PRODUCTION AND COLLECTIVE MOTION OF $q\bar{q}$ PLASMA IN HEAVY ION COLLISIONS*

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(*Received April 1, 1986*)

The Heinz transport equations for $q\bar{q}$ plasma are generalized to include a term responsible for $q\bar{q}$ tunneling from vacuum in chromoelectric field. They are used to study the time dependence of the particle and energy densities and other characteristics of $q\bar{q}$ production in a color flux tube. The energy density larger than 4 GeV/fm^3 is found at the "formation time" 0.3 fm for a tube five times stronger than the elementary one. The oscillations of the system survive the damping effects brought about by the process of production of $q\bar{q}$ pairs.

PACS numbers: 05.60.+w, 12.38.Mh

1. *Introduction*

Possibilities of production of the quark-gluon plasma in high energy collisions of heavy ions were recently discussed by many authors [1]. It was soon recognized that its materialization requires large densities and sizable volumes of produced hadronic matter at the first stages of the production process. As its mechanism is not well understood by the present theory, the problem is faced with substantial uncertainties [2–9]. This situation stimulated investigations of some simple models of the first stages of particle production with the hope of obtaining at least a hint on the behavior of hadronic matter at these early times. One class of such models, which attracted some attention recently, were the

* This research was supported in part by the M. Skłodowska-Curie Foundation, Grant No F7F-071-P.

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generalizations of Schwinger's one-dimensional electrodynamics [10–14]. These models assume that after a high-energy collision the two receeding colored hadrons span in the space between them a quasi-uniform chromoelectric field $\mathscr{E}$ [15, 16]. Quarks, antiquarks and gluons are to be produced by tunneling from vacuum under the influence of this chromoelectric field [17, 18]. The value of $\mathscr{E}$ determines the main characteristics of the production process.

In the present paper we continue investigations of the hypothesis that, in collisions of high-energy heavy ions, chromoelectric fields $\mathscr{E}$ much stronger than those in elementary (e.g. $e^+e^- \to$ hadrons) collisions may be created [10, 11, 12]. In the previous work on this problem a stochastic description was attempted [10, 12]. Here we employ the Heinz-Vlasov transport equations [3] which we have already used for later stages of plasma evolution [19, 20]. By introducing a source term responsible for $q\bar{q}$ generation [13] we obtain a possibility of describing of the very beginning of production process and of the collective motions at these early times.

The model we are going to discuss was described in detail in Ref. [12]: The two opposite color charges receed from each other with velocity of light and span a chromoelectric field between them. This field fills a colored flux tube of radius r taken to be the same as the radius of the elementary color tube spanned between quark and antiquark (for example in $e^+e^- \to q\bar{q}$). Therefore, the predictions of the model can be given in terms of two parameters: the radius of the tube and the size of the color charges which span the field. We expect the tube radius, r, to be some universal parameter of the order of 1 fm (to be calculated from QCD, eventually). On the other hand the color charges can be, to some extent, controlled in experiments: collisions of heavier nuclei are likely to create larger color charges [10–14]. We hope that our calculations shall turn out to be a relevant contribution in constructing a viable model of particle production in high energy nuclear collisions.

In Section 2 we construct transport equations in the presence of a source of $q\bar{q}$ pairs. These equations are solved analytically in Section 3. The Vlasov equations for the self-consistent field and their solutions are described in Section 4. The numerical results are discussed in Section 5. Section 6 contains conclusions and an outlook. In Appendix A relations between several parameters relevant to production of $q\bar{q}$ pairs in a color tube are derived. The algorithm for solving the field equations is spelled out in Appendix B. Finally, Appendix C contains formulae for the momentum and energy densities in terms of the quark distribution functions.

2. *Transport equations for the $q\bar{q}$ plasma in the presence of a source of $q\bar{q}$ pairs*

In this Section we show how to incorporate production of $q\bar{q}$ pairs into the Heinz version [3] of the $q\bar{q}$ plasma. To this end let us first write the Heinz equations in the matrix form

$$p^\mu \partial_\mu G = \mp \frac{\lambda}{2} p^\nu \partial_p^\mu \{F_{\mu\nu}, G\} - i\lambda p^\mu [A_\mu, G]. \tag{2.1}$$

Here G, $F_{\mu\nu}$ and A_μ are 3×3 matrices in color space denoting, respectively, the quark (antiquark) density, the color field strength and the color potential. λ is the color coupling constant. The upper sign refers to quarks and the lower one to antiquarks. The left-hand side of (2.1) represents the rate of change of the quark (antiquark) phase space density per unit time caused by the action of the colored field given by the right-hand side of the equation. When production of quarks (antiquarks) takes place one must add to the right--hand side of (2.1) the rate of change of the phase space density due to this production. The formula for the production rate of $q\bar{q}$ pairs (integrated over longitudinal momenta) in the field $\mathcal{E}$ reads [11, 12, 17] (see Appendix A for details)

$$rd^2p_\perp = \frac{\Lambda}{4\pi^3}\sum_{\text{flavors}} |\ln(1-e^{-\frac{\pi m_\perp^2}{\Lambda}})|d^2p_\perp$$

$$= 0, \quad \text{if} \quad \Lambda \leqslant 0. \tag{2.2}$$

Here

$$\Lambda = \tfrac{1}{2}\lambda|\mathcal{E}|-\sigma, \tag{2.3}$$

with σ being the $(3\ \bar{3})$ string tension and $m_\perp^2 = m^2+p_\perp^2$ where m is the quark mass.

For generalization of Eq. (2.1) we need the production rate in the phase space element

$$dP = d^4p\delta(p^2-m^2) = \frac{dw}{2v}\, d^2p_\perp, \tag{2.4}$$

where w and v are the boost invariant variables introduced in [19]:

$$w = p_\parallel t - Ez, \tag{2.5}$$

$$v = \sqrt{w^2+m^2u}, \tag{2.6}$$

with

$$u = t^2-z^2. \tag{2.7}$$

Thus we have to guess the longitudinal momentum distribution of the produced quarks and antiquarks. Following the result of WKB formula for tunneling [17, 18] we assume that, on the plane $z = 0$, they are produced at rest [13]. It follows then — from boost invariance — that

$$rdP = \frac{\Lambda}{4\pi^3}\sum_{\text{flavor}} |\ln(1-e^{-\frac{\pi m_\perp^2}{\Lambda}})|2v\delta(w)dP. \tag{2.8}$$

The next problem we face is to generalize (2.8) to the three different colors. We observe that, since the $q\bar{q}$ pairs tunnel independently from vacuum, it seems natural to assume that the quarks and the antiquarks are of opposite colors. Consequently, the production rate contributes only to the diagonal terms of Eq. (2.1) and we obtain

$$p^\mu \partial_\mu G = \mp \frac{\lambda}{2} p^\nu \partial^\mu_p \{F_{\mu\nu}, \dot{G}\} - i\lambda p^\mu [A_\mu, G] + 2vR\delta(w), \tag{2.9}$$

where R is the diagonal matrix whose elements R_i are given by

$$R_i = \frac{A_i}{4\pi^3} \sum_{\text{flavor}} |\ln(1-e^{-\frac{\pi m_\perp^2}{A_i}})|, \quad R_i = 0 \quad \text{for} \quad A_i \leqslant 0, \tag{2.10}$$

with $A_i = \frac{1}{2}\lambda|\mathscr{E}_i| - \sigma$.

