SINGLET FORM FACTORS AND LOCAL OBSERVABLES IN THE GLASHOW-WEINBERG-SALAM MODEL*

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Within the Glashow-Weinberg-Salam model local observables like the electromagnetic current and field strength tensor must be defined as singlet fields in order to be gauge invariant and to satisfy Maxwell's equations. We show that S-matrix elements of the physical electromagnetic current (lepton form factors) are non renormalizable due to "anomalous" short distance behaviour. An explicit calculation demonstrates that there is a one to one correspondence between gauge invariant short distance anomalous terms and gauge dependent terms in the standard R-gauge formulation. As a result, the gauge invariant electron current is an observable only on the classical level. When quantum corrections become relevant unphysical short distance behaviour is exhibited by the form factors and thus quantities like the charge radius of the electron or the neutrino become ambiguous concepts.

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1. *Introduction*

The aim of this article is a study of properties of local observables in the Glashow-Weinberg-Salam (GSW) model [1]. Explicitly, we investigate the electromagnetic current, the electromagnetic field strength and the validity of Maxwell's equations. The short distance properties of electromagnetic lepton form factors are computed in the one-loop approximation using the singlet field current. A brief account of the results obtained for the high energy behavior of the physical electromagnetic current has been presented in Ref. [2].

Locality is one of the basic properties (together with relativistic covariance, unitarity and renormalizability [3]) of quantum gauge field theory models which rather successfully describe elementary particle dynamics. Physical local fields like currents, field strength tensors and the energy momentum tensor are the quantities which admit direct tests of causal space-time properties of interactions. Though actual measurements always take place in finite space-time regions, experimental data are fitted usually, with good accuracy,

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to S-matrix elements evaluated using LSZ-asymptotic conditions for scattering states (we disregard the infrared problem of QED here). We are interested here to analyse S-matrix elements of local observables.

Apart from the conceptual questions, there is a direct practical interest in studying physical form factors since these are supposed to define physical effective (running) couplings. The latter are important in electroweak theory because the GWS model describes processes at extremely different energy scales μ, like low energy lepton processes and vector boson processes, and a suitable reparametrisation of the theory admits to keep radiative corrections small at very different scales.

The common approach to the concept of running parameters uses the renormalization group. This approach suffers, either from a lack of decoupling of heavy particles $m \gg \mu$ (which should not influence physics at lower energy scales $E < \mu$) in the gauge invariant and infrared finite minimal subtraction ($\overline{\text{MS}}$) scheme, or from lack of gauge invariance in the momentum subtraction schemes (off shell renormalization) [4].

One could expect that a more physical definition of the effective electrical charge $e(\mu^2)$, for example, is directly given by the universal lepton form factor

$$e(q^2) = e(1+\Delta F_{\text{E}}(q^2)), \tag{1.1}$$

which exhibits the proper physical thresholds (excitation of virtual pairs). Our investigation has led to an obstruction of this expectation. In a renormalizable gauge $F_{\text{E}}(q^2)$ is gauge dependent and turns out to have no unitary gauge limit when $q^2 \neq 0$. In this gauge the gauge parameter functions as an ultraviolet cut off (in a similar way as in the gauge boson propagators). In a physical gauge, the singlet form factor turns out not to be renormalizable.

Since the S-matrix is renormalizable it seems doubtful that non-renormalizable physical fields are observable though renormalizability often is considered to be merely a technical requirement rather than a physical principle. Indeed, it seems impossible to measure the electromagnetic current and the field strength tensor beyond the validity of the one photon exchange approximation. The latter breaks down at high momentum transfer, due to the additional exchange of the Z-boson (mixing of currents), two photons, γZ and so on. Apparently the range of the weak interactions, which is given by the inverse vector boson mass M_{W}^{-1}, sets the scale beyond which local observables cease to be measurable quantities. In addition actual observability of form factors of a particle is limited by the Compton wavelength m^{-1} of the particle. As a low energy concept local observables have their physical meaning and describe perfectly charge sectors, the presence of the massless photon and the infrared structure of the theory.

The proper definition of local observables and physical form factors in non abelian gauge theories is not completely trivial. In principle, physical fields are given by the classical expressions in the unitary gauge. Since renormalization in this gauge is rather obscure we shall follow a different approach and use Kibble's singlet fields[1] (unitary gauge orbit fields) [5] and perform our analysis in the renormalizable 't Hooft gauge. The relevance

[1] Kibble has noted the "absence of symmetry" when using these variables.

of singlet fields in gauge theories has been pointed out by many authors. Sucher and Woo [6] use them implicitly to argue about the physical interpretation of the Higgs mechanism and the absence of spontaneous breaking of local gauge symmetries.

Within the context of the manifestly gauge invariant lattice gauge models Wegner [7] first proposed the Z_2-lattice gauge model as a statistical mechanics system without local order parameter exhibiting a phase transition. It is shown that only singlet fields (non local in this case) have non trivial correlation functions. Elitzur [8] pointed out the impossibility of spontaneously breaking local symmetries and Banks and Rabinovici [9] stressed the singlet field nature of the GWS model.

Independently, 't Hooft [10] exhibited explicitly composite singlet fields as interpolating fields for the physical particles (confinement phase picture).

It is well known that the Green functions of interpolating singlet particle fields (which coincide with the standard fields in the unitary gauge) are not renormalizable. The main result of our investigation is that the Green functions of (some) physical observables are not renormalizable either.

Since explicit perturbative calculations using singlet fields are uncommon, we will briefly review the singlet field formulation of the GWS model and show the equivalence of different formal representations in Sections 2 and 3. The electromagnetic quantities are discussed in Section 4. In Section 5 we analyse the electromagnetic form factors of the electron and present the analytic results for large momentum transfers. A discussion of our results and of possible implications follows in Section 6.

2. *U-gauge orbit fields*

We consider the GWS model in the renormalizable 't Hooft gauge (R-gauge). Associated with the local gauge group $SU(2)_L \otimes U(1)_Y$ we have the gauge fields $W_{\mu a}$ and B_μ. The gauge couplings are g and $g'/2$, respectively. The field strength tensors are $G_{\mu\nu a} = \partial_\mu W_{\nu a} - \partial_\nu W_{\mu a} + g\varepsilon_{abc} W_{\mu b} W_{\nu c}$ and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ with associated conserved currents $\partial_\mu G_a^{\mu\nu} = -gJ_a^\nu$ and $\partial_\mu B^{\mu\nu} = -g'/2 J_Y^\nu$ (the currents considered will always include the terms from the gauge fixing part $\mathscr{L}_{GF}$ and the Faddeev-Popov part $\mathscr{L}_{FP}$ of the effective Lagrangian $\mathscr{L}_R$). Obviously the conserved current J_a^ν is not covariant whereas the covariant matter field current $D_\mu G_a^{\mu\nu} = \partial_\mu G_a^{\mu\nu} + g\varepsilon_{abc} W_{\mu b} G_c^{\mu\nu} = -gj_a^\nu$ is not conserved. J_Y^ν is a conserved singlet current.

The charges associated with the non-abelian gauge group $SU(2)_L$ must be screened in a phase where the gauge bosons $W_{\mu a}$ are massive (we suppose $g' = 0$ for simplicity of the argument here) since in this case

$$Q_a = \int d^3x J_a^0 = -\frac{1}{g}\int d^3x \partial_i G_a^{i0} = 0$$

by virtue of Gauss' law. This infers the existence of an effective scalar field Φ_b (elementary or composite) which screens the $SU(2)_L$ quantum numbers of the physical fields. Φ_b must transform as a true representation of the gauge group. In principle a non-linear realisation with 3 fields and $\Phi_b^+ \Phi_b = \frac{v^2}{2}$ = const. would be possible but this would not lead to a renormalizable theory. The case of a linear realisation yields the standard Higgs doublet Φ_b with 3+1 fields. We shall discuss the screening of $SU(2)_L$ in the GWS model in detail in the following.

