THE STANDARD SIX-QUARK MODEL WITH HIERARCHICAL SYMMETRY BREAKING

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The simultaneous mixing of quarks in both negative and positive electric charge subspaces is considered. Quark mixing in each space is described by the Kobayashi-Maskawa matrix. In order to get a right number of independent mixing parameters only one angle θ_7 common for both subspaces has been adjusted. Since the electromagnetic mass splitting of u and d quarks has been taken into account the real K-M mixing angles can be calculated explicitly. The Gell-Mann, Oakes and Renner model has been used. As an input only meson masses and f_x factors (treated as factors in matrix elements between one meson state and vacuum according to PCAC) are needed. Physical quark mixing is realized for maximal allowed symmetry breaking and it corresponds to vanishing of θ_7, which implies that only quark mixings with mass generation are permitted. Bounds of the phase δ have been also found.

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The Kobayashi-Maskawa mixing matrix [1]

$$U = \begin{bmatrix} c_1, & s_1c_3, & s_1s_3 \\ -s_1c_2, & c_1c_2c_3-s_2s_3e^{i\delta}, & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2, & -c_1s_2c_3-c_2s_3e^{i\delta}, & -c_1s_2s_3+c_2c_3e^{i\delta} \end{bmatrix} \tag{1}$$

is usually considered to mix quarks in the negative electric charge subspace. The matrix (1) can be written also as follows

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{bmatrix}, \tag{2}$$

$$U = U_2U_\delta U_1U_3. \tag{2a}$$

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From the form of the charged weak current

$$J_\mu = (\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1-\gamma_5)U\begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{3}$$

the following variants of quark mixing are allowed:

$$\text{A:}\quad U = U_2(s-b)U_\delta U_1(d-s)U_3(s-b), \tag{4a}$$

$$\text{B:}\quad U = U_2(c-t)U_\delta U_1(d-s)U_3(s-b), \tag{4b}$$

$$\text{C:}\quad U = U_2(c-t)U_\delta U_1(u-c)U_3(s-b), \tag{4c}$$

$$\text{D:}\quad U = U_2(c-t)U_\delta U_1(u-c)U_3(c-t), \tag{4d}$$

where $U_k(x-y)$ denotes the mixing of x and y quarks by the matrix U_k. The variants A and B were considered in detail in Ref. [2]. The variants C and D were rejected due to the fact that the Cabibbo angle value could not be explained by the mixing in the (u$-$c) sector only [3, 4]. In variant A, in order to get the value of the angle θ_3 and limits for the angle θ_2 and the phase δ, the experimental Cabibbo angle value was put as an input. The introduction of the quark mixing in the (u, c, t) sector allowed the author to get the Cabibbo angle value from the model.

Let us assume that quarks are mixed in both subspaces simultaneously, then the charged weak current is expressed as follows

$$J_\mu = (\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1-\gamma_5)U_+U_-\begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{5}$$

where

$$U_+ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{bmatrix}\begin{bmatrix} c_5 & s_5 & 0 \\ -s_5 & c_5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_6 & s_6 \\ 0 & -s_6 & c_6 \end{bmatrix}, \tag{6a}$$

$$U_- = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_7 & s_7 \\ 0 & -s_7 & c_7 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_1} \end{bmatrix}\begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{bmatrix}. \tag{6b}$$

The matrices U_+ and U_- mix quarks in spaces with charges $+2/3$ and $-1/3$ respectively.

The Kobayashi-Maskawa mixing matrix is parametrized by four independent parameters only, while in the product of the matrices (6a) and (6b) in the current (5) there are eight mixing angles. In order to get the effective mixing matrix in the K$-$M form with the right number of independent mixing parameters, we must adjust the angles in such a way as to get the effective matrix with only four independent angles. We shall demand the following elements U_{11}, U_{12}, U_{13}, U_{21}, U_{31} of the effective matrix to be real and the complex phase to exist in the elements U_{22}, U_{23}, U_{32}, U_{33} only, as in the original K−M matrix. The only solution is

$$\theta_6 = -\theta_7. \tag{7}$$

Hence in the matrix U_+U_- there will be effectively only four parameters: $\theta_2, \theta_C = \theta_1 + \theta_5, \theta_3$, $\delta = \delta_1 + \delta_2$.