In the abelian approximation the matrices G and $F_{\mu\nu}$ can be taken diagonal and, using boost invariance [20], (2.9) can be reduced to

$$\dot{G}_i \mp \frac{\lambda}{4} \mathscr{E}_i G_i' = R_i \delta(w), \quad i = 1, 2, 3, \tag{2.11}$$

where G_i are the diagonal elements of the density matrix and no summation of repeated indices is implied. The dot denotes partial differentiation with respect to u and the prime with respect to w.

In the next Section we derive the general solution of Eq. (2.11).

3. *The general solution of the transport equation*

By introducing the abbreviation

$$\frac{\lambda}{4} \mathscr{E}_i = \dot{h}_i, \tag{3.1}$$

Eqs (2.11) can be written in the form

$$\dot{G}_i \mp \dot{h}_i G_i' = R_i \delta(w), \tag{3.2}$$

where R_i is given by (2.10) with $A_i = 2|\dot{h}_i| - \sigma$.

It can be verified that the general solution of (3.2) is

$$G_i(u, w, m_\perp^2) = H_i(w \pm h_i(u), m_\perp^2) \mp F_i(w \pm h_i(u), m_\perp^2) \theta\left(\frac{\dot{h}_i}{|\dot{h}_i|} w\right), \tag{3.3}$$

where H_i is an arbitrary function (to be fixed by the initial conditions) and

$$F_i(\pm h_i(u)) \equiv R_i(\dot{h}_i(u))\,(|\dot{h}_i(u)|)^{-1}. \tag{3.4}$$

This solution is valid at all points where $\dot{h}_i(u) \neq 0$.

As is seen from Eq. (3.3) the quark density is given in terms of the function $h(u)$, related by (3.1) to the field strength. In accordance with Vlasov's selfconsistency requirement $h(u)$ must be obtained by solving the field equatinos. At this step it is treated as a given function.

Let us implement now the initial conditions. Assuming that the collision takes place at $u = 0$, we have $G_i(0, w, m_\perp^2) = 0$. $\dot{h}(0)$ is determined by the strength of the initial color field. Let us assume that $\dot{h}(0) > 0$, $h(0) = 0$. Then, Eq. (3.3) gives for the quarks (we omit the index i)

$$G(0, w, m_\perp^2) = 0 = H(w, m_\perp^2) - F^{(0)}(w, m_\perp^2)\theta(w). \tag{3.5}$$

Hence $H(w, m_\perp^2) = F^{(0)}(w, m_\perp^2)\theta(w)$ and we obtain

$$G(u, w, m_\perp^2) = F^{(0)}(w+h(u), m_\perp^2)\,[\theta(w+h(u))-\theta(w)]. \tag{3.6}$$

Similarly, we get for the antiquarks

$$\bar{G}(u, w, m_\perp^2) = F^{(0)}(w-h(u), m_\perp^2)\,[\theta(w)-\theta(w-h(u))]. \tag{3.6'}$$

Solutions (3.6) and (3.6') are valid as long as $\dot{h}(u) > 0$. In this region also $h(u) > 0$ and we obtain the situations shown schematically in Fig. 1. One notices the symmetry between the quark and the antiquark distributions [19]:

$$G(u, w, m_\perp^2) = \bar{G}(u, -w, m_\perp^2). \tag{3.7}$$

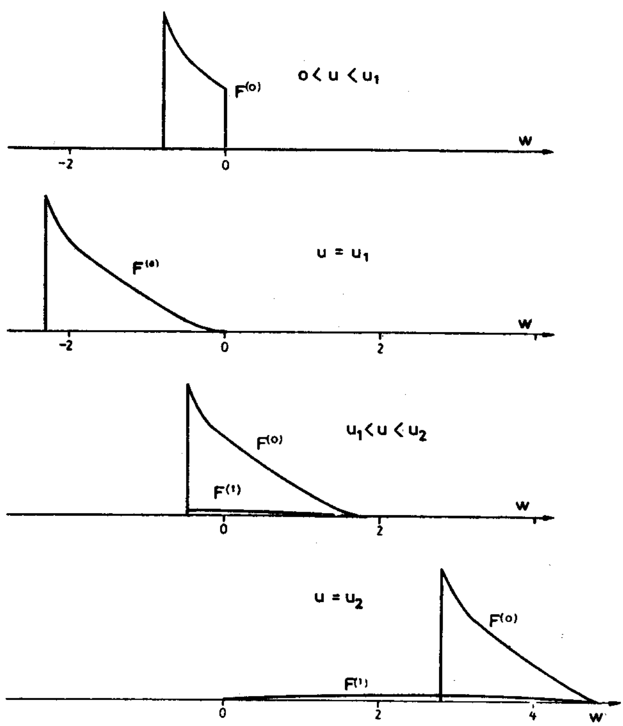

Fig. 1. Evolution of the quark distribution G with increasing time, for $K = 5$ and the tube radius $r = 0.7$ fm. The antiquark distribution $\bar{G}$ is a mirror reflection of G with respect to $w = 0$. One sees the oscillations of G and an increase of its complexity with time

To obtain the solution beyond the point u_1 where $\dot{h}(u_1) = 0$, we have to repeat the procedure of matching the solutions at the points $u_1, u_2 \dots u_l \dots$ which are determined by vanishing of the field strength $\dot{h}(u_l) = 0$ (we anticipate the fact that $h(u)$ is an oscillating function of u). Thus, for the sector $u_1 < u < u_2$ we have (for the quarks, we also skip $m_\perp^2$ dependence)

$$G(u_1, w) = F^{(0)}(w+h(u_1))\,[\theta(w+h(u_1))-\theta(w)]$$
$$= H(w+h(u_1))-F^{(1)}(w+h(u_1))\theta(-w), \tag{3.8}$$

where we have assumed that for $u_1 < u < u_2$ $\dot{h}(u) < 0$. Here $F^{(1)}$ denotes the function calculated from (3.3) for $u > u_1$ which is, in general, different from $F^{(0)}$ calculated for $u < u_1$ because the relation between $h(u)$ and $\dot{h}(u)$ is different in different sectors. We obtain now

$$H(w+h(u_1)) = F^{(0)}(w+h(u_1))\,[\theta(w+h(u_1))-\theta(w)]+F^{(1)}(w+h(u_1))\theta(-w), \tag{3.9}$$

which, substituted into (3.3) with $w \to w+h(u)-h(u_1)$ gives

$$G(u, w) = F^{(0)}(w+h(u))\,[\theta(w+h(u))-\theta(w+h(u)-h(u_1))]$$
$$+F^{(1)}(w+h(u))\,[\theta(-w-h(u)+h(u_1))-\theta(-w)]. \tag{3.10}$$

Continuing this piecemeal construction to the next sector $u_2 < u < u_3$ one obtains for the quark distribution function

$$G(u, w) = F^{(0)}(w+h(u))\,[\theta(w+h(u))-\theta(w+h(u)-h(u_1))]$$
$$+F^{(1)}(w+h(u))\,[\theta(-w-h(u)+h(u_1))-\theta(-w-h(u)+h(u_2))]$$
$$+F^{(2)}(w+h(u))\,[\theta(w+h(u)-h(u_2))-\theta(w)], \tag{3.11}$$

and so on — to the following sectors. One can convince oneself that (3.7) is generally valid, hence one needs to construct only the quark distribution functions. Fig. 1 shows the evolution of $\int d^2p_\perp G(u, w, m_\perp^2)$ with increasing u for the first two sectors.

