts $\delta_\varepsilon H$ and $\delta_{\bar\varepsilon} H$ vanish. Therefore, we can examine transformations εQ and $\bar\varepsilon\bar{Q}$ separately. Both increments of density take the following form:

$$\delta_\varepsilon \mathscr{H} = \tfrac{1}{4}\,\varepsilon^\alpha \not{p}_{\alpha\dot\alpha}(2D_\beta \bar{D}^{\dot\alpha}\Pi\psi^\beta + \bar{D}^{\dot\alpha}\Pi F + D^2\bar{D}^{\dot\alpha}\Pi A$$

$$-2D_\beta\tilde{\Pi}\not{p}^{\beta\dot\alpha}A^* + D^2\tilde{\Pi}\bar\psi^{\dot\alpha} + D^2\bar{D}^{\dot\alpha}\tilde{\Pi}A^* + 2\psi_\beta\not{p}^{\beta\dot\alpha}A^* - F\bar\psi^{\dot\alpha}),$$

$$\delta_{\bar\varepsilon} \mathscr{H} = \tfrac{1}{4}\,\bar\varepsilon^{\dot\alpha} \not{p}_{\alpha\dot\alpha}(2\bar{D}_{\dot\beta}\Pi\not{p}^{\alpha\dot\beta}A + \bar{D}^2\Pi\psi^\alpha + \bar{D}^2D^\alpha\Pi A$$

$$+2\bar{D}^{\dot\beta}D^\alpha\tilde{\Pi}\bar\psi_{\dot\beta} + D^\alpha\tilde{\Pi}F^* + \bar{D}^2D^\alpha\tilde{\Pi}A^* - 2\bar\psi_{\dot\beta}\not{p}^{\alpha\dot\beta}A - F^*\psi^\alpha). \tag{5.7}$$

In the previous sections of the paper we employed the method of making the coefficients at the field velocities and accelerations as well as at time-derivatives of momenta, equal zero. Now, we can make use of the same method. Thus we can get the components of multiplets Π and $\tilde{\Pi}$ and the constraints of the model. The components of superfields Π and $\tilde{\Pi}$ are the following:

$$\begin{aligned}
&\bar{D}^{\dot\alpha}\Pi|_{\theta=\bar\theta=0} = \bar\psi^{\dot\alpha}, && D^\alpha\tilde{\Pi}|_{\theta=\bar\theta=0} = \psi^\alpha,\\
&D^\alpha\bar{D}^{\dot\alpha}\Pi|_{\theta=\bar\theta=0} = \mathring{\not{p}}^{\alpha\dot\alpha}A^*, && \bar{D}^{\dot\alpha}D^\alpha\tilde{\Pi}|_{\theta=\bar\theta=0} = \mathring{\not{p}}^{\alpha\dot\alpha}A,\\
&\bar{D}^2\Pi|_{\theta=\bar\theta=0} = F^*, && D^2\tilde{\Pi}|_{\theta=\bar\theta=0} = F,\\
&D^2\bar{D}^{\dot\alpha}\Pi|_{\theta=\bar\theta=0} = 0, && D^2\bar{D}^{\dot\alpha}\tilde{\Pi}|_{\theta=\bar\theta=0} = \mathring{\not{p}}^{\alpha\dot\alpha}\psi_\alpha,\\
&\bar{D}^2D^\alpha\Pi|_{\theta=\bar\theta=0} = -\mathring{\not{p}}^{\alpha\dot\alpha}\bar\psi_{\dot\alpha}, && \bar{D}^2D^\alpha\tilde{\Pi}|_{\theta=\bar\theta=0} = 0.
\end{aligned} \tag{5.8}$$

Simultaneously, equations of motion were obtained:

$$\begin{aligned}
F &= 0, \quad F^* = 0,\\
\not{p}\psi &= 0, \quad \not{p}\bar\psi = 0,\\
\Box A &= 0, \quad \Box A^* = 0.
\end{aligned} \tag{5.9}$$