The current (5) can be expressed as follows

$$J_\mu = R_2 R_1 J_\mu(0) R_1^{-1} R_2^{-1}, \tag{8}$$

where

$$J_\mu(0) = (\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1-\gamma_5) I \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{9}$$

$$R_1 = e^{-2i\theta_3 Q^{21}} e^{-2i\theta_1 Q^7} e^{-iX\delta_1} e^{-2i\theta_7 Q^{21}}, \tag{10a}$$

$$R_2 = e^{2i\theta_2 Q^{32}} e^{-iY\delta_2} e^{2i\theta_5 Q^{10}} e^{-2i\theta_7 Q^{32}}, \tag{10b}$$

$$X = \frac{4}{\sqrt{10}} Q^{24} - \frac{1}{\sqrt{15}} Q^{35}, \tag{11a}$$

$$Y = \frac{\sqrt{15}}{3} Q^{35}, \tag{11b}$$

Q^k is the 6×6 matrix representation of the k-th generator of SU_6 group. To get the values of the angles θ_i the Gell-Mann, Oakes and Renner model will be used. In variants A and B [2], because of quark mixing in (d, s, b) sector only, the electromagnetic mass splitting of u and d quarks was neglected. For the simultaneous mixing in both (d, s, b) and (u, c, t) sectors the calculation of the angles θ_i explicitly is not possible (see below formulae (24) and (25)). The Hamiltonian density breaking the chiral $SU_6 \times SU_6$ symmetry is given as follows

$$H_0 = \sum_{j=1}^{6} c_{j^2-1} u^{j^2-1}, \tag{12}$$

where c_i are the symmetry breaking parameters, u^i — the scalar components of the $(\bar{6}, 6)+(6, \bar{6})$ of the chiral $SU_6 \times SU_6$ group.

From the GMOR model [5] we obtain the following relations for masses of pseudoscalar mesons for $SU_6 \times SU_6$ symmetry:

$$\pi = m_\pi^2 f_\pi^2 = Z\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{\sqrt{3}} + \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right), \tag{13a}$$

$$K^+ = m_{K^+}^2 f_{K^+}^2 = Z\left(\frac{c_0}{\sqrt{3}} + \frac{c_3}{2} - \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right), \tag{13b}$$

$$K^0 = m_{K^0}^2 f_{K^0}^2 = Z\left(\frac{c_0}{\sqrt{3}} - \frac{c_3}{2} - \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right), \tag{13c}$$

$$D^+ = m_{D^+}^2 f_{D^+}^2 = Z\left(\frac{c_0}{\sqrt{3}} - \frac{c_3}{2} + \frac{c_8}{2\sqrt{3}} - \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right), \tag{13d}$$

$$D^0 = m_{D^0}^2 f_{D^0}^2 = Z\left(\frac{c_0}{\sqrt{3}} + \frac{c_3}{2} + \frac{c_8}{2\sqrt{3}} - \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right), \tag{13e}$$

$$B^+ = m_{B^+}^2 f_{B^+}^2 = Z\left(\frac{c_0}{\sqrt{3}} + \frac{c_3}{2} + \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{2\sqrt{6}} - \frac{3c_{24}}{2\sqrt{10}} + \frac{c_{25}}{\sqrt{15}}\right), \tag{13f}$$

$$T^+ = m_{T^+}^2 f_{T^+}^2 = Z\left(\frac{c_0}{\sqrt{3}} - \frac{c_3}{2} + \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{2\sqrt{6}} + \frac{c_{24}}{2\sqrt{10}} - \frac{2c_{35}}{\sqrt{15}}\right). \tag{13g}$$

In a model with hierarchical symmetry breaking the highest exact symmetry, which can be assumed, is the $SU_4 \times SU_4$ one. At least one quark in each sector must be massive [6]. Following the procedure described in Ref. [2] the Hamiltonian density breaking the chiral $SU_6 \times SU_6$ symmetry will be rotated in the opposite direction by comparison with the rotation of the weak charged current.