4. *The solution of the field equation*

The field equations for the diagonal case and in absence of particle production were written down explicitly in Ref. [20]. Here we have to generalize them to include the tunneling term (2.8). We shall consider the special case when only one chromoelectric field is present: In the notation of Ref. [20] we take $\psi_3 = 0$ and $\psi_8 \neq 0$ which means that only the part of the quark-field interaction proportional to the generator $\frac{1}{2}\lambda_8$ is active. We set

$$h = h_3 = -2h_1 = -2h_2 = -\frac{\lambda}{\sqrt{3}}\psi_8. \tag{4.1}$$

We have now the quarks of all three colors present, since they all couple to ψ_8, however only two distributions are different because $G_1 = G_2$. Using (4.1) and Eq. (2.16) of Ref. [20]

the field equation takes the form

$$u\ddot{h}(u) = \tfrac{1}{12}\lambda^2 \int dPw\{G_3(u,w)-G_1(u,w)-\overline{G}_3(u,w)+\overline{G}_1(u,w)\}$$

$$-\frac{\lambda^2\sqrt{u}}{12\dot{h}}\int d^2p_\perp p_\perp \sum_{i=1}^{3} R_i$$

$$= \tfrac{2}{3}\pi\alpha_s \int dPw\{G_3(u,w)-G_1(u,w)\} - \frac{\pi\alpha_s\sqrt{u}}{3\dot{h}}\int d^2p_\perp p_\perp \sum_{i=1}^{3} R_i, \tag{4.2}$$

where the last term represents a change of the field due to particle production (an analogon of the displacement current in electrodynamics). Its derivation is outlined in Appendix A. In the second part of Eq. (4.2) we have used the symmetry relation (3.7) explicitly. The strong coupling constant $\alpha_s = \dfrac{\lambda^2}{4\pi}$. Since $G_i(u, w)$ are expressed in terms of $h(u)$ by the

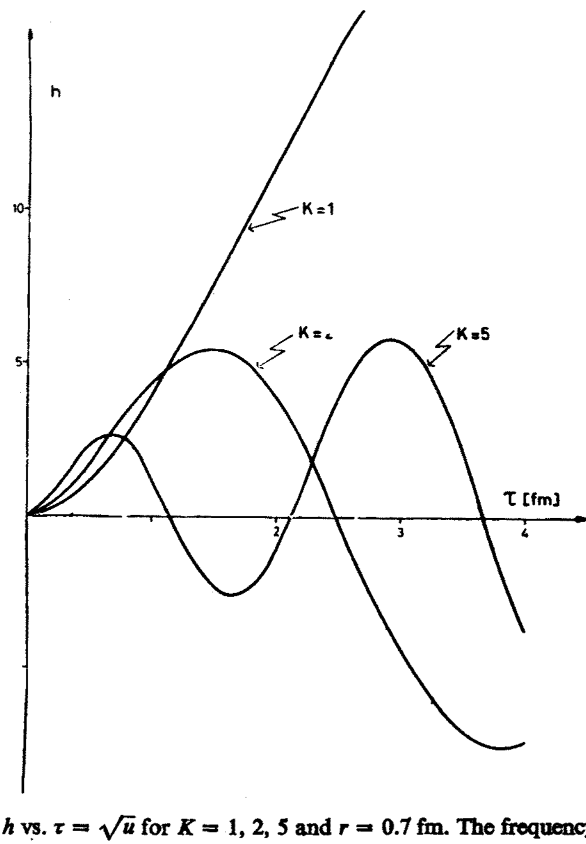

Fig. 2. The potential h vs. $\tau = \sqrt{u}$ for $K = 1, 2, 5$ and $r = 0.7$ fm. The frequency of oscillations increases with K. Note that production of $q\bar{q}$ pairs is determined by the time derivative of h, see Fig. 3

formulae of the previous Section, (4.2) represents a second order integro-differential equation for $h(u)$.

Using Eqs (2.10), (3.3) and (3.4) one can rewrite Eq. (4.2) in the form

$$u\ddot{h}(u) = -\frac{\alpha_s}{6\pi^2}\int_0^u d\tilde{u}[\mathscr{H}_3(u,\tilde{u})-\mathscr{H}_1(u,\tilde{u})]-\frac{\alpha_s\zeta(\frac{5}{2})\sqrt{u}}{12\pi^2\dot{h}}\sum_{i=1}^{3}A_i^{5/2}(u). \tag{4.3}$$

The functions $\mathscr{H}_i(u, \tilde{u})$ are explicitly given in Appendix B. They are expressed in terms of $h_i(u)$ for all $\tilde{u} \leqslant u$. Consequently, Eq. (4.3) can be solved by iteration starting from $u = 0$.

The solutions of (4.3) are determined, essentially, by two parameters: the value of the field strength at $u = 0$, and the coupling constant α_s. The initial value of the field strength can be expressed in terms of the string tension (c.f. (A. 15)). We have

$$\dot{h}(0) \equiv K\sigma,$$

with $K = 1$ for the elementary $\bar{3}3$ staring. For σ we take the standard value 1 GeV/fm $= 5\,\text{fm}^{-2}$. We consider only 2 quark flavors ($N_f = 2$) and neglect the quark masses.

In Fig. 2 we plot the function $h(u)$ (obtained from (4.4)) versus $\tau = \sqrt{u}$ for a few values of K and α_s. One sees clearly the oscillating character of the solution. Hence we can say that introduction of $q\bar{q}$ production by tunneling does not remove the oscillations of plasma found in Refs [19, 20].

5. *Results*

Once the solution $h(u)$ is obtained from Eq. (4.2), other quantities of physical interest can be calculated. We have performed several such calculations. The results are described below.

In Fig. 3 the chromoelectric field strength $\mathscr{E} = \mathscr{E}_3$ is plotted versus $\tau = \sqrt{u}$. One sees again the oscillating character of the process. The frequency of oscillations increases rapidly with the initial field strength (i.e. with the parameter K). The amplitude decreases with time. Both features are qualitatively consistent with the results obtained in Refs [18] and [19].