Constraints for the components of multiplets Π and $\tilde{\Pi}$ are conserved in time. It results from the condition of Hamiltonian invariance under global supersymmetry:

$$\partial_0\bar{D}^2\Pi = \partial_0F^* \approx 0, \qquad \partial_0D^2\tilde{\Pi} = \partial_0F \approx 0,$$

$$\partial_0D^2\bar{D}^{\dot\alpha}\Pi \approx 0, \qquad \partial_0\bar{D}^2D^\alpha\tilde{\Pi} \approx 0. \tag{5.10}$$

On the surface of constraints (5.8)–(5.10), chiral Hamiltonian is invariant under supersymmetry transformation, i.e. the increment of its density is a total spatial divergence. It is interesting to notice that the components of fields (5.8) give correct expressions for conjugate momenta. Having applied the formulas (5.8)–(5.10), the part of Hamiltonian which contains multiplets Π and $\tilde{\Pi}$ takes the form:

$$Z\Pi\phi|_{\theta=\bar{\theta}=0} \approx Z\Pi A+\dot{A}\Pi_A+\dot{\bar{\psi}}^{\dot{\alpha}}\Pi_{\dot{\alpha}},$$

$$Z\tilde{\Pi}\phi^+|_{\theta=\bar{\theta}=0} \approx Z\tilde{\Pi}A^*+\dot{A}^*\Pi_{A^*}+\dot{\psi}^{\alpha}\Pi_{\alpha}. \tag{5.11}$$

The components $Z\Pi$ and $Z\tilde{\Pi}$ were not obtained by the conditions $\delta_{\varepsilon}\mathscr{H}$ = 3s-div, $\delta_{\bar{\varepsilon}}\mathscr{H}$ = 3s-div. Now we can assume:

$$Z\Pi|_{\theta=\bar{\theta}=0} \approx 0, \quad Z\tilde{\Pi}|_{\theta=\bar{\theta}=0} \approx 0.$$

On such an assumption we shall get the Hamiltonian in component form. The superfield formulation confirms the conclusions from Chapter 2. In addition we obtained equations of motion of the scalar fields A and A^* as well as the conservation of secondary constraints in time and the expressions for higher components of superfields Π and $\tilde{\Pi}$.

6. Conclusions

The aim of this paper was to investigate, on the basis of a few examples, the relation between the dynamical constraints and the invariance of a canonical Hamiltonian under global supersymmetry.

First, we calculated the conjugate momenta and the dynamical constraints according to the Dirac-Bergmann algorithm. We considered two models with Lagrangians invariant off-shell, i.e. the chiral model and the supersymmetric extension of the electromagnetic field. We discovered that in both of these cases an equivalence takes place:

(*) fulfilled dynamical constraints $\Leftrightarrow$ su-sy invariance of Hamiltonian (+ gauge invariance for electromagnetic model)

Then, we checked the supersymmetric extension of electrodynamics. Lagrangian of this model gives an action which is invariant on the surface of equations of motion. All the calculations conducted revealed that the generation of dynamical constraints takes place by applying the requirement that variation of Hamiltonian density is independent of velocities and accelerations of fields. In Chapter 5, on the example of chiral model, we showed that the component Hamiltonian can be obtained from the superfield Hamiltonian.

It is possible that for the models of $N = 1$ supersymmetry, which are invariant under supersymmetry off the surface of the equations of motion, the equivalence (*) is generally fulfilled. This possibility will be the subject of our future research.

APPENDIX A

The su-sy transformations of components of chiral superfield (Chapter 2) are of the form:

$$\delta A = i\bar{\zeta}\psi, \quad \delta B = i\bar{\zeta}\gamma_5\psi,$$

$$\delta\psi = \partial_\mu(A-\gamma_5 B)\gamma^\mu\zeta+(F+\gamma_5 G)\zeta,$$

$$\delta F = i\bar{\zeta}\partial\!\!\!/\psi, \quad \delta G = i\bar{\zeta}\gamma_5\partial\!\!\!/\psi.$$