$$H_{SB} = R_{21} R_{11} H_E R_{11}^{-1} R_{21}^{-1}, \tag{14}$$

where

$$R_{11} = e^{-2i\theta_7 Q^{21}} e^{-iX\delta_1} e^{-2i\theta_1 Q^7} e^{-2i\theta_3 Q^{21}}, \tag{15a}$$

$$R_{21} = e^{-2i\theta_7 Q^{32}} e^{-iY\delta_2} e^{2i\theta_5 Q^{10}} e^{2i\theta_2 Q^{32}}. \tag{15b}$$

The exact $SU_4 \times SU_4$ symmetry implies that

$$c_3 = c_8 = c_{15} = \sqrt{5}\, c_0 + c_{35} = 0. \tag{16}$$

The $SU_4 \times SU_4$ invariant Hamiltonian density is given as

$$H_E = P\bar{q}_6 q_6 - V\bar{q}_5 q_5, \tag{17}$$

where

$$P = \sqrt{12}\, c_0 + V, \qquad V = \frac{5}{\sqrt{10}} c_{24}. \tag{18}$$

We shall assume that in the model with hierarchical symmetry breaking the flavor will not be conserved in the intermediate stages of the symmetry breaking, but it will be conserved in the broken symmetry taken as a whole. The symmetry breaking Hamiltonian density retaining only the flavor-conserving part is given as follows

$$H_{(\Delta F=0)} = \bar{q}_6 q_6 P(\lambda - M) - \bar{q}_5 q_5 V(\alpha - A)$$
$$+ \bar{q}_4 q_4 P(\varrho + M) - \bar{q}_3 q_3 V(\beta + A) - \bar{q}_2 q_2 V\gamma + \bar{q}_1 q_1 P\tau, \tag{19}$$

where

$$\alpha = c_1^2 s_3^2 s_7^2 + c_3^2 c_7^2, \tag{20a}$$

$$\beta = c_1^2 s_3^2 c_7^2 + c_3^2 s_7^2, \tag{20b}$$

$$\gamma = s_1^2 s_3^2, \tag{20c}$$

$$A = \tfrac{1}{2} \sin 2\theta_7 \sin 2\theta_3 c_1 \cos \delta_1, \tag{20d}$$

$$\lambda = s_2^2 c_5^2 s_7^2 + c_2^2 c_7^2, \tag{21a}$$

$$\varrho = s_2^2 c_5^2 c_7^2 + c_2^2 s_7^2, \tag{21b}$$

$$\tau = s_2^2 s_5^2, \tag{21c}$$

$$M = -\tfrac{1}{2} \sin 2\theta_7 \sin 2\theta_2 c_5 \cos \delta_2. \tag{21d}$$

The broken Hamiltonian density (19) can be expressed as a function of operators u^k ($k = 0, 3, 8, 15, 24, 35$). The coefficients of the operators u^k are as follows

$$c_0' = c_0, \tag{22a}$$

$$c_3' = \tfrac{1}{2}(P\tau + V\gamma), \tag{22b}$$

$$c_8' = \frac{1}{2\sqrt{3}}(P\tau + V(2\beta' - \gamma)), \tag{22c}$$

$$c_{15}' = \frac{1}{2\sqrt{6}}(P(\tau - 3\varrho') - V(\beta' + \gamma)), \tag{22d}$$

$$c_{24}' = \frac{1}{2\sqrt{10}}(P(\tau + \varrho') - V(5\beta' + 5\gamma - 4)), \tag{22e}$$

$$c_{35}' = \frac{1}{2\sqrt{15}}(6P(\tau + \varrho') - 5P - V), \tag{22f}$$

where

$$\beta' = \beta + A, \qquad \varrho' = \varrho + M. \tag{23}$$

After symmetry breaking the pseudoscalar masses (13) will be described as functions of the coefficients c_i' [4].