The most important characteristics of the process is the energy density per unit volume, since it determines the conditions for a phase transition, hence the possibility of formation of $q\bar{q}$ plasma. dE/dV at $z = 0$ is plotted in Fig. 4 versus time $t = \tau = \sqrt{u}$. One sees that the energy transferred into $q\bar{q}$ pairs increases rapidly with increasing K, as expected: from 150 MeV/fm^3 for elementary tube ($K = 1$) to more than 4 GeV/fm^3 for $K = 5$. The increase is thus much faster than the one one would get from a simple addition of K elementary tubes. One sees also that the time at which the maximum of the $q\bar{q}$ energy density is reached (which we identify with the formation time of $q\bar{q}$ pairs) shortens considerably with increasing K: for $K = 5$ we obtain a rather short formation time of about 0.3 fm. This confirms the speculations formulated in Refs. [11, 12] and [4]. The oscillating character of the process is also evident: the energy is transferred from $q\bar{q}$ pairs into the field and back. For comparison we show also in Fig. 4 the total energy density, dE_{TOT}/dV, which is the sum of the energy densities of the $q\bar{q}$ pairs and of the chromoelectric field. For small times dE_{TOT}/dV is varying slowly (it would be constant in absence of $q\bar{q}$ pairs) and then decreases approximately as $1/t$ due to the longitudinal expansion.

It is important to realize that in our model even at $z = 0$ both longitudinal and transverse momenta of quarks contribute to the $q\bar{q}$ energy density. This is to be contrasted with the standard approach [2] where the point $z = 0$ corresponds to $y = 0$ and thus only transverse momentum is relevant. Consequently, if our approach is the correct one, i.e. if quarks and antiquarks are indeed accelerated by the chromoelectric field in the tube, the estimates of the energy density based on the extrapolation from measurements at $y = 0$ [6] are to be reconsidered.

In Fig. 5 the average longitudinal momentum of quarks of color 3 at $z = 0$ is plotted versus time. It reaches the values of the order of 1 GeV, significantly greater than the average

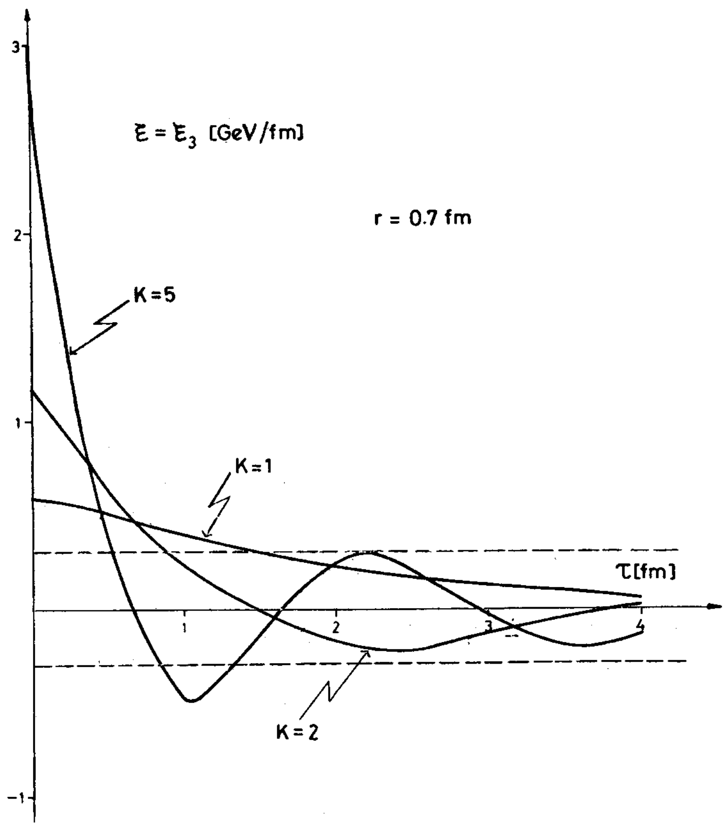

Fig. 3. The field strength $\mathscr{E} = \frac{4}{\lambda}\dot{h}$ vs. $\tau = \sqrt{u}$ for $K = 1, 2, 5$ and $r = 0.7$ fm. The frequency of oscillations increases with the initial field strength given by K. The amplitudes decrease with time. The horizontal lines denote the value of the field below which there is no $q\bar{q}$ tunneling

transverse momentum shown in Fig. 6. This stresses again the importance of the longitudinal motion.

As seen from Fig. 6, the transverse momentum of the produced quarks and antiquarks increases with increasing value of the chromoelectric field. This confirms the results of Refs [11–14]. The value of $\langle p_\perp \rangle$ is fairly low for elementary tube, suggesting that hadronization of quarks and antiquarks must play a significant role [14] and thus the observed value of the average transverse momentum, contrary to the earlier expectations [11, 12], gives only qualitative, indirect information on transverse momenta of $q\bar{q}$ pairs at the formation time.

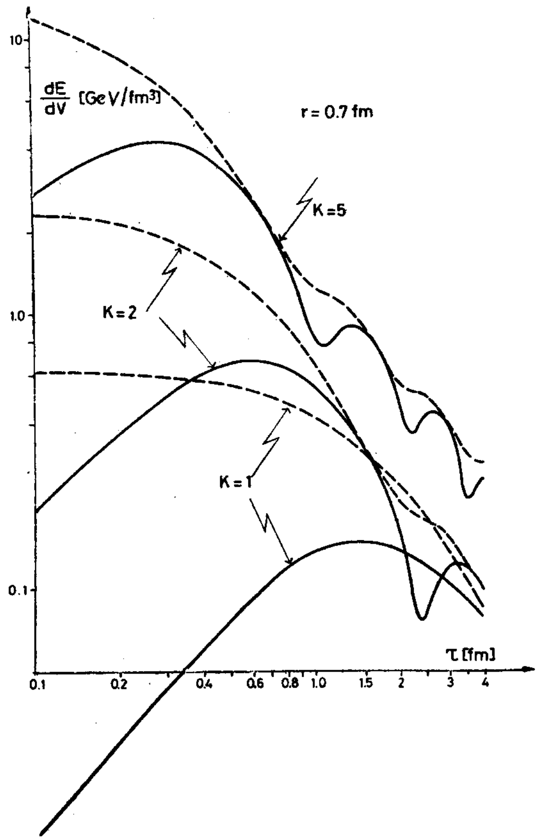

Fig. 4. The energy density of the $q\bar{q}$ pairs at $z = 0$ $\left(\text{solid lines, } \frac{dE_{q\bar{q}}}{dV}\right)$ and the sum of the energy densities of the $q\bar{q}$ pairs and of the chromoelectric field $\left(\text{dashed lines, } \frac{dE_{\mathrm{TOT}}}{dV}\right)$ for $K = 1, 2, 5$ and $r = 0.7$ fm. $\frac{dE_{q\bar{q}}}{dV}$ increases dramatically, while the formation time $\left(\text{given by the position of the first maximum of } \frac{dE_{q\bar{q}}}{dV}\right)$ decreases with increasing K. The oscillations are clearly visible. One sees also the energy being transferred from the $q\bar{q}$ pairs into the field and back

In Fig. 7 the density of quarks and antiquarks per unit of rapidity at $z = 0$ is plotted versus time. One sees a very rapid increase of dN/dy with increasing K, much faster than one would obtain by addition of the contributions of K elementary tubes.