The su-sy transformations of components of electromagnetic multiplet (Chapters 3, 4) are:

$$\delta V_\mu = i\bar{\zeta}\gamma_\mu\lambda,$$

$$\delta\lambda = -i\zeta\sigma^{\mu\nu}\partial_\mu V_\nu - \gamma_5\zeta D,$$

$$\delta D = -i\bar{\zeta}\not{\partial}\gamma_5\lambda.$$

The su-sy transformations of matter fields (Chapter 4) are given by the formulae:

$$\delta A_i = \bar{\zeta}\psi_i, \quad \delta B_i = \bar{\zeta}\gamma_5\psi_i,$$

$$\delta\psi_i = -(F_i+\gamma_5 G_i)\zeta - i\not{\partial}(A_i+\gamma_5 B_i)\zeta,$$

$$\delta F_i = i\bar{\xi}\not{\partial}\psi_i, \quad \delta G_i = i\bar{\zeta}\gamma_5\not{\partial}\psi_i.$$

The su-sy transformations of components of multiplet contragradient to electromagnetic multiplet (Chapter 4) are:

$$\delta C = \bar{\zeta}\gamma_5\chi,$$

$$\delta\chi = (M+\gamma_5 N)\zeta - i\gamma^\mu(j_\mu+\gamma_5\partial_\mu C)\zeta,$$

$$\delta j_\mu = i\bar{\zeta}\sigma_{\mu\nu}\partial^\nu\chi + i\bar{\zeta}\gamma_\mu\lambda',$$

where the components of multiplet are dependent on matter-fields

$$C = A_1 B_2 - B_1 A_2,$$

$$\chi = (A_1+\gamma_5 B_1)\psi_2 - (A_2+\gamma_5 B_2)\psi_1,$$

$$j_\mu = A_1\overleftrightarrow{\partial}_\mu A_2 + B_1\overleftrightarrow{\partial}_\mu B_2 - i\bar{\psi}_1\gamma_\mu\psi_2,$$

$$M = -A_1F_2 + A_2F_1 + B_1G_2 - B_2G_1, \quad N = -A_1G_2 - B_1F_2 + A_2G_1 + B_2F_1,$$

$$\lambda' = (F_1-\gamma_5 G_1)\psi_2 - (F_2-\gamma_5 G_2)\psi_1 - i(A_1-\gamma_5 B_1)\not{\partial}\psi_2 + i(A_2-\gamma_5 B_2)\not{\partial}\psi_1.$$

The explicit forms of the coefficients X_i, Y_j are (Chapter 5):

$$Y_{A_i} = i\pi_i\gamma^0\zeta - \tfrac{1}{2}\bar{\zeta}(\gamma^0)^2\psi_i,$$

$$Y_{B_i} = i\pi_i\gamma^0\gamma_5\zeta - \tfrac{1}{2}\bar{\zeta}\gamma_5(\gamma^0)^2\psi_i$$

$$Y_{\psi_i} = -i\pi_{F_i}\bar{\zeta}\gamma^0 - i\pi_{G_i}\bar{\zeta}\gamma_5\gamma^0$$

$$Y_{V_k} = -i\pi\sigma^{0k}\zeta + \frac{i}{2}\bar{\zeta}\gamma^k\lambda$$

$$Y_\lambda = i\pi_D\bar{\zeta}\gamma^0\gamma_5$$

$$X_\lambda = \delta\pi - i\pi^\mu\bar{\zeta}\gamma_\mu - i\partial_k\pi_D\bar{\zeta}\gamma^k\gamma_5 - \tfrac{1}{2}\,\partial_k V_l\bar{\zeta}\sigma^{kl}\gamma^0 - \frac{i}{2}\,D\bar{\zeta}\gamma^0\gamma_5 - iC\bar{\zeta}\gamma^0\gamma_5 - \frac{i}{2}f^{0\mu}\bar{\zeta}\gamma_\mu$$

$$X_{\psi_i} = \delta\pi_i - \bar{\zeta}(\pi_{A_i} + \gamma_5\pi_{B_i}) + i\bar{\zeta}\gamma^k\partial_k(\pi_{F_i} - \gamma_5\pi_{G_i}) + \tfrac{1}{2}\,\bar{\zeta}\gamma^0\not{\partial}(A_i + \gamma_5 B_i)$$

$$+ \frac{i}{2}\,\bar{\zeta}\gamma^0(F_i - \gamma_5 G_i) + im\bar{\zeta}\gamma^0(A_i - \gamma_5 B_i) - \delta^{i2}\bar{\zeta}(A_1 + \gamma_5 B_1)V^0 + \delta^{i1}\bar{\zeta}(A_2 + \gamma_5 B_2)V^0$$