$$\pi = \frac{Z}{2}(P\tau - V\gamma), \tag{24a}$$

$$K^+ = \frac{Z}{2}(P\tau - V\beta'), \tag{24b}$$

$$K^0 = -\frac{Z}{2}V(\beta'+\gamma), \tag{24c}$$

$$D^0 = \frac{Z}{2}P(\varrho'+\tau), \tag{24d}$$

$$D^+ = \frac{Z}{2}(P\varrho'-V\gamma), \tag{24e}$$

$$B^+ = \frac{Z}{2}(P\tau+V(\beta'+\gamma-1)), \tag{24f}$$

$$T^+ = \frac{Z}{2}(P(1-\tau-\varrho')-V\gamma). \tag{24g}$$

From (20), (21), (23) and (24) we get

$$\beta' = \frac{K^0+K^+-\pi}{2B^++3K^0-K^+-\pi}, \tag{25a}$$

$$\gamma = \frac{K^0-K^++\pi}{2B^++3K^0-K^+-\pi}, \tag{25b}$$

$$\varrho' = \frac{D^0+D^+-\pi}{2T^++3D^0-D^+-\pi}, \tag{25c}$$

$$\tau = \frac{D^0-D^++\pi}{2T^++3D^0-D^+-\pi} \tag{25d}$$

(contrary to the case of mixing in (d, s, b) sector only (variant A in Ref. [2]) the electromagnetic mass splitting of u and d quarks cannot be neglected; if we put arbitrarily $c_3 = 0$ in Eq. (12) as in variants A and B in Ref. [2], the parameters γ, β', τ, ϱ' could not be calculated separately. We would obtain only three nonlinear relations connecting these parameters with meson masses.)

Since

$$\alpha+\beta+\gamma = \lambda+\varrho+\tau = 1, \tag{26}$$

putting (20, 21, 23) to (25) and eliminating θ_2 and θ_3 from the obtained set of four equations we get

$$f_1(\theta_1, \delta_1) = \tan\theta_7 = -f_5(\theta_5, \delta_2), \tag{27}$$

where

$$f_1(\theta_1, \delta_1) = \frac{B_1 \mp \sqrt{B_1^2-A_1C_1}}{A_1}, \tag{28}$$

$$A_1 = s_1^2(1-\beta')-\gamma, \tag{29a}$$

$$B_1 = \sqrt{\gamma}\sqrt{s_1^2-\gamma}\, c_1 \cos\delta_1, \tag{29b}$$

$$C_1 = \gamma - s_1^2(\beta'+\gamma), \tag{29c}$$

$$f_5(\theta_5, \delta_2) = \frac{B_5 \mp \sqrt{B_5^2 - A_5 C_5}}{A_5}, \tag{30}$$

$$A_5 = s_5^2(1-\varrho')-\tau, \tag{31a}$$

$$B_5 = \sqrt{\tau}\sqrt{s_5^2-\tau}\, c_5 \cos\delta_2, \tag{31b}$$

$$C_5 = \tau - s_5^2(\varrho'+\tau). \tag{31c}$$

In Ref. [4] the simultaneous mixing in (d, s) and (u, c) sectors in the $SU_4 \times SU_4$ symmetry was considered. The mixing angles θ and ϕ could not be calculated separately, however the nonlinear formula connecting both angles and pseudoscalar masses was found

$$2\pi + 2(K+D)\sin^2\theta\sin^2\phi = (2K+\pi)\sin^2\theta + (2D+\pi)\sin^2\phi. \tag{32}$$

A numerical calculation showed that there is an extremum (a maximum) of the function

$$f(\theta, \phi) = \sin(\theta+\phi) \tag{33}$$

with condition (32) for the angles $\theta_m + \phi_m$ very close to the experimentally measured Cabibbo angle. This fact suggests that the symmetry breaking is realized in the maximal allowed case, so the effective angle of mixing would correspond to the maximum of function (33).

As in Ref. [4] we shall look for the extremum of the function

$$f(\theta_1, \theta_5) + \sin(\theta_1+\theta_5) \tag{34}$$

with condition (27). The following set of equations must be obeyed

$$f_1(\theta_1, \delta_1) + f_5(\theta_5, \delta_2) = 0, \tag{35a}$$

$$\frac{\partial f_1(\theta_1, \delta_1)}{\partial \theta_1} - \frac{\partial f_5(\theta_5, \delta_2)}{\partial \theta_5} = 0, \tag{35b}$$

$$\frac{\partial f_1(\theta_1, \delta_1)}{\partial \delta_1} = 0, \tag{35c}$$

$$\frac{\partial f_5(\theta_5, \delta_2)}{\partial \delta_2} = 0. \tag{35d}$$

From (35c) and (35d) we get

$$C_1 = 0, \quad C_5 = 0 \tag{36}$$

respectively, which implies that the separation constant

$$\tan\theta_7 = 0. \tag{37}$$

This means that the maximal allowed symmetry breaking occurs only for independent mixing of quarks in both sectors.