All results shown in Figs 1–7 were obtained for $r = 0.7$ fm. We have also investigated their sensitivity to the value of r: We found no qualitative changes in the behavior of the system. Decreasing r slows down the whole process somewhat, that is to say, the frequency of oscillations goes down and the formation time increases. The maximal energy density

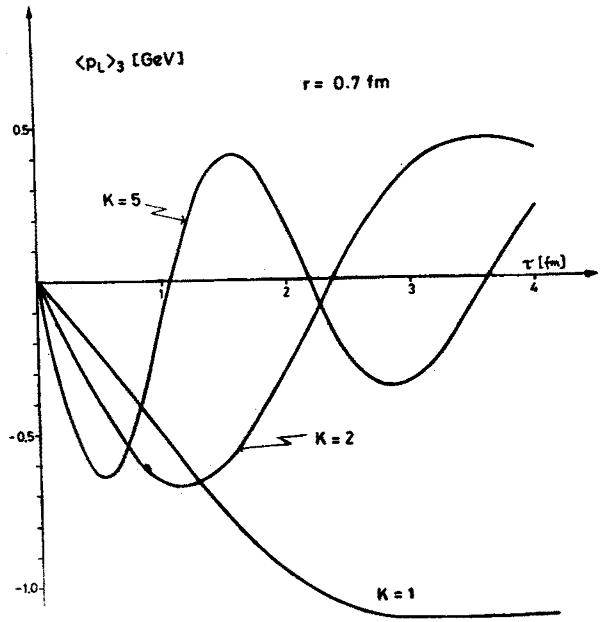

Fig. 5. The average longitudinal momentum of quarks of color 3 at $z = 0$ vs. time. It reaches $\sim$ 1 GeV, which is considerably more than the average transverse momentum (see Fig. 6)

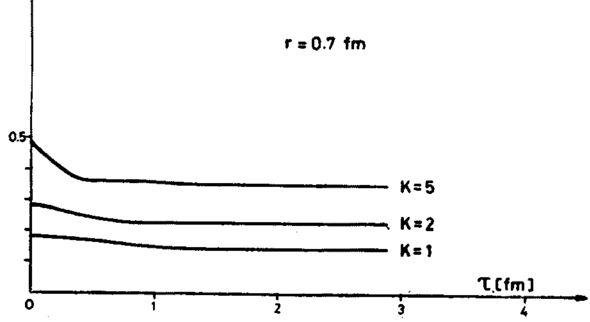

Fig. 6. The average transverse momentum of quarks and antiquarks. It increases with increasing K

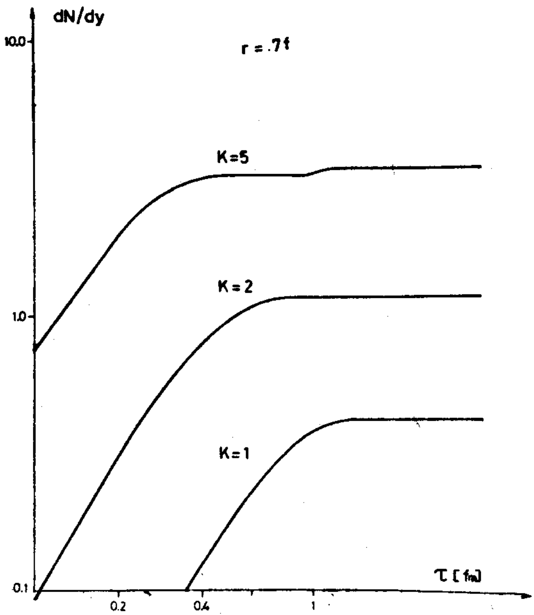

Fig. 7. Density of quarks (antiquarks) at $z = 0$ per unit of rapidity vs. time. One sees a rapid increase of dN/dy with K

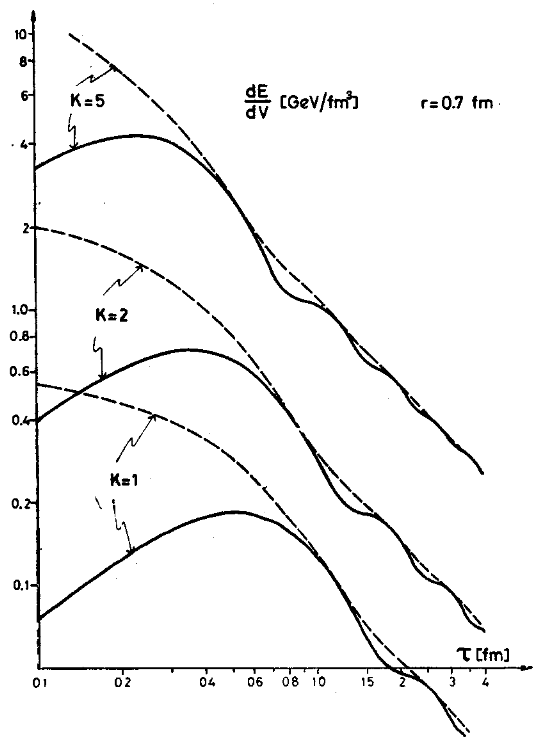

Fig. 8. The same as in Fig. 4 but with $q\bar{q}$ interaction term set to zero (i.e. Eq. (5.1) is employed)

increases, while the density of particles per unit of rapidity is fairly insensitive to variations of r.

Finally, we have studied the effect of the quark-antiquark interactions [18]. To this end we repeated the calculations without the $q\bar{q}$ interaction term, i.e. taking

$$A_i = \frac{1}{2}\lambda|\mathscr{E}_i|, \tag{5.1}$$

instead of (2.10). This removes the treshold for particle production: The particle densities are larger and, consequently, the $q\bar{q}$ oscillations are strongly damped by the displacement current. This is illustrated in Figs 8 and 9 where the energy and particle densities, respectively, are shown. Comparing Figs 8, 9 with Figs 4,7 one sees that, as expected from the relative magnitudes of the $q\bar{q}$ interaction and the initial field strength given by K, the removal of the $q\bar{q}$ interaction influences little the rate of the process for large K, while for small K it results in considerable acceleration of the evolution of the system.

Let us finish this Section with a brief comment on the recent work of Kajantie and Matsui [13]. The main difference between the two approaches is that we included the interaction of quarks and antiquarks with the field while they did not. The field introduced selfconsistently leads to oscillations of the system which do not exist in Ref. [13]. On the other hand in Ref. [13] the relaxation effects are introduced through a relaxation time parameter. This was not done in our paper because inclusion of the relaxation term makes impossible an explicit solution of the Heinz equations. Thus the results of the two papers escape a direct comparison.

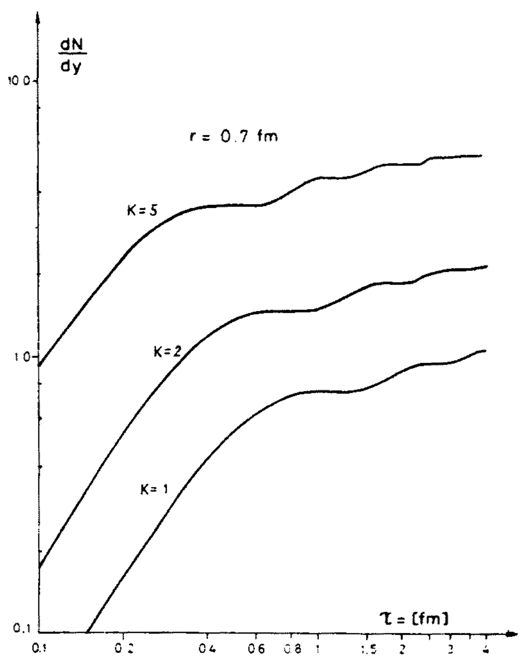

Fig. 9. The same as in Fig. 7 but with the $q\bar{q}$ interaction term set to zero (i.e. Eq. (5.1) is employed)

6. *Conclusions and outlook*

We have investigated production and subsequent motion of $q\bar{q}$ pairs in an uniform chromoelectric field spanned between two opposite color charges receeding from each other with velocity close to that of light. We hope that this picture can serve as a first step towards constructing a realistic model of the first moments of particle production in high energy collisions.