$$X_{V_k} = -\delta\pi^k - i\partial_l\pi\sigma^{lk}\zeta + i\bar{\zeta}\sigma^{0k}\chi + \tfrac{1}{2}\,\partial_l\bar{\zeta}\sigma^{lk}\gamma^0\lambda$$

$$X_{V_0} = -\delta\pi^0 - i\partial_k\pi\sigma^{k0}\zeta + \frac{i}{2}\,\bar{\zeta}\hat{\not{\partial}}\lambda$$

$$X_{A_i} = \delta\pi_{A_i} + i\partial_k\pi_i\gamma^k\zeta + \tfrac{1}{2}\,\bar{\zeta}\gamma^0\hat{\not{\partial}}\psi_i - im\bar{\zeta}\gamma^0\psi_i + \delta^{i1}\bar{\zeta}\gamma^0\gamma_5\lambda B_2 - \delta^{i2}\bar{\zeta}\gamma^0\gamma_5\lambda B_1$$

$$-i\delta^{i1}\bar{\zeta}\sigma^{0\nu}\psi_2 V_\nu + i\delta^{i2}\bar{\zeta}\sigma^{0\nu}\psi_1 V_\nu$$

$$X_{F_i} = \delta\pi_{F_i} - \pi_i\zeta + \frac{i}{2}\,\bar{\zeta}\gamma^0\psi_i$$

$$X_{G_i} = \delta\pi_{G_i} - \pi_i\gamma_5\zeta - \frac{i}{2}\,\bar{\zeta}\gamma^0\gamma_5\psi_i$$

$$X_D = \delta\pi_D - \pi\gamma_5\zeta - \frac{i}{2}\,\bar{\zeta}\gamma^0\gamma_5\lambda.$$

The equations of motion of fields ψ_i (Chapter 5) are:

$$R\psi_1 = (i\not{\partial} + m)\psi_1 - (A_2 + \gamma_5 B_2)\lambda + i\gamma_\mu V^\mu\psi_2 = 0,$$

$$R\psi_2 = (i\not{\partial} + m)\psi_2 + (A_1 + \gamma_5 B_1)\lambda - i\gamma_\mu V^\mu\psi_1 = 0.$$

APPENDIX B

The properties of covariant derivatives D_α, $\bar{D}_{\dot{\alpha}}$:

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = \not{p}_{\alpha\dot{\alpha}},$$

$$[D_\alpha, \bar{D}^2] = 2\not{p}_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}},$$

$$[\bar{D}_{\dot{\alpha}}, D^2] = -2\not{p}_{\alpha\dot{\alpha}}D^\alpha.$$

The properties of σ^μ matrices:

$$\not{p}_{\alpha\dot{\alpha}} = \sigma^\mu_{\alpha\dot{\alpha}}p_\mu,$$

$$\sigma^\mu = (1, \vec{\sigma}), \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\bar{\sigma}^{\mu\alpha\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\sigma^{\mu}_{\beta\dot{\beta}}\varepsilon^{\alpha\beta},$$

$$\bar{\sigma}^{\mu} = (1, -\vec{\sigma}),$$

$$\mathrm{Tr}\,[\sigma^{\mu}\bar{\sigma}^{\nu}] = -2\eta^{\mu\nu}, \qquad \eta = (-, +, +, +).$$

The denotations of components:

— for chiral superfield ϕ, $\bar{D}_{\dot{\alpha}}\phi = 0$,

$$\phi|_{\theta=\bar{\theta}=0} = A,$$

$$D_{\alpha}\phi|_{\theta=\bar{\theta}=0} = \psi_{\alpha},$$

$$D^2\phi|_{\theta=\bar{\theta}=0} = F,$$

— for antichiral superfield ϕ^{+}, $D_{\alpha}\phi^{+} = 0$,

$$\phi^{+}|_{\theta=\bar{\theta}=0} = A^{*},$$

$$\bar{D}_{\dot{\alpha}}\phi^{+}|_{\theta=\bar{\theta}=0} = \bar{\psi}_{\dot{\alpha}},$$

$$\bar{D}^2\phi^{+}|_{\theta=\bar{\theta}=0} = F^{*}.$$

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