Let us consider the action of the operators R_{11}, R_{21} on quarks. The operator R_{11} mixes quarks in the negative electric charge subspace in the following sequence: (s−b) (θ_3), (d−s) (θ_1), a phase rotation (δ_1), (s−b) (θ_7), however the operator R_{21} mixes quarks as follows: (c−t) (θ_2), (u−c) (θ_5), a phase rotation (δ_2), (c−t) (θ_7). By the exact $SU_4 \times SU_4$ symmetry only b and t quarks are massive. After the symmetry breaking a massless quark can become massive if it mixes with the other massive one [6]. By the mixing in the sector with the charge $-1/3$ the quark s has become massive in the first stage of the hierarchical symmetry breaking, after mixing with the quark b (the rotation on the angle θ_3 generated by the operator Q^{21}), the quark d has become massive in the second stage after mixing with the already massive quark s (the rotation on the angle θ_1). The next rotation by the angle θ_7 and mixing of s and b quarks are not connected with the symmetry breaking, because the mixing quarks have been already massive. There is analogical situation in the sector with the charge $+2/3$. The c and u quarks have become massive due to the hierarchical symmetry breaking (rotations on angles θ_2, θ_5 respectively), however the rotation by the angle θ_7 and mixing of c and t quarks are also not connected with the symmetry breaking. Thus, from (37) it results that the physical quark mixing is realized only in the symmetry breaking with the quark masses generation. Putting (37) to (20) and (21) and comparing with (25) we get

$$\sin^2\theta_1 = \frac{K^0 - K^+ + \pi}{2K^0}, \tag{38a}$$

$$\sin^2\theta_5 = \frac{D^0 - D^+ + \pi}{2D^0}, \tag{38b}$$

$$\sin^2\theta_3 = \frac{2K^0}{2B^+ + 3K^0 - K^+ - \pi}, \tag{38c}$$

$$\sin^2\theta_2 = \frac{2D^0}{2T^+ + 3D^0 - D^+ - \pi}, \tag{38d}$$

so $\theta_C = \theta_1 + \theta_5$ depends on the parameters of mesons belonging only to the SU_4 multiplet.

Let us notice that in comparison to the variant A in Ref. [2], taking into account the quark mixing in the (u, c, t) sector allowed the author to calculate the Cabibbo angle from the model and the angles θ_2 and θ_3.

Let us compare the value of the calculated angle $\theta_C = \theta_1 + \theta_5$ realized for the maximal symmetry breaking with the experimentally measured Cabibbo angle value

$$\cos\theta = 0.9737 \pm 0.0025 \quad [11]. \tag{39}$$

The well known values of meson masses were taken from [7]. $f_\pi, f_{K^+}, \ldots$ were assumed as the factors in the matrix elements between one meson state and the vacuum according to PCAC, so for meson multiplets with the isospin 1/2 the factors for charged and neutral mesons are the same [8]. There exist many conjectures concerning the values of f_x. They widely differ in magnitude, depending on the particular approach to the estimation of the matrix element $\langle 0|v_x|x\rangle$ and so far they have no reliable experimental support. Only in the case of f_K there is a fair consensus that the value is around 1.28 [8, 9, 10, 12]. For a calculation we took as f_D for comparison's sake values significantly different

$$f_D = 0.974 \quad [9], \tag{40a}$$

$$f_D = 0.65 \quad [12], \tag{40b}$$

which give

$$\cos(\theta_1+\theta_5) = 0.9799, \tag{41a}$$

$$\cos(\theta_1+\theta_5) = 0.9709, \tag{41b}$$

respectively, very close to the experimental value (39), as in the case of the $SU_4\times SU_4$ broken symmetry [4]. It seems to us that such a well agreement in both $SU_4\times SU_4$ and $SU_6\times SU_6$ symmetries is not accidental and the symmetry breaking is indeed realized for the maximal allowed case.