Our main result is the generalization of the boost-invariant Boltzmann-Vlasov equations by including a term responsible for $q\bar{q}$ tunneling from vacuum. In this way we obtain a fairly complete description of the early stages of particle production depending only on two parameters: the initial field strength and the radius of the color tube (or — equivalently — the value of the strong coupling constant). This is a substantial improvement over our earlier calculations, where a number of free parameters (which were difficult to control) had to be introduced. Consequently, the model gives rather well-defined predictions for many characteristics of the production process which can be tested in future experiments with heavy ion beams, where appearance of strong chromoelectric fields is expected [10–13].

As described in Section 5, we have calculated several relevant quantities. Our main conclusions are as follows.

(*i*) The oscillating character of the process, first discovered in [19] is not destroyed by the presence of the $q\bar{q}$ production term. The quarks and antiquarks are accelerated and subsequently de-accelerated in the chromoelectic field. The frequency of the oscillations increases with increasing value of the initial chromoelectric field.

(*ii*) The energy density of $q\bar{q}$ pairs increases rapidly after the collision, as a consequence of both copious production and acceleration of quarks and antiquarks in the field. It reaches maximum (equal to about 1/4 of the initial energy density of the field) and falls subsequently as a result of the longitudinal expansion. The time t_0 at which the maximum of $q\bar{q}$ energy density is reached, which we identify with formation time, turns out to be well below 0.5 fm for strong initial fields. The value of the energy density at $t = t_0$ varies strongly with increasing of the initial field and reaches values exceeding 4 GeV/fm^3 for strong fields, suggesting a possibility of plasma formation. The effect is significantly stronger than simple incoherent addition of elementary processes.

(*iii*) The longitudinal motion of quarks and antiquarks gives important contribution to the energy density of the $q\bar{q}$ pairs, even at $z = 0$. This feature is very different from the standard picture [2] where it is assumed that particles at $z = 0$ stay at rest ($y = 0$) in the c.m. system of the collision. The difference is caused by the action of the chromoelectric field: although the quarks and antiquarks are indeed produced at rest in the co-moving frame, they are subsequently accelerated by the field and the simple relation between z and y [2] is lost. Consequently, the observed energy density of particles at $y = 0$ does not necessarily reflect the actual energy density of $q\bar{q}$ pairs at $z = 0$. This may have consequences for phenomenological estimates of the possibilities of formation of $q\bar{q}$ plasma [6].

We would like to finish with a few remarks on limitations of our present calculation and on possible future improvements. First we note that the following two important phenomena are missing at the present stage: (*i*) the model ignores the possibility of production of gluons and (*ii*) it ignores the collisions between quarks and antiquarks and is thus unable to account for thermalization. While inclusion of gluons seems to be just a technical complication which we hope to overcome in the future, the effects of $q\bar{q}$ collisions represent a rather difficult problem which is, moreover, crucial for relevance of our present work. Indeed, we may expect our calculations to be valid only for times shorter than the time t_{Th} needed for thermalization of the system. Since at present there exist no reliable estimate of t_{Th}, one cannot determine its relation to the formation time t_0, characteristic for the process we describe in the present paper. This is a very interesting and important problem for the future.

Another point which is not entirely clear is the physical reality of the oscillations between the $q\bar{q}$ pairs and the field. We feel that whereas it is hard to doubt that the initial chromoelectric field can create and accelerate quarks and antiquarks, it is not obvious that the transfer of energy from $q\bar{q}$ pairs back into the color field can be correctly described in terms of interactions with a classical field. On the contrary, it seems likely that the deceleration of quarks may be dominated by the gluon bremsstrahlung. This is also an interesting problem which can be undertaken, however, only after gluons are introduced into the picture.

Let us end up with an obvious remark that it would be very useful to estimate production of photons and lepton pairs from the oscillating plasma, using the solutions presented in the present paper. In view of the small number of free parameters, such calculation should be more reliable than the existing estimates [19, 20, 21] which, nevertheless, already indicated that preequilibrium plasma may give non-negligible signals of photons and lepton pairs.

We thank K. Kajantie and T. Matsui for sending us their unpublished results. The interesting discussions with J. P. Blaizot, P. Danielewicz and J. Knoll are gratefully acknowledged.

APPENDIX A

Production of $q\bar{q}$ pairs in a uniform chromoelectric field

In this Appendix we fix the numerical coefficients in various relations between fields, coupling constants, densities, string tensions etc. We start with normalizations of the constant chromoelectric fields from which tunneling of $q\bar{q}$ pairs takes place.

The interaction energy of quarks with the field is (see e.g. Ref. [22])

$$W = -\lambda\bar{\psi}_i\gamma_\mu Q^a_{ij}\psi_j A^a_\mu. \tag{A.1}$$

Let us take a uniform, constant chromoelectric field

$$A^a_0 = -E^a z, \quad \vec{A}^a = 0. \tag{A.2}$$

The interaction energy becomes

$$W = \lambda z E^a \psi_i^+ Q^a_{ij}\psi_j. \tag{A.3}$$

We know that E^8 field is coupled to all three colored quarks and that we can have a diagonal form of G when E^3 and E^8 are different from zero (E^3 couples to just two colors) [20]. We set $E^3 = 0$ for the sake of simplicity but our calculations can be easily generalized to the case $E^3 \neq 0$, $E^8 \neq 0$.

We take

$$\psi_j = \begin{pmatrix} \delta_{j1}\psi_1 \\ \delta_{j2}\psi_2 \\ \delta_{j3}\psi_3 \end{pmatrix},$$

$$Q^8_{ij} = \tfrac{1}{2}\lambda^8_{ij} = \begin{bmatrix} \frac{1}{2\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{2\sqrt{3}} & 0 \\ 0 & 0 & -\frac{1}{\sqrt{3}} \end{bmatrix}. \tag{A.4}$$

Thus

$$W = \lambda z E^8 \left(g_{11}\frac{1}{2\sqrt{3}} + g_{22}\frac{1}{2\sqrt{3}} - g_{33}\frac{1}{\sqrt{3}} \right)$$

$$= \lambda z (g_{11}\tfrac{1}{2}\mathscr{E}_1 + g_{22}\tfrac{1}{2}\mathscr{E}_2 + g_{33}\tfrac{1}{2}\mathscr{E}_3), \tag{A.5}$$

where $g_{ii} = \psi_i^+\psi_i$ are the quark densities (hence the zeroth components of the current) and

$$\mathscr{E}_i = 2Q^8_{ii}E^8; \quad \text{hence } \mathscr{E}_1 = \mathscr{E}_2 = \frac{1}{\sqrt{3}}E^8 \quad \text{and} \quad \mathscr{E}_3 = -\frac{2}{\sqrt{3}}E^8. \tag{A.6}$$

Eq. (A. 5) implies that the energy balance in the tunneling process of a $q\bar{q}$ pair of color i is the same as the one used in Ref. [11]:

$$2\sqrt{p_L^2 + p_\perp^2 + m^2} - \lambda z \mathscr{E}_i + 2z\sigma = 0, \tag{A.7}$$

(for the definition of the elementary string tension σ see below). Consequently, also the rate of production of pairs of color i is given by the formula of Ref. [12]

$$\frac{dg_{ii}}{dtdN} = r_i d^2 p_\perp \doteq \frac{\Lambda_i}{4\pi^3} \sum_{\text{flavors}} |\ln(1 - e^{-\frac{\pi m_\perp^2}{\Lambda_i}})| d^2 p_\perp, \tag{A.8}$$

with $\Lambda_i = \frac{1}{2}\lambda|\mathscr{E}_i| - \sigma$.