The Higgs field[2]

$$\Phi_b = \begin{pmatrix} i\varphi^+ \\ \varphi_0 \end{pmatrix} = \frac{1}{\sqrt{2}}(H_s + i\tau_i \varphi_i)\chi_b;\ \chi_b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.1a}$$

and its Y-charge conjugate $\Phi_t = i\tau_2 \Phi_b^*$

$$\Phi_t = \begin{pmatrix} \varphi_0^* \\ i\varphi^- \end{pmatrix} = \frac{1}{\sqrt{2}}(H_s + i\tau_i \varphi_i)\chi_t;\ \chi_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.1b}$$

with covariant derivatives

$$D_\mu \Phi_b = \left(\partial_\mu - \frac{g'}{2} B_\mu - \frac{g}{2} W_{\mu a}\tau_a\right) \Phi_b$$

and

$$D_\mu \Phi_t = \left(\partial_\mu + \frac{g'}{2} B_\mu - \frac{g}{2} W_{\mu a}\tau_a\right) \Phi_t,$$

respectively, enter the terms

$$\mathscr{L}_{\text{Higgs}} = (D_\mu \Phi_b)^+ (D^\mu \Phi_b) - V(\Phi_b^+ \Phi_b)$$

and

$$\mathscr{L}_{\text{Yukawa}} = -(\bar{L}_\psi(\Phi_t, \Phi_b) G_\psi R_\psi + \text{h.c.})$$

of the invariant Lagrangian. In the Yukawa term G_ψ is a diagonal coupling matrix $G_\psi \doteq \text{diag}(G_t, G_b)$, L_ψ is the left handed fermion doublet and R_ψ are the two associated right handed singlets:

$$\Psi = \begin{pmatrix} \psi_t \\ \psi_b \end{pmatrix}, \quad L_\psi = \Pi_- \Psi, \quad R_\psi = \Pi_+ \Psi; \quad \Pi_\pm \doteq \frac{1 \pm \gamma_5}{2}. \tag{2.2}$$

In the Higgs phase the $SU(2)_L \otimes U(1)_Y$ symmetry is broken to the residual electromagnetic $U(1)_{em}$. The ground state is characterized in *a particular gauge* by a translationally invariant classical background field

$$(\Phi_b)_0 = \Phi_0 = \frac{v}{\sqrt{2}} \chi_b; \qquad v > 0 \tag{2.3}$$

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[2] τ_i are the Pauli matrices; $\varphi^\pm = (\varphi_1 \mp i\varphi_2)/\sqrt{2}$, $\varphi_0 = (H_s - i\varphi)/\sqrt{2}$ and $\varphi = \varphi_3$.

such that the shifted field $\hat{\Phi}_b = \Phi_b - \Phi_0$ has zero vacuum expectation value $\langle \hat{\Phi}_b \rangle = 0$. Equivalently, for the "physical" Higgs field $H = H_s - v$, $\langle H \rangle = 0$.

In the particular gauge considered the gauge boson mass-term $(D_\mu \Phi_0)^+(D^\mu \Phi_0)$ is diagonalized, on the classical level, by the orthogonal transformation to the standard "physical" gauge fields

$$A_\mu = \frac{gB_\mu + g' W_\mu^3}{\sqrt{g^2+g'^2}}, \quad Z_\mu = \frac{gW_\mu^3 - g' B_\mu}{\sqrt{g^2+g'^2}}, \quad W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}. \tag{2.4}$$

The massless photon field A_μ defined by (2.4) is not abelian and thus cannot be the proper $U(1)_{em}$ gauge field beyond the tree approximation.

The reason why the photon field is not obtained correctly by the above argument is that no care has been taken of the fact that all ground states $|0\rangle_{\Phi_0}$ associated with gauge equivalent background fields

$$\Phi_0 = \frac{v}{\sqrt{2}} \hat{U} \chi_b, \quad \hat{U} = U U_0,$$

$$U_0 = e^{ig'/2\omega_0} \in U(1)_Y, \quad U = e^{ig/2\tau_i \omega_i} \in SU(2)_L \tag{2.5}$$

are physically equivalent.

Kibble [5] has constructed the fields which account for the gauge equivalence of the ground states $|0\rangle_{\Phi_0}$. Writing the Higgs field in polar form

$$\Phi_b = \frac{\varrho + v}{\sqrt{2}} U(\theta) \chi_b, \quad U(\theta) = \exp\left(i \frac{\tau_i}{2} \frac{\theta_i}{v} \right) \tag{2.6}$$

we observe that relative to (2.3)

$$\Phi_0 = \frac{v}{\sqrt{2}} U(\theta) \chi_b \tag{2.7}$$

is an equivalent background field. The physical fields are obtained now by the replacement of the standard R-gauge fields by the U-gauge fields[3] [11]:

$$\Phi_b^u = U^+(\theta) \Phi_b = \frac{\varrho + v}{\sqrt{2}} \chi_b,$$

$$W_{\mu a}^u \tau_a = U^+(\theta) W_{\mu a} \tau_a U(\theta) + \frac{2i}{g} U^+(\theta) \partial_\mu U(\theta),$$

$$L_\psi^u = U^+(\theta) L_\psi. \tag{2.8}$$

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[3] With $T_a = \frac{\tau_a}{2}$, $\bar{\theta}_a = \frac{\theta_a}{v}$ and $[T_a, T_b] = (t_b)_{ac} T_c$ Kibble's original form $W_{\mu b}^u = (\exp(-it_i \bar{\theta}_i))_{ba} W_{\mu a} + ig^{-1} \Lambda_{ba}(\bar{\theta}) \partial_\mu \bar{\theta}_a$ follows from (2.8) using the identities $U^+(\theta) T_a U(\theta) = T_b (\exp(-it_i \bar{\theta}_i))_{ba}$ and $U^+(\theta) \partial_\mu U(\theta) = T_b \left(\frac{1 - \exp(-it_i \bar{\theta}_i)}{t_i \bar{\theta}_i} \right)_{ba} \partial_\mu \bar{\theta}_a = T_b \Lambda_{ba}(\bar{\theta}) \partial_\mu \bar{\theta}_a$.

Eq. (2.8) defines an operator gauge-transformation by which the Higgs ghosts are eliminated from the invariant part of the Lagrangian:

$$\mathscr{L}_{\rm inv}(W, B, \Phi_b, \Phi_t, L_\psi, R_\psi) = \mathscr{L}_{\rm inv}(W^u, B, \Phi_b^u, \Phi_t^u, L_\psi^u, R_\psi). \tag{2.9}$$

In particular the Higgs Lagrangian takes the form

$$\mathscr{L}_{\rm Higgs} = \tfrac{1}{2}(\partial_\mu\varrho\partial^\mu\varrho) + \frac{(\varrho+v)^2}{2v^2}(M_Z^2 Z_\mu^u Z^{u\mu} + 2M_W^2 W_\mu^{u+} W^{u\mu-}) - V\left(\frac{(\varrho+v)^2}{2}\right) \tag{2.10}$$

and

$$\mathscr{L}_{\rm Yukawa} = -\bar{\psi}_i^u \psi_i^u m_i \left(1+\frac{\varrho}{v}\right).$$

Thus, on the one hand (2.8) provides a transformation to the unitary gauge (U-gauge) characterized by $\varphi_i = \frac{\theta_i}{2} = 0$ and $H = \varrho$, on the other hand (2.8) defines the *U-gauge field orbits* in the R-gauge.

It may be worthwhile to notice that in the physical gauge it is the *discrete* symmetry $\varrho \to -\varrho$ which is spontaneously broken. This also explains the absence of physical Goldstone bosons. We also notice that $\langle \varrho_s \rangle = v$ is a physical gauge singlet order parameter ($\varrho_s = \varrho + v$). Indeed, the Higgs mechanism is not a spontaneous breaking of a local gauge symmetry in the physical Hilbert space [6, 8].

3. $SU(2)_L \otimes U(1)_Y$ *screening to* $U(1)_{em}$

The crucial observation is that under $SU(2)_L \otimes U(1)_Y$ the U-gauge fields W^u, Φ^u, L_ψ^u transform completely different from the standard R-gauge fields W, Φ, L_ψ. In fact, the $SU(2)_L \otimes U(1)_Y$ charges of the R-gauge fields are screened up to the electric charge by the Higgs ghost fields $U(\theta)$ [9].