Putting (36) to (35b) we find the relation connecting both phase parameters

$$\cos\delta_2 = \xi\cos\delta_1, \tag{42}$$

where

$$\xi = \sqrt{\frac{\gamma\varrho'(1-\gamma-\beta')(\varrho'+\tau)}{\tau\beta'(1-\tau-\varrho')(\beta'+\gamma)}}. \tag{43}$$

The effective phase parameter $\delta = \delta_1+\delta_2$ is bounded for $\xi \neq 1$. Indeed, the equation (42) has solution only for

$$|\delta| > \arccos\left(\frac{1}{\xi}\right) \quad \text{if} \quad (\xi > 1), \tag{44a}$$

or

$$|\delta| > \arccos(\xi) \quad \text{if} \quad (\xi < 1). \tag{44b}$$

It is worth noticing that even for $\xi \to \infty$ or $\xi \to 0$ the second and third quadrant for δ is still allowed.

APPENDIX

From the GMOR model for $SU_6\times SU_6$ symmetry we obtain the following relation for masses of pseudoscalar mesons

$$m_a^2 f_a^2 \delta^{ab} + \int \frac{dq^2}{q^2}\varrho^{ab} = i\langle 0|\,[\bar{Q}^a, \bar{D}^b]\,|0\rangle$$

$$= \sum_{i=1}^{6}\left(\sum_{j=1}^{6} c_{j^2-1} d_{i^2-1,a,c} d_{j^2-1,b,c}\right)(u^{i^2-1})_0, \tag{A1}$$

where

$$\varrho^{ab} = (2\pi)^3 \sum_{n \neq a} \delta^4(p_n - q) \langle 0|\bar{D}^a|n\rangle \langle n|\bar{D}^b|0\rangle, \tag{A2}$$

d_{abc} symmetric constants of the SU_6 group, $(u^i)_0$ — vacuum expectation value of the operator u^i, $Q^i \pm \bar{Q}^i = \int d^3x V_0^a(x) \pm \int d^3x A_0^a(x)$ — the generators of the $SU_6 \times SU_6$ group

$$D^a = \partial^\mu V_\mu^a(x), \quad \bar{D}^a = \partial^\mu A_\mu^a(x) \quad [10].$$

Because the vacuum expectation values of operators u^i: $i = 3, 8, 15, 24, 35$ and the spectral density ϱ^{ab} are proportional to the squared parameters of symmetry breaking, they were neglected. Approximately we obtain

$$m_a^2 f_a^2 = Z \sum_{j=1}^{6} c_{j^2-1} d_{j^2-1,a,a}, \tag{A3}$$

where

$$Z = \frac{1}{\sqrt{3}} (u^0)_0. \tag{A4}$$

Because the symmetric constants of SU_6 group

$$d_{113} = d_{223} = d_{333} = 0 \tag{A5}$$

the masses of neutral and charged pions are not differentiated, however there is the electromagnetic mass splitting of the other meson multiplets (see Eqs. (13)).

The experimental data [7] gives

$$\Delta m_K = m_{K^0} - m_{K^+} = 4.003 \text{ MeV}, \tag{A6a}$$

$$\Delta m_D = m_{D^0} - m_{D^+} = -5.3 \text{ MeV}, \tag{A6b}$$

so

$$\text{sign } \Delta m_K = -\text{sign } \Delta m_D. \tag{A7}$$

Let us notice that from (13) we get

$$\text{sign } (K^0 - K^+) = -\text{sign } (D^0 - D^+), \tag{A8}$$

so the direction of the electromagnetic mass splitting by thefac tor c_3 responsible for this effect is consistent with the experimental data. On the other hand

$$\Delta m_\pi = m_{\pi^0} - m_{\pi^+} = -4.603 \text{ MeV}, \tag{A9}$$

so the electromagnetic mass splitting of pions is of the same order as kaons or D mesons. It suggests that the neglected terms in approximate formula (A3) are of the order of the factor c_3. This means that such an approximation does not generate error greater than the electromagnetic mass splitting of pion in meson masses description.

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