Now, let us relate the field E^8 to the color charges of quarks and antiquarks at the ends of the color tube. We start from the field equations

$$\partial^\mu F^a_{\mu\nu} = \lambda\bar{\psi}\gamma_\nu Q^a \psi = \lambda j_{\nu 33} Q^a_{33}, \tag{A.9}$$

where we assume that only the quarks (antiquarks) of a given color (anticolor) $3(\bar{3})$ span the field. From (A.9) we get the Gauss theorem for a color tube

$$E^8 A = \lambda K Q^8_{33}, \tag{A.10}$$

with K equal to the number of quarks (antiquarks) (of color 3) at the end, and A being the area of the transverse cross section of the tube. In terms of $\mathscr{E}_3$ we get the relation

$$\mathscr{E}_3 = 2\lambda K (Q^8_{33})^2 \frac{1}{A}. \tag{A.11}$$

It is also useful to calculate the string tension. From the definition of the string tension and Eqs (A. 10) we get

$$\sigma_K = \tfrac{1}{2}(E^8)^2 A = \tfrac{1}{8}\frac{\mathscr{E}_3^2}{(Q^8_{33})^2} A = \tfrac{3}{8}\mathscr{E}_3^2 A. \tag{A.12}$$

Thus from (A. 11) we obtain

$$\sigma_K = \tfrac{1}{4}\lambda K \mathscr{E}_3. \tag{A.13}$$

Let us also note that (A. 11) and (A. 12) imply a relation between the strong coupling constant, the radius of the tube and the elementary string tension $\sigma(K = 1)$

$$\sigma = \tfrac{1}{2}\frac{\lambda^2}{A}(Q^8_{33})^2 = \frac{2\lambda^2}{4\pi r^2}(Q^8_{33})^2 = \frac{2\alpha_s}{3r^2}. \tag{A.14}$$

Finally we observe that from (A. 13) and (A. 11) we get

$$\frac{\lambda \mathscr{E}_3}{4} = \tfrac{1}{2}\lambda^2 K \frac{(Q^8_{33})^2}{A} = K\sigma. \tag{A.15}$$

As the last point we outline briefly the derivation of the field equation (4.2). The energy and momentum conservation condition implies

$$\partial_\mu T^{\mu 0}_{\text{field}} = \tfrac{1}{2}\frac{\partial}{\partial t}(E^8)^2 = -\partial_\mu T^{\mu 0}_{\text{particles}}, \tag{A.16}$$

where

$$T^{\mu\nu}_{\text{particles}} = \int dP p^\mu p^\nu \sum_{i=1}^{3} (G_i + \bar{G}_i). \tag{A.17}$$

Using transport equations (2.9) one can show that

$$\partial_\mu T^{\mu\nu}_{\text{particles}} = -\lambda F^{\mu\nu}_8 j^8_\mu + 2\int d^2 p_\perp p^\nu \sum_{i=1}^{3} R_i, \tag{A.18}$$

where

$$j_\mu = \int dP p_\mu (g^8 - \bar{g}^8). \tag{A.19}$$

Substituting (A. 18) and (A. 19) into (A. 16) we obtain, after some algebra, Eq. (4.2)

APPENDIX B

Algorithm for solving the field equations (4.2)

Using Eqs (2.4), (2.10), (3.3) and (3.4), and introducing a new integration variable $\tilde{u}_i$ through the equation

$$-\Delta h_i = h_i(\tilde{u}_i) - h_i(u), \tag{B.1}$$

the integrals in (4.2) can be written as follows

$$I_i(u) = -\frac{N_f}{4\pi^3} \int_0^u d\tilde{u}_i \Delta h_i \tilde{\Lambda}_i \int d^2 p_\perp \frac{\ln\left(1 - e^{-\frac{\pi m_\perp^2}{\tilde{\Lambda}_i}}\right)}{\sqrt{(\Delta h_i)^2 + m_\perp^2 u}}, \tag{B.2}$$

where $\tilde{\Lambda}_i = \Lambda[h_i(\tilde{u}_i)]$. The integral over the transverse momentum we express as

$$\int d^2 p_\perp \frac{\ln\left(1 - e^{-\frac{\pi m_\perp^2}{\tilde{\Lambda}_i}}\right)}{\sqrt{(\Delta h_i)^2 + m_\perp^2 u}} = \frac{\tilde{\Lambda}_i C(\beta_i)}{\sqrt{(\Delta h_i)^2 + \frac{u\tilde{\Lambda}_i}{\pi}}}, \tag{B.3}$$

where, neglecting quark masses,

$$C(\beta) = \int_0^\infty dx \frac{\ln(1 - e^{-x})}{\sqrt{\beta^2 + (1-\beta^2)x}}, \tag{B.4}$$

and $\beta_i = \Delta h_i \Big/ \sqrt{(\Delta h_i)^2 + \frac{u\tilde{\Lambda}_i}{\pi}}$.

Finally, we obtain

$$I_i(u) = -\frac{N_f}{4\pi^3} \int_0^u d\tilde{u}_i \Delta h_i \tilde{\Lambda}_i^2 C(\beta_i) \Big/ \sqrt{(\Delta h_i)^2 + \frac{u\tilde{\Lambda}_i}{\pi}}, \tag{B.5}$$

and the field equations become

$$u\ddot{h}(u) = -\frac{\alpha}{6\pi^2} \int_0^u \left[d\tilde{u}_3 \Delta h_3 \tilde{\Lambda}_3^2 C(\beta_3) \Big/ \sqrt{(\Delta h_3)^2 + \frac{u\tilde{\Lambda}_3}{\pi}} - (3 \to 1) \right]. \tag{B.6}$$

The integrals on the r.h.s. of (B. 6) can be performed numerically step by step if the solution $h(\tilde{u}_i)$ is known for all $\tilde{u}_i \leqslant u$.