In order to see this we have to consider the transformation properties of $U(\theta)$ under the gauge transformations (2.5). We first consider the matrix field

$$\tilde{\phi} = \frac{1}{\sqrt{2}}(H+v+i\tau_i\varphi_i) = (\Phi_t, \Phi_b) \tag{3.1}$$

which transforms according to

$$\tilde{\phi}' = (UU_0^{-1}\Phi_t, UU_0\Phi_b) = U\tilde{\phi}\tilde{U}_0; \quad \tilde{U}_0 = \exp(-ig'/2\omega_0\tau_3) \tag{3.2}$$

and hence has a covariant derivative

$$\tilde{D}_\mu\tilde{\phi} \doteq \partial_\mu\tilde{\phi} + i\frac{g'}{2}B_\mu\tilde{\phi}\tau_3 - i\frac{g}{2}W_{\mu a}\tau_a\tilde{\phi}. \tag{3.3}$$

By definition the relations

$$\tilde{\phi}\chi_b = \Phi_b, \quad \tilde{\phi}\chi_t = \Phi_t, \quad \tilde{D}_\mu\tilde{\phi}\chi_b = D_\mu\Phi_b \quad \text{and} \quad \tilde{D}_\mu\tilde{\phi}\chi_t = D_\mu\Phi_t \tag{3.4}$$

hold[4]. Since in the Higgs phase ($v > 0$) the singlet field

$$\Phi_b^+\Phi_b = \frac{(\varrho+v)^2}{2} = \frac{v^2}{2}(1+X); \qquad X = 2\frac{H}{v} + \frac{H^2+\varphi^2}{v^2} + 2\frac{\varphi^+\varphi^-}{v^2} \tag{3.5}$$

is strictly positive we may relate $U(\theta)$ to $\tilde{\phi}$ by the regular transformation

$$U(\theta) = (\sqrt{\Phi_b^+\Phi_b})^{-1}\tilde{\phi} \tag{3.6}$$

(for the components: $H+v = (\varrho+v)\cos\frac{\theta}{2v}$, $\varphi_i = (\varrho+v)\frac{\theta_i}{\theta}\sin\frac{\theta}{2v}$ with $\theta = \sqrt{\theta_i\theta_i}$). According to (3.2), $U(\theta)$ then transforms as

$$U(\theta)' = UU(\theta)\tilde{U}_0.$$

We observe that we may write, using (2.8) and (3.3),

$$X^u \doteq -g'B_\mu\tau_3 + gW^u_{\mu a}\tau_a = i(U^+(\theta)\tilde{D}_\mu U(\theta) - \text{h.c.})$$

such that the "physical" gauge fields are given by

$$\chi_b^+ X^u \chi_b = -\sqrt{g^2+g'^2}\, Z^u_\mu = (\Phi_b^+\Phi_b)^{-1}\cdot i(\Phi_b^+ D_\mu\Phi_b - \text{h.c.})$$

and

$$\chi_t^+ X^u \chi_b = \sqrt{2}\, gW^{u+}_\mu = (\Phi_b^+\Phi_b)^{-1}\cdot i(\Phi_t^+ D_\mu\Phi_b - (D_\mu\Phi_t)^+\Phi_b). \tag{3.7}$$

Evidently these fields transform under $SU(2)_L \otimes U(1)$ as $U(1)_{em}$ fields, since

$$(Z^u_\mu)' = Z^u_\mu; \qquad (W^{u\pm}_\mu)' = e^{\pm ie\omega_A}W^{u\pm}_\mu, \tag{3.8}$$

where we have identified $g'\omega_0 = e\omega_A$ (see below). It turns out, in particular, that the U-gauge fields are *SU(2)-singlet fields*. The Higgs field is also a singlet now:

$$\chi_b^+\Phi_b^u = \sqrt{\Phi_b^+\Phi_b} = \frac{\varrho+v}{\sqrt{2}}; \qquad \chi_t^+\Phi_b^u = 0. \tag{3.9}$$

Similarly for the fermi fields (2.2), which transform as

$$L'_\psi = U(U_0)^{y_-}L_\psi, \qquad R'_\psi = \tilde{U}_0^+(U_0)^{y_-}R_\psi,$$

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[4] In the limit $g' = 0$ and $G_\psi = 0$ (massless fermions) a *global* $SU(2)_L \otimes SU(2)_R$ symmetry: $\chi_i = a_i\chi_b + b_i\chi_t$, $|a_i|^2+|b_i|^2 = 1$ would be present as follows from (3.3) and (3.4) and the form of $\mathscr{L}_{inv}$ [12]. In this case $W^u \to U_L W^u U_L^+$($SU(2)_L$ triplet), $\psi_L^u \to U_L\psi_L^u$($SU(2)_L$ doublet), $\psi_R^u \to U_R\psi_R^u$($SU(2)_R$ doublet) and $\varrho \to \varrho$ ($SU(2)_L \otimes SU(2)_R$ singlet) under the global group. The associated Noether currents are $J^{uv}_{Li} = -\varepsilon_{ikl}G^{u\mu\nu}_k W^u_{\mu l} + \bar{L}^u_\psi \frac{\tau_i}{2}\gamma^\nu L^u_\psi = -\frac{1}{g}\partial_\mu G^{u\mu\nu}_i - \frac{(\varrho+v)^2}{4}W^{u\nu}_i$ (by the equation of motion) and $J^{uv}_{Ri} = \bar{R}^u_\psi\frac{\tau_i}{2}\gamma^\nu R^u_\psi$. In the GWS model this symmetry is broken, but it nevertheless has its physical drawback in the screening of Higgs effects [13].

with y_- the Y-charge of $L_\varphi (y_- = -1$ for leptons, $y_- = 1/3$ for quarks), we obtain

$$\chi_b^+(L_\varphi^u + R_\varphi) = (\sqrt{\Phi_b^+ \Phi_b})^{-1} \Phi_b^+ L_\varphi + \psi_{bR} \doteq \psi_b^u$$

and

$$\chi_t^+(L_\varphi^u + R_\varphi) = (\sqrt{\Phi_b^+ \Phi_b})^{-1} \Phi_t^+ L_\varphi + \psi_{tR} \doteq \psi_t^u. \tag{3.10}$$

Again the U-gauge fermi-fields are SU(2)-singlets transforming as $U(1)_{em}$ fields:

$$(\psi_i^u)' = e^{ie\omega_A q_i} \psi_i^u; \quad (i = b, t) \tag{3.11}$$

with q_i the electric charges of the fields in units of e.

Some remarks are in order concerning the unusual non-polynomial composite form of the U-gauge fields:

(i) In the Higgs phase ($v > 0$), according to (3.5), the singlet field

$$S = \Phi_b^+ \Phi_b = \frac{v^2}{2}(1+X) > 0 \tag{3.12}$$

is strictly positive and $X = O(v^{-1})$. Thus functions $F(S)$ analytic in Re $S > 0$, as S^{-1} and $(\sqrt{S})^{\pm 1}$ appearing in (3.7), (3.9) and (3.10), are well-defined as perturbation series in the loop-expansion parameter v^{-1}. In lowest order, with $S = \frac{v^2}{2}$, $\Phi_b = \frac{v}{\sqrt{2}} X_b$ and $\Phi_t = \frac{v}{\sqrt{2}} \chi_t$, the U-gauge fields X_i^u agree with the corresponding R-gauge fields X_i^R.

(ii) The U-gauge fields are linear in the corresponding R-gauge multiplet fields.

(iii) Formally, in the U-gauge $\left(\varphi_i = \frac{\theta_i}{2} = 0, H = \varrho\right)$, with $S = \frac{v^2}{2}\left(1 + \frac{\varrho}{v}\right)^2$, $\Phi_b = \frac{\varrho + v}{\sqrt{2}} \chi_b$ and $\Phi_t = \frac{\varrho + v}{\sqrt{2}} \chi_t$, the U-gauge fields coincide with the standard (classical) expressions. This property together with the singlet nature of the physical fields uniquely determines the U-gauge fields. The aim of the non-polynomial singlet normalization factors obviously is to eliminate the *physical composite* fields which would be present otherwise.

As seen from the physical U-gauge, all the physical particles are singlets and therefore have arbitrary masses. How does the gauge group $SU(2)_L \otimes U(1)_Y$ manifest itself physically? Obviously it still determines the dynamics (notice that apart from $\mathscr{L}_{\text{Higgs}}$ and $\mathscr{L}_{\text{Yukawa}}$ the form of $\mathscr{L}_{\text{inv}}$ remains unchanged). Typically, given the electromagnetic coupling e and the particle masses, all the interaction vertices are determined by the *mass-coupling relations* which are such that a decent high energy behavior for the S-matrix elements emerges (power counting rules typical for a renormalizable field theory). In fact, the GWS model in the U-gauge can be derived, using power counting arguments for the on-shell tree amplitudes, starting from an effective Lagrangian

$$\mathscr{L}_{\text{eff,int}} = \frac{g}{\sqrt{2}}(J_\mu^+ W^{\mu-} + \text{h.c.}) + \frac{g}{\cos \theta_W} J_{\mu Z} Z^\mu + e j_{\mu \text{em}} A^\mu$$

with $J^{\mu\pm} = J_1^\mu \pm iJ_2^\mu$, $J_Z^\mu = J_3^\mu - \sin^2\theta_W j_{em}^\mu$, J_i^μ the phenomenological $SU(2)_L$ fermion currents ($e = g\cos\theta_W$ unification condition) [14]. On-shell renormalizability requires the existence of a physical scalar Higgs particle. Off-shell renormalizability may be achieved upon introduction of three scalar Higgs ghosts and leads back to the standard approach of the GWS model.

4. *The photon field*

We are prepared now to discuss the electromagnetic quantities within the GWS-model. The essential point is that in order to recover local QED, with the electromagnetic field strength tensor $F_{\mu\nu}$ and the conserved electromagnetic current j_{em}^ν as observable fields, the formulation in terms of the U-gauge fields is inevitable.

By virtue of $gW_{\mu 3}^u - g'B_\mu = \sqrt{g^2+g'^2}\, Z_\mu^u$ and (3.7) we find that the correct photon field must be

$$A_\mu^u = \frac{gB_\mu + g'W_{\mu 3}^u}{\sqrt{g^2+g'^2}} = \frac{\sqrt{g^2+g'^2}}{g} B_\mu + \frac{g'}{g} Z_\mu^u = \frac{\sqrt{g^2+g'^2}}{g} B_\mu - \frac{e}{2M_W^2} J_{\mu Y}^\Phi \frac{(\Phi_0^+\Phi_0)}{(\Phi_b^+\Phi_b)} \tag{4.1}$$

with

$$J_{\mu Y}^\Phi = i(\Phi_b^+ D_\mu \Phi_b - \text{h.c.}) \tag{4.2}$$

the contribution from $\mathscr{L}_{\text{Higgs}}$ to the $U(1)_Y$ singlet current. Now, since the second term in (4.1) is a neutral singlet and $B_\mu' = B_\mu + \partial_\mu \omega_0$ under the gauge transformations (2.5), we have the proper abelian transformation law

$$(A_\mu^u)' = A_\mu^u + \partial_\mu \omega_A; \qquad \omega_A = \frac{g'}{e}\omega_0 \tag{4.3}$$

for the photon field. Therefore

$$F_{\mu\nu}^u = \partial_\mu A_\nu^u - \partial_\nu A_\mu^u \quad \text{and} \quad j_{em}^{u\nu} = -\frac{1}{e}\partial_\mu F^{u\mu\nu} \tag{4.4}$$

are singlet fields. They define the correct electromagnetic field and conserved current, respectively, and the Maxwell equations are satisfied.

According to our considerations in the last part of the preceding section we may rewrite (4.1) as

$$A_\mu^u = A_\mu - \frac{e}{2M_W^2}\mathcal{O}_\mu, \tag{4.5}$$

where $\mathcal{O}_\mu$ is a ghost operator. Explicitly we have

$$J_{\mu Y}^\Phi = -\sqrt{g^2+g'^2}\left(\frac{v^2}{2} + vH + \tfrac{1}{2}(H^2+\varphi^2)Z_\mu\right)$$

$$+\varphi^+\varphi^-\left(2eA_\mu-\frac{g}{\cos\theta_W}(\sin^2\theta_W-\cos^2\theta_W)Z_\mu\right)$$

$$-g\varphi(\varphi^+W_\mu^-+\varphi^-W_\mu^+)+ig(H+v)(\varphi^+W_\mu^--\varphi^-W_\mu^+)$$

$$+v\overleftrightarrow{\partial}_\mu\varphi+H\overleftrightarrow{\partial}_\mu\varphi-i\varphi^+\overleftrightarrow{\partial}_\mu\varphi^- \qquad (4.6)$$

with the leading term

$$\overset{\circ}{j}{}^{\Phi}_{\mu Y}=-\frac{v^2}{2}\sqrt{g^2+g'^2}\,Z_\mu. \qquad (4.7)$$

For the ghost operator we obtain from (4.1), using (3.12) and (3.5),

$$\mathcal{O}_\mu=\left(J^{\Phi}_{\mu Y}\sum_{n=0}^{\infty}(-1)^nX^n+\frac{v^2}{2}\sqrt{g^2+g'^2}\,Z_\mu\right), \qquad (4.8)$$

or up to the one-loop order

$$\mathcal{O}_\mu=-\partial_\mu((H-v)\varphi)-i\varphi^+\overleftrightarrow{\partial}_\mu\varphi^--ig(H-v)(\varphi^+W_\mu^--\varphi^-W_\mu^+)$$

$$-g\varphi(\varphi^+W_\mu^-+\varphi^-W_\mu^+)+2\varphi^+\varphi^-(eA_\mu+g\cos\theta_WZ_\mu)+O(v^{-1}). \qquad (4.9)$$

We observe that the physical vertices HZ_μ and H^2Z_μ, present in $J^{\Phi}_{\mu Y}$, have disappeared from $\mathcal{O}_\mu$. Now, the observable singlet fields (4.4) are given by

$$F^{\mu}_{\mu\nu}=F_{\mu\nu}-\frac{e}{2M_W^2}(\partial_\mu\mathcal{O}_\nu-\partial_\nu\mathcal{O}_\mu)$$

and

$$j^{\mu\nu}_{\mathrm{em}}=j^{\nu}_{\mathrm{em}}+\frac{1}{2M_W^2}(\Box g^{\nu\mu}-\partial^\nu\partial^\mu)\mathcal{O}_\mu \qquad (4.10)$$

with all physical vertices contained in the standard field expressions $F_{\mu\nu}$ and j^{ν}_{em}. Explicitly, the standard conserved electromagnetic current reads

$$j^{\nu}_{\mathrm{em}}=\sum_f q_f\bar\psi_f\gamma^\nu\psi_f+i\left(\Phi_b^+\frac{1+\tau_3}{2}D^\nu\Phi_b\right)$$

$$+\varepsilon_{3bc}(W_{\mu b}G_c^{\mu\nu}-\partial_\mu(W_b^\mu W_c^\nu))-i((\partial^\nu\bar\eta^-)\eta^+-(\partial^\nu\bar\eta^+)\eta^-)+\frac{1}{\alpha e}\partial^\nu(\partial_\mu A^\mu), \qquad (4.11)$$

where the last two terms are the contributions from the Faddeev-Popov part $\mathscr{L}_{\mathrm{FP}}$ and the gauge-fixing part $\mathscr{L}_{\mathrm{GF}}$ of the effective Lagrangian.

The electromagnetic form-factors

$$\langle \cdot \, | j^{u}_{\mu\,\mathrm{em}}(x) | \, \cdot \rangle$$

are gauge invariant now and may be computed in the renormalizable 't Hooft gauge[5].

5. The electromagnetic singlet form factor of the electron

Using the singlet field formalism, we now investigate the S-matrix element

$$-\Gamma_{j\mu} \doteq \langle e^-(p_2) | \tilde{j}^{u}_{\mu\mathrm{em}}(q) | e^-(p_1) \rangle \tag{5.1}$$

of the electromagnetic current (4.10) in the one-loop approximation. Since the standard QED (virtual photon) contributions and the infrared problem are the same as in pure QED we shall focus our analysis on the "weak" part and ignore the QED contribution [15]. We may evaluate $\Gamma_{j\mu}$ by virtue of Maxwell's equations from the matrix element

$$\langle e^-(p_2) | \tilde{A}^{u}_{\mu}(q) | e^-(p_1) \rangle$$

$$= \langle e^-(p_2) | \tilde{A}_{\mu}(q) | e^-(p_1) \rangle - \frac{e}{2M_W^2} \langle e^-(p_2) | \tilde{\mathcal{O}}_{\mu}(q) | e^-(p_1) \rangle$$

$$\doteq -ie\Gamma^{\varrho}_{A} D_{\varrho\mu}(q) - \frac{e}{2M_W^2} \Gamma_{\mathcal{O}\mu}, \tag{5.2}$$

which is given diagrammatically by Fig. 1.

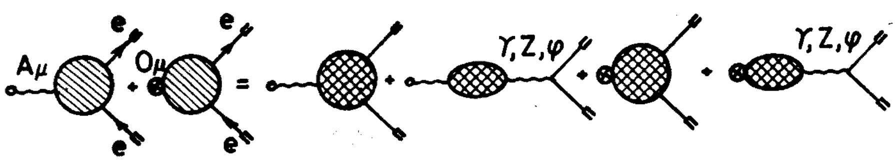


Fig. 1. Contributions to the matrix element of the abelian photon field

In (5.2) $D_{\mu\nu}(q) = i\left(-g_{\mu\nu} + (1-\alpha)\frac{q_\mu q_\nu}{q^2}\right)\frac{1}{q^2+i\varepsilon}$ is the standard photon propagator (α the gauge parameter). The CP-invariant ($\Gamma^{\mu c}(p_1, p_2) = \gamma^0 \Gamma^{\mu +}(-p_2, -p_1)\gamma^0 = \Gamma^{\mu}(p_1, p_2)$) covariant decomposition for on shell electrons[6] reads ($P \doteq p_1 + p_2$, $q \doteq (p_1 - p_2)$, $P_q = 0$)

$$\Gamma^{\varrho}_{x} = \gamma^{\varrho} A^{x}_{1} + \gamma^{\varrho}\gamma_5 A^{x}_{2} + \frac{p^{\varrho}}{2m_e} A^{x}_{3} + \frac{q^{\varrho}}{2m_e} \gamma_5 A^{x}_{4}; \; x = A, \mathcal{O} \tag{5.3}$$

---

[5] The propagators then have the form $-\left(g_{\mu\nu} - (1-\alpha)\frac{p_\mu p_\nu}{p^2 - \alpha M^2 + i\varepsilon}\right)(p^2 - M^2 + i\varepsilon)^{-1}$ for the vector bosons and $(p^2 - \alpha M^2 + i\varepsilon)^{-1}$ for the related Higgs and Faddeev-Popov ghosts.

[6] All equations are to be considered as sandwiched between the electron spinors $\bar{u}(p_2) \ldots u(p_1)$ ($\not{p}_1 u(p_1) = m_e u(p_1)$, $\bar{u}(p_2)\not{p}_2 = \bar{u}(p_2) m_e$).

and hence

$$\Gamma_{j\mu} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left(\gamma^\nu F_1 + \gamma^\nu\gamma_5 F_2 + \frac{p^\nu}{2m_e} F_3\right), \tag{5.4}$$

with

$$F_i = A_i^A + \frac{q^2}{2M_W^2} A_i^\varphi.$$

Using the Gordon decomposition we may write the electromagnetic vertex in the form:

$$\Gamma_j^\mu = \gamma^\mu F_E(q^2) + \left(\gamma^\mu + \frac{2m_e q^\mu}{q^2}\right)\gamma_5 F_A(q^2) + i\sigma^{\mu\varrho}\frac{q_\varrho}{2m_e} F_M(q^2), \tag{5.5}$$

where

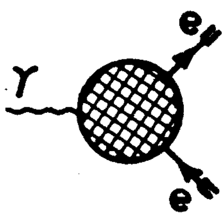
$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

and

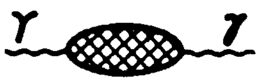
$$F_E = F_1 + F_3, \quad F_A = F_2, \quad F_M = F_3.$$

Since $F_A(0) = 0$ (see Eq. (5.9) below) Eq. (5.5) exhibits no pole at $q^2 = 0$.

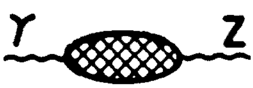
According to Fig. 1 we may express the form factors in terms of irreducible amplitudes, defined by



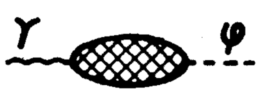
$$: -ie\left(\gamma^\mu A_{10}^{\gamma ee} + \gamma^\mu\gamma_5 A_{20}^{\gamma ee} + \frac{p^\mu}{2m_e} A_{30}^{\gamma ee} + \frac{q^\mu}{2m_e}\gamma_5 A_{40}^{\gamma ee}\right)$$



$$: i(ev)^2(g^{\mu\nu}A_{10}^{\gamma\gamma} + q^\mu q^\nu A_{20}^{\gamma\gamma})$$



$$: i(ev)M_Z(g^{\mu\nu}A_{10}^{\gamma Z} + q^\mu q^\nu A_{20}^{\gamma Z})$$



$$: -(ev)q^\mu B_{10}^{\gamma\varphi}; B_{10}^{\gamma\varphi} = -(A_{10}^{\gamma Z} + q^2 A_{20}^{\gamma Z}). \tag{5.6}$$

Including renormalization counterterms we obtain

$$A_1^A = A_{1r}^{\gamma ee} - 2a\frac{M_Z^2}{q^2 - M_Z^2} A_{1r}^{\gamma Z} - (ev)^2\frac{1}{q^2} A_{1r}^{\gamma\gamma},$$

$$A_2^A = A_{2r}^{\gamma ee} - 2b\,\frac{M_Z^2}{q^2 - M_Z^2}\,A_{1r}^{\gamma Z},$$

$$A_3^A = A_{30}^{\gamma ee},$$

$$A_4^A = A_{40}^{\gamma ee} - 2m_e^2\left(\frac{A_{10}^{\gamma Z}}{q^2 - M_Z^2} + A_{20}^{\gamma Z}\right), \tag{5.7}$$

with the "renormalized" amplitudes

$$A_{1r}^{\gamma ee} = A_{10}^{\gamma ee} + z_a + \tfrac{1}{2}(Z_\gamma - 1) + \frac{\delta e}{e} + 2aA_{10}^{\gamma Z}(0),$$

$$A_{2r}^{\gamma ee} = A_{20}^{\gamma ee} + z_b + 2bA_{10}^{\gamma Z}(0),$$

$$A_{1r}^{\gamma\gamma} = A_{10}^{\gamma\gamma} - q^2\,\frac{dA_{10}^{\gamma\gamma}}{dq^2}(0); \qquad Z_\gamma - 1 = (ev)^2\,\frac{dA_{10}^{\gamma\gamma}}{dq^2}(0),$$

$$A_{1r}^{\gamma Z} = A_{10}^{\gamma Z} - A_{10}^{\gamma Z}(0) - \frac{q^2}{M_Z^2}\left(A_{10}^{\gamma Z}(M_Z^2) - A_{10}^{\gamma Z}(0)\right). \tag{5.8}$$

Expressions analogous to (5.7) are obtained for the amplitudes $A_i^{\mathcal{V}}$. The renormalization counterterms are all included in A_i^A. $a = \sin^2\theta_W - \frac{1}{4}$ and $b = \frac{1}{4}$ are the vector and axial vector couplings of the Zee-vertex, z_a and z_b are given by the electron wave function renormalization factor $Z_e - 1 = z_a + z_b\gamma_5$ and $\frac{\delta e}{e}$ is the charge renormalization counterterm.

The renormalization conditions are chosen such that the physical particle masses m_f, M_W, M_Z and m_H and the fine structure constant $\alpha = e^2/4\pi$ are the independent parameters. This QED like on shell renormalization scheme has been discussed in detail in Ref. [16]. The parameters $\sin^2\theta_W \doteq 1 - M_W^2/M_Z^2$ and $v^{-1} \doteq e/(2M_W \sin\theta_W)$ are dependent parameters. The wave function renormalization factors for the external lines are determined by the LSZ-asymptotic condition for scattering states. Furthermore, the $\gamma-Z$ mixing propagator is renormalized such that it is diagonal at $q^2 = 0$ and at $q^2 = M_Z^2$ (see $A_{1r}^{\gamma Z}$ in Eq. (5.8)). Whereas the parameter counterterms are gauge invariant (if properly defined), the wave function renormalizations depend on the gauge. In our case the latter must be determined for the singlet field propagators. It is important to notice that in the U-gauge $A_{10}^{\gamma Z}(0) = 0$, such that no mixing mass counterterm is present.[7]

[7] This indicates that our gauge dependent choice $b = -\frac{ev}{M_Z}A_{10}^{\gamma Z}(0)$ for the γ-Z mixing mass counterterm is correct. In contrast, Sirlin [16] uses the gauge invariant quantity

$$b = \frac{g\delta g' - g'\delta g}{g'^2 + g^2} = -\frac{(ev)}{M_Z}\,\frac{1}{4\sin^2\theta_W}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right)$$

which does not even agree in the UV singular terms with our prescription.

Before we are going to discuss our results for the renormalized form factors, (5.5) let us add the following remarks: As in QED the form factors are real analytic functions. The singlet electromagnetic current is trivially conserved by (5.4): $q_\mu \Gamma_J^\mu = 0$. On the other hand, the Slavnov-Taylor identity $\langle e^-(p_2)|\partial_\mu A^\mu(x)|e^-(p_1)\rangle = 0$ yields $q_\mu \Gamma_A^\mu = \Sigma_e(p_2) - \Sigma_e(p_1)$, for on shell electrons, where $\Sigma_e(p) = \not{p}(A + B\gamma_5) + m_e C$ is the bare electron self-energy operator. For the irreducible amplitudes this implies

$$A_{20}^{\gamma ee} - \frac{q^2}{4m_e^2} A_{40}^{\gamma ee} + \tfrac{1}{2}(A_{10}^{\gamma Z} + q^2 A_{20}^{\gamma Z}) = -B(m_e^2) = -z_b. \tag{5.9}$$

This yields the physical condition $F_A(0) = 0$ for the axial form factor. The charge renormalization condition, $F_E(0) = 1$, determines

$$\begin{aligned}\frac{\delta e}{e} &= -(A_{10}^{\gamma ee}(0) + A_{30}^{\gamma ee}(0) + z_a + \tfrac{1}{2}(Z_\gamma - 1) + 2aA_{10}^{\gamma Z}(0)) \\ &= \frac{e^2}{16\pi^2}\left(\tfrac{7}{2}\ln\frac{M_W^2}{\mu^2} - \tfrac{1}{3} - \tfrac{2}{3}\sum_f q_f^2 \ln\frac{m_f^2}{\mu^2}\right),\end{aligned} \tag{5.10}$$

in the $\overline{\text{MS}}$ scheme ($\ln\mu^2 = \frac{2}{\varepsilon} - \gamma + \ln 4\pi$, γ Euler's constant, $\varepsilon = 4-d$, d space dimensions). The sum extends over the charged fermions (flavors and colors).

We have performed the one-loop calculation for the gauge invariant form factors in the singlet field formalism. The singlet field Feynman rules and some rules for the evaluation of the non-standard contributions are given in Appendix A. We have used dimensional regularization together with an anticommuting γ_5 [17] in order to preserve the canonical Slavnov-Taylor-identities. The complete one-loop expressions are somewhat lengthy and will not be given here. The result of a numerical evaluation of $F_E(q^2)$ in the region 1 GeV $\lesssim \sqrt{|q^2|} \lesssim 1$ TeV is depicted in Fig. 2.

Our main interest concerns the asymptotic behavior of the form factors for large (spacelike or timelike) momentum transfers. Terms which vanish in the limit $m_e \to 0$ are neglected. In order to make clear the origin of different kinds of terms we present the one-loop results for the form factors (5.5) as a sum

$$F_i = F_i^W + F_i^Z + F_i^\gamma + F_i^f \tag{5.11}$$

of contributions from loops involving virtual $W^\pm$, Z, γ and fermions (f), respectively. The Higgs contributions are suppressed by factors m_e^2/M_W^2 and thus may be ignored.

For $|q^2| \gg M_W^2$ we find ($c_\theta^2 \doteq M_W^2/M_Z^2$; $s_\theta^2 \doteq 1 - c_\theta^2$)

$$\begin{aligned}F_i^W(q^2) = \frac{\alpha}{16\pi s_\theta^2 c_\theta^2}\Bigg\{& a_{i0} + b_{i0}\ln\frac{|q^2|}{M_W^2} \\ &+ a_{i1}\frac{q^2}{M_W^2}\left(\ln\frac{|q^2|}{\mu^2} - \tfrac{8}{3}\right) - i\pi\theta(q^2 - 4M_W^2)\left(b_{i0} + a_{i1}\frac{q^2}{M_W^2}\right)\Bigg\}\end{aligned} \tag{5.12}$$

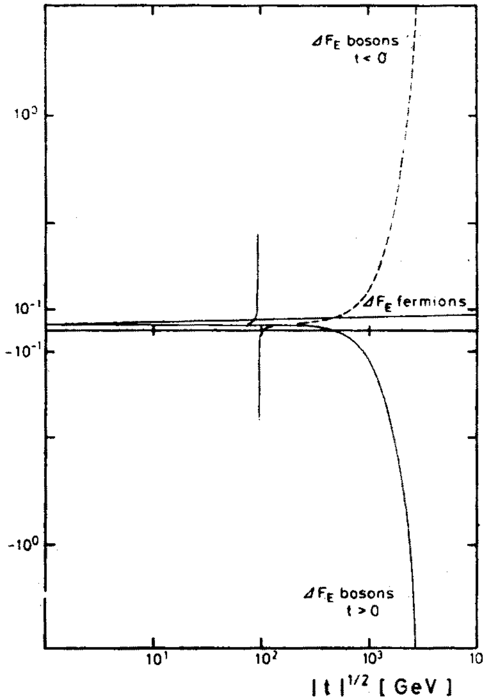


Fig. 2. Radiative correction to the singlet charge form factor F_E (Eqs. (1.1) and (5.5)) as a function of the momentum transfer $\sqrt{|t|}$ ($t = q^2$), for a cut off $\mu = 1$ GeV. For increasing values of μ the region where corrections grow rapidly (breakdown of perturbation expansion) moves to the right. The pole indicated at $\sqrt{t} = M_Z \simeq 93$ GeV would be there if the γ-Z mixing amplitude $A_1^{\gamma Z}$ would not be renormalized according to Eq. (5.8)

for the "renormalized" virtual $W^\pm$ contributions in the $\overline{\mathrm{MS}}$ subtraction scheme (see Eq. (5.10)). The coefficients are given by ($g(c_\theta^2) \doteq \sqrt{4c_\theta^2-1}$ arc ctg $\sqrt{4c_\theta^2-1}$)

$$a_{E0} = -32c_\theta^6 - \tfrac{56}{3}c_\theta^4 + \tfrac{253}{6}c_\theta^2 + \tfrac{1}{2}$$
$$-(-32c_\theta^6 - \tfrac{64}{3}c_\theta^4 + \tfrac{134}{3}c_\theta^2 - \tfrac{22}{3} - \tfrac{1}{2}c_\theta^{-2})g(c_\theta^2) \simeq 7.972,$$
$$b_{E0} = \tfrac{5}{3}c_\theta^2 - \tfrac{19}{6} - \tfrac{1}{4}c_\theta^{-2} \simeq -2.181, \quad a_{E1} = \tfrac{1}{6}c_\theta^2 - \tfrac{1}{4} \simeq -0.120,$$

for the charge form factor F_E, and by

$$a_{A0} = 8c_\theta^4 - \tfrac{1}{6}c_\theta^2 + \tfrac{1}{6} - (8c_\theta^4 + \tfrac{34}{3}c_\theta^2 - \tfrac{8}{3} - \tfrac{1}{6}c_\theta^{-2})g(c_\theta^2) \simeq -4.616,$$
$$b_{A0} = 3c_\theta^2 - \tfrac{7}{6} - \tfrac{1}{12}c_\theta^{-2} \simeq 1.076, \quad a_{A1} = \tfrac{1}{6}c_\theta^2 - \tfrac{1}{12} \simeq 0.047$$

for the axial form factor F_A. Numerical values are for $s_\theta^2 = 0.217$.

On the other hand,

$$F_i^Z(q^2) = \frac{\alpha}{16\pi s_\theta^2 c_\theta^2}\left\{a'_{i0} + b'_{i0}\ln\frac{|q^2|}{M_Z^2} + \tfrac{2}{3}b'_{i0}\left(Sp\left(1+\frac{q^2}{M_Z^2}\right) - \frac{\pi^2}{6}\right) - i\pi\theta(q^2-4m_e^2)b'_{i0}\right\} \tag{5.13}$$

is the contribution from virtual Z bosons with coefficients

$$a'_{E0} = -14c_\theta^4+21c_\theta^2-\tfrac{35}{4} \simeq -0.890, \quad b'_{E0} = 12c_\theta^4-18c_\theta^2+\tfrac{15}{2} \simeq 0.763,$$

and

$$a'_{A0} = 7c_\theta^2-\tfrac{21}{4} \simeq 0.231, \quad b'_{A0} = -6c_\theta^2+\tfrac{9}{2} \simeq -0.198.$$

The Spence function $Sp(x) = -\int_0^1 \frac{dt}{t} \ln(1-xt)$ is given asymptotically by

$$Sp\left(1+\frac{q^2}{M_Z^2}\right)-\frac{\pi^2}{6} \simeq \begin{cases} \frac{\pi^2}{6}-\frac{1}{2}\ln^2\frac{q^2}{M_Z^2}+i\pi\ln\frac{q^2}{M_Z^2}\,; & q^2 \gg M_Z^2 \\ -\frac{\pi^2}{3}-\frac{1}{2}\ln^2\left(-\frac{q^2}{M_Z^2}\right); & -q^2 \gg M_Z^2. \end{cases}$$

For comparison the standard QED (virtual photon) contribution to F_E is ($|q^2| \gg m_e^2$)

$$F_E^\gamma(q^2) = \frac{\alpha}{4\pi}\Bigg\{4\ln\frac{m_e}{m_\gamma}-2\ln\frac{q^2}{m_e^2}\ln\frac{q^2}{m_\gamma^2}+\ln^2\frac{q^2}{m_e^2}+3\ln\frac{q^2}{m_e^2}-4+\frac{4\pi^2}{3}$$
$$-i\pi\theta(q^2-4m_e^2)\left(3-2\ln\frac{q^2}{m_\gamma^2}\right)\Bigg\} \tag{5.14}$$

with m_γ a fictitious photon mass. The inclusion of the real soft photons in the electron states, necessary in order to obtain an infrared finite result, will not be considered here. Apart from the infrared problem F_E^γ is gauge invariant and finite.

For completeness we add the fermion loop terms which come in through the self-energies $A_1^{\gamma\gamma}$ and $A_1^{\gamma Z}$. According to Eqs. (5.7) and (5.8) we have

$$F_E^f(q^2) = -2a\frac{M_Z^2}{q^2-M_Z^2}(A_{1r}^{\gamma Z})_f-(ev)^2\frac{1}{q^2}(A_{1r}^{\gamma\gamma})_f$$

$$F_A^f(q^2) = -2b\frac{M_Z^2}{q^2-M_Z^2}(A_{1r}^{\gamma Z})_f, \tag{5.15}$$

where

$$((A_1^{\gamma Z}(0))_f = 0, \quad \left(\frac{\delta e}{e}\right)_f+\tfrac{1}{2}(Z_\gamma-1)_f = 0)$$

$$-(ev)^2\frac{1}{q^2}(A_{1r}^{\gamma\gamma})_f = \frac{\alpha}{\pi}\frac{1}{3}\sum_f q_f^2 H_f\left(\frac{4m_f^2}{q^2}\right)$$

is the QED vacuum polarisation contribution to F_E and

$$(A_{1r}^{\gamma Z})_f = -\frac{\alpha}{8\pi s_\theta^2}\frac{q^2}{M_W^2}\frac{1}{3}\sum_f (4a_f q_f)\left[H_f\left(\frac{4m_f^2}{q^2}\right)-H_f\left(\frac{4m_f^2}{M_Z^2}\right)\right]$$

is the γ-Z mixing contribution. The function H_f is given by

$$H_f(y_f) = \tfrac{5}{3}+y_f+\left(1+\frac{y_f}{2}\right)\sqrt{1-y_f}\,\ln\frac{\sqrt{1-y_f}-1}{\sqrt{1-y_f}+1}\,; \quad (H_f(\infty)=0)$$

and $a_f = -q_f\sin^2\theta_W \pm \frac{1}{4}$ for the upper and lower components of the weak isodoublets, respectively (q_f is the charge of the fermion f). These contributions are given by the standard expressions which are gauge invariant and finite.

The magnetic form factor F_M vanishes in the limit $m_e \to 0$. The leading terms, for $m_e^2 \ll M_W^2 \ll |q^2|$, are given by

$$F_M^W(q^2) = \frac{\alpha}{4\pi s_\theta^2}\frac{m_e^2}{M_W^2}\left[1-\ln\frac{|q^2|}{M_W^2}+i\pi\theta(q^2-4M_W^2)\right]$$

$$+\frac{\alpha}{4\pi s_\theta^2}\frac{m_e^2}{q^2}\left[-\tfrac{1}{2}+5\left(\ln\frac{|q^2|}{M_W^2}-i\pi\theta(q^2-4M_W^2)\right)+2I\left(\frac{4M_W^2}{q^2}\right)\right], \tag{5.16}$$

$$F_M^Z(q^2) = \frac{\alpha}{8\pi s_\theta^2}\frac{m_e^2}{q^2}\left[(-20c_\theta^2+30-\tfrac{33}{2}c_\theta^{-2})\right.$$

$$+(8c_\theta^2-12+9c_\theta^{-2})\left(\ln\frac{|q^2|}{M_Z^2}-i\pi\theta(q^2-4m_e^2)\right)$$

$$\left.+2c_\theta^{-2}\left(Sp\left(1+\frac{q^2}{M_Z^2}\right)-\frac{\pi^2}{6}\right)\right] \tag{5.17}$$

and

$$F_M^\gamma(q^2) = \frac{\alpha}{\pi}\frac{m_e^2}{q^2}\frac{1}{\sqrt{1-y}}\ln\frac{\sqrt{1-y}+1}{\sqrt{1-y}-1}\,; \quad y=\frac{4m_e^2}{q^2}. \tag{5.18}$$

The function $I(y)$ in (5.16) is given by

$$I(y) = Sp\left(\frac{2}{1+\sqrt{1-y}}\right)+Sp\left(\frac{2}{1-\sqrt{1-y}}\right) = \begin{cases} 2\left(\operatorname{arctg}\dfrac{1}{\sqrt{y-1}}\right)^2; & y>1 \\ -\frac{1}{2}\left(\ln\dfrac{\sqrt{1-y}-1}{\sqrt{1-y}+1}\right)^2; & y<1. \end{cases}$$

An important observation is the presence of terms which survive for large $|q^2|$ in F_M^W.

From these results we see that the contributions F_i^Z, F_i^γ and F_i^f are gauge invariant and finite. Beyond that they also agree with the standard R-gauge field result. This is apparently not the case for F_i^W, i.e. this contribution (see Fig. 3) is not ultraviolet finite. Since the singlet fields formally coincide with the standard fields in the unitary gauge (U-gauge), it is interesting to compare our results with the asymptotic expressions obtained for F_i^A in the 't Hooft gauge as $\alpha \to \infty$ (unitary gauge limit). In fact, the results agree up to

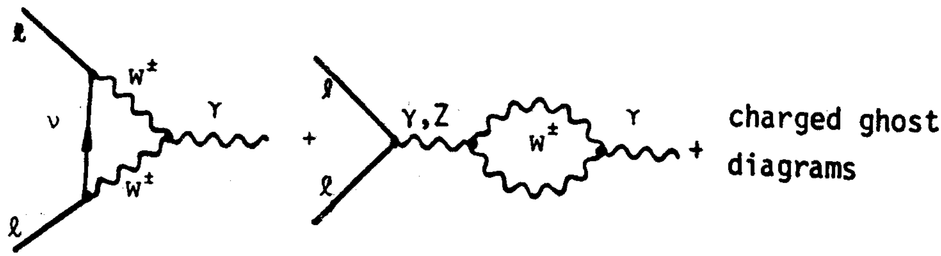


Fig. 3. Diagrams involving virtual charged bosons

terms proportional to q^2/M_W^2:

$$\Delta F_i = F_i(q^2, \mu^2) - F_i^A(q^2, \alpha)|_{\alpha\to\infty} = K \cdot \frac{q^2}{M_W^2}\left[c_{i0} + c_{i1}\left(\ln\alpha + \ln\frac{M_W^2}{\mu^2}\right)\right] \qquad (5.19)$$

for arbitrary values of m_l^2 and q^2 (full one-loop result). The coefficients are

$$c_{E0} = \tfrac{5}{12}s_\theta^2 - \tfrac{5}{8}\frac{m_l^2}{M_Z^2}, \quad c_{A0} = \tfrac{5}{36}s_\theta^2 - \tfrac{5}{8}\frac{m_l^2}{M_Z^2}, \quad c_{i1} = a_{i1} + \tfrac{1}{4}\frac{m_l^2}{M_Z^2} \qquad (5.19a)$$

and $\Delta F_M = 0$. Eq. (5.19) also exhibits the full cut-off dependence of F_i, and formally, for $\alpha = \dfrac{\mu^2}{M_W^2}$, the divergent parts cancel. A detailed discussion of our results follows in the next section.

6. *Discussion*

Each of the contributions $F_i^{\mathcal{L}}$, F_i^{γ} and F_i^{f} (Eq. (5.11)) to the "renormalized" electromagnetic form factors (5.5) is gauge invariant and finite and coincides with the standard R-gauge field result in the one-loop approximation. In contrast, the virtual charged vector boson term F_i^W either depends on the gauge and has no unitary gauge limit $\alpha \to \infty$ (standard current (4.11)) or it is not ultraviolet finite (physical singlet current (4.10)). The asymptotic expression for F_i^W, Eq. (5.12), for large $|q^2|$ is given in the minimal subtraction ($\overline{\text{MS}}$) scheme (see also Eq. (5.10)) and depends on the superficial scale parameter μ. Some numerical values obtained for F_E^W are presented in Table I.

The short distance behavior is "anomalous" since, up to logarithms, $F_i = O(q^2)$ whereas $F_i = O(1)$ for the standard current of canonical dimension 3. The ghost operator $\mathcal{O}_\mu$, apart from cancelling the gauge dependent terms of the standard current, induces gauge invariant terms of order $O(q^2)$ which are not ultraviolet finite. In view of the non--polynomial form of the singlet current (which has no canonical dimension) this seems not to be surprising. Because of the bad high energy behavior it seems to make not much sense to try to construct a finite singlet current by inventing some renormalization prescription for $j_{em}^{\mu\nu}$ as an external current. All counterterms needed to renormalize the S-matrix elements and the standard field Green functions have already been taken into account.

TABLE I

Percentage correction of the charge form factor as a function of $\sqrt{|q^2|}$ for different cut-offs μ (for comparison also the Feynman-'t Hooft ($\alpha = 1$) values are given). Values in brackets are for $q^2 < 0$

	$\sqrt{|q^2|}$ (GeV)	100	316	1000	3162
Re F_E^W	μ (GeV) = 1	0.5 (0.7)	−1.1(1.5)	−16.9(16.4)	−201.4(200.0)
$q^2 > 0$	M_W	0.65(0.6)	0.2(0.2)	−3.7(3.2)	−69.6(68.2)
($q^2 < 0$)	1000	0.7 (0.5)	0.9(−0.6)	3.7(−4.2)	4.8(−6.1)
	$\alpha = 1$	0.5 (0.5)	0.2(0.2)	−0.2(−0.2)	−0.5(−0.5)
Im F_E^W	"$\alpha = \infty$"	0.6	1.1	5.3	47.5
$q^2 > 0$	$\alpha = 1$	0.4	0.4	0.4	0.4

It is obvious that the renormalization conditions for S-matrix elements cannot remove ultraviolet divergencies originating from the ghost-operator $\mathcal{O}_\mu$, since the latter does not contribute to on shell quantities.

The question to be asked is whether the electromagnetic singlet current as a "local observable" is actually observable. For example, we could try to measure the electromagnetic form factor of the muon in a μ-e Coulomb scattering experiment. The μ-e scattering matrix element is known to have renormalizable high energy behavior. Obviously the "anomalous" singlet field terms (or the corresponding gauge dependent standard field terms) in the muon form factors cancel upon including consistently *all* diagrams contributing to the scattering process to a given order of perturbation theory. This cancellation mechanism for terms of bad high energy behavior in the U-gauge is completely equivalent to the cancellation of gauge dependent terms in the R-gauge [18]. The complementary of the U-gauge formulation, which is adapted to the physical Hilbert space but not to the dynamics, and the R-gauge formulation, which is adapted to the dynamics but not to the physical Hilbert space, has already been pointed out by Kibble [5] and is commonly known.

However, there seems to be common agreement, too, that a satisfactory understanding of the structure of quantum field theories should rely on a formulation in terms of local observables [19] avoiding unphysical concepts, like gauge potentials, which necessarily lead to mathematical inconsistencies of the basic principles when applied to unphysical degrees of freedom [20]. For quantum electrodynamics such an attempt has been carried to a satisfactory stage on a perturbative level only recently [21]. For non abelian gauge theories, at least for models like the GWS electroweak theory, the above mentioned complementarity raises doubts on whether local observables are the preferred "coordinates" for understanding all aspects of such models. For dynamical reasons one has to expect severe difficulties in controlling the short distance aspects in terms of gauge invariant fields on which the lattice gauge theory approach or the canonical quantization approach rely, for example. Also the stochastic quantization method [22] does not circumvent the problems addressed here.

It may be helpful to keep in mind the close resemblance of local gauge symmetries and the equivalence principle (general covariance) of gravity [23] for a better structural understanding of non abelian gauge theories.

If S-matrix elements would be the only measurable quantities, the choice of interpolating fields to the one particle states (off shell extrapolations) would be rather arbitrary according to the equivalence theorem for the S-matrix [24] and the question, in terms of which fields the theory was (equivalently) formulated, would be essentially a technical one. In fact, actual measurements always take place within bounded space-time regions and "one particle states" are identified as "configurations" (prototype: charged particle plus soft photon cloud) depending on experimental resolutions (and possibly also on assumptions about the expected particle spectrum and the range of interactions). Thus, rather than S-matrix elements, local observables "near" the mass shell are measured and the problem of proper off shell extrapolation[8] of the S-matrix becomes physically relevant, at least in principle. Rather than trying to give a proper characterization of local observables we use this term *formally* for local gauge invariant fields like the electromagnetic current.

Our analysis leads to the conclusion that "local observables" are physical only in the low momentum transfer approximation. The detailed one-loop calculation demonstrates that the electromagnetic form factors are dominated for $|q^2| \ll M_W^2$ by the standard QED contributions (virtual photons and fermionic contributions to the vacuum polarization). Actually for $|q^2| \lesssim M_W^2$ the cut-off dependence is insignificant for cut-offs μ in the range $1\ \mathrm{GeV} \lesssim \mu \lesssim 1\ \mathrm{TeV}$. In this approximate sense we fairly well control the local structure, which is important to account for the local nature of actual measurements and which provides the possibility of direct tests of causal space-time properties.

This result is by no means satisfactory. It rather raises a number of questions. The major question is whether the model uniquely predicts the quantities which are actually (precisely) measured or which are measurable in principle. This would require the existence of a class of gauge invariant and finite quantities other than the S-matrix elements.

We should mention here, that gauge invariant fields with a decent (renormalizable) high energy behavior need not be observable. Such fields exist within the GWS model. In fact the abelian $U(1)_Y$ field B_μ has associated a field strength tensor $B_{\mu\nu}$ and a conserved current J_Y^ν which are gauge invariant in the R-gauge. Since right handed fermions do not couple to $W_{\mu a}$, the physical γ-Z mixture $B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$ can be measured, in principle, on right handed lepton states. However, matrix elements like $\langle e_R^-(p_2)|\tilde{J}_Y^\nu(q)|e_R^-(p_1)\rangle$

[8] We should point out that the physical interpolating singlet fields are by no means unique. A physical *composite* singlet representation (confinement form) [10] may be obtained by the replacement $\Phi_b/|\Phi_b| \to \frac{\sqrt{2}}{v}\Phi_b$ in the U-gauge singlet fields Eqs. (3.7), (3.10) and (4.1). In the R-gauge the composite fields A_μ^c, Z_μ^c and $W_\mu^{c\pm}$ are of canonical dimensions 3. Explicitly we have $Z_\mu^c = Z_\mu^u\left(1+\frac{\varrho}{v}\right)^2$, $W_\mu^{c\pm} = W_\mu^{u\pm}\left(1+\frac{\varrho}{v}\right)^2$ and $\psi_i^c = \left(1+\Pi_-\frac{\varrho}{v}\right)\psi_i^u$ in the unitary gauge. v appears as a compositeness scale.

are measurable only in the high energy limit where chirality is a good quantum number and provided the one-particle (B) exchange is predominant at the same time.

We cannot exclude the possibility that the actually measured (off shell) quantities turn out to exhibit "anomalous" (non renormalizable) high energy behaviour (U-gauge) or to be gauge dependent (R-gauge) within the GWS-model. This could mean either that perturbation theory breaks down for physical off shell quantities at scales $\mu \gtrsim M_W$, or that the model is "incomplete" (a low energy effective form) and should be imbedded in a grand unified or a supersymmetric theory.

In any case, since the physical off shell behavior need not be "smooth" at momentum transfers $|q^2| \gtrsim M_W^2$ a study of related problems may be relevant for the analysis of vector boson processes going on at the CERN $p\bar{p}$-collider [25].

We are grateful for stimulating discussions with O