APPENDIX C

Some average densities in terms of the quark distribution functions

To calculate the quark density per unit volume of space we start from the general formula for the number of quarks and antiquarks in the phase-space volume $dVdP$ [19]

$$dN = \int_{dVdP} dP d\sigma_\mu p^\mu \{G(x, p) + \bar{G}(x, p)\}. \tag{C.1}$$

Observing that (note: $t^2 - z^2 = \text{inv.}$ hence $zdz = tdt$)

$$d\sigma_\mu p^\mu = d^2 s(dzE - dtp_\parallel) = d^2 sdz \frac{v}{t} = dV \frac{v}{t}, \tag{C.2}$$

where $\vec{s}$ is the transverse distance, we have for the density of quarks and antiquarks

$$\frac{dN}{dV} = \frac{dN}{d^2 sdz} = \frac{1}{t} \int dwd^2 p_\perp G(x, p), \tag{C.3}$$

where we have also employed Eq. (2.4).

Similarly, the average value of any power of the transverse momentum can be expressed as

$$\langle p_\perp^m \rangle = \int_{dVdP} dP d\sigma_\mu p^\mu p_\perp^m G(x, p) / \int_{dVdP} dP d\sigma_\mu p^\mu G(x, p)$$
$$= \int dwd^2 p_\perp p_\perp^m G(x, p) / \int dwd^2 p_\perp G(x, p). \tag{C.4}$$

Using Eqs (2.10), (3.3) and (3.4) the interaction over $d^2 p_\perp$ in (C.3) and (C. 4) can be explicitly performed giving

$$\frac{dN}{dV} = \frac{N_f \zeta(2)}{4\pi^3 t} \int dw \frac{\Lambda^2}{h}, \tag{C.5}$$

$$\langle p_\perp^m \rangle = \frac{\zeta\left(2 + \frac{m}{2}\right)}{\zeta(2)} \frac{\Gamma\left(1 + \frac{m}{2}\right)}{\pi^{m/2}} \int dw \frac{\Lambda^{2+\frac{1}{2}m}}{h} \Big/ \int dw \frac{\Lambda^2}{h}, \tag{C.6}$$

where Λ is given by Eq. (2.10), $\zeta(x)$ is the Riemann ζ function, N_f is the number of flavors, and we neglected the quark masses. The integral over w is to be performed numerically. More details about evaluating integrals of this kind are given in Appendix B.

The quark density per unit space volume (C.5) is an interesting quantity for description of plasma, but it is not directly observable. It is therefore interesting to find its relation to the density of quarks per unit of rapidity which is easier to estimate experimentally (through measurements of the particle density per unit of rapidity). To this end we introduce a new set of boost invariant variables (see. e.g. [13]). First we introduce

$$\eta = \tfrac{1}{2} \ln \frac{t+z}{t-z} \quad \text{and} \quad y = \tfrac{1}{2} \ln \frac{E + p_\parallel}{E - p_\parallel}. \tag{C.7}$$

We have then

$$\frac{dz}{t} = d\eta, \qquad \frac{dw}{v} = \frac{dp_{\parallel}}{E} = dy, \tag{C.8}$$

and

$$w = -\tau m_{\perp} \sinh(\eta - y), \qquad v = \tau m_{\perp} \cosh(\eta - y). \tag{C.9}$$

Using (C.7)-(C.9) Eq. (C.1) gives

$$dN = \int d^2 s d\eta d^2 p_{\perp} dy v G. \tag{C.10}$$

Now, boost invariance implies that G must be a function of the difference $\eta - y$, so we find that

$$\frac{dN}{d^2 s dy} = \frac{dN}{d^2 s d\eta} = t \frac{dN}{d^2 s dz}. \tag{C.11}$$

Let us now proceed to the energy density. Using (C.1) we find for the energy contained in a phase-space volume $dVdP$

$$dE = \int_{dVdP} dP d\sigma_{\mu} p^{\mu} E\{G(x, p) + \bar{G}(x, p)\}, \tag{C.12}$$

thus, employing (C.12), we obtain the energy density per unit volume of space:

$$\frac{dE}{dV} = \frac{1}{t}\int dPEv\{G(x, p) + \bar{G}(x, p)\} = \frac{1}{2t}\int dwd^2 p_{\perp}$$

$$\times \frac{zw + tv}{u}(G(x, p) + \bar{G}(x, p)). \tag{C.13}$$

Using the symmetry relation (3.7) we have

$$\frac{dE}{dV} = \frac{1}{u}\int dwd^2 p_{\perp} v G(x, p). \tag{C.14}$$

From Eqs (2.10), (3.3) and (3.4) we obtain finally

$$\frac{dE}{dV} = \frac{N_f}{4\pi^3 u}\int dw \frac{\Lambda^2}{\dot{h}} \sqrt{(w-h)^2 + \frac{\Lambda u}{\pi}} B(\beta), \tag{C.15}$$

with $\beta^2 = (w-h)^2 \Big/ \left[(w-h)^2 + \dfrac{\Lambda u}{\pi}\right]$ and $B(\beta) = \int_0^{\infty} dx \sqrt{\beta^2 + (1-\beta^2)x} \ln(1 - e^{-x})$. This formula was used in numerical estimates of dE/dV given in Section 5.

To calculate the energy density, per unit of rapidity we again employ (C.12). Taking into account (C.7)-(C.9) we have

$$\frac{dE}{d^2 s d\eta} = \int dy d^2 p_\perp v m_\perp G(x, y-\eta) \cosh y = t \frac{dE}{dV}, \tag{C.16}$$

and

$$\frac{dE}{d^2 s dy} = \int d\eta d^2 p_\perp v m_\perp G(x, y-\eta) \cosh y = \langle m_\perp \rangle \cosh y \frac{dN}{d^2 s dy}. \tag{C.17}$$

Finally, let us give the formula for the longitudinal momentum density of quarks

$$\frac{dp_\parallel}{dV} = \frac{1}{t} \int d^4 P v p_\parallel G(x, p) = \frac{t}{\tau} \{\cosh \eta T^{03} + \sinh \eta T^{33}\} \tag{C.18}$$

with $T^{\mu\nu} = \int d^4 P p^\mu p^\nu G$ which at $z = 0$, gives

$$\frac{dp_\parallel}{dV} = \frac{1}{2t^2} \int dw d^2 p_\perp w G(x, p) = \frac{N_f \zeta(2)}{8\pi^3 t^2} \int dw (w-h) \frac{\Lambda^2}{\dot{h}} \tag{C.19}$$

The integral is taken as indicated in Appendix B.

Using (C.5) we obtain for the average value of the longitudinal momentum of quarks at $z = 0$

$$\langle p_\parallel \rangle = \frac{dp_\parallel}{dV} \bigg/ \frac{dN}{dV} = \frac{1}{t} \int dw (w-h) \frac{\Lambda^2}{\dot{h}} \bigg/ \int dw \frac{\Lambda^2}{\dot{h}}, \tag{C.20}$$

which was used for numerical estimates.

Note added in proof: The problem of $q\bar{q}$ pair creation and gluon pair creation in a constant color field of arbitrary direction was discussed by Gyulassy and Iwasaki (*Phys. Lett.* **165B**, 157 (1985). We thank U. Heinz for calling our attention to this paper which has not been accesible to us so far (see also: U. Heinz, *Color response and color transport in quark-gluon plasma.* Invited talk presented at the second International Workshop "Local Equlibrium in Strong Interaction Physics", Santa Fe, New Mexico, April 9–12, 1986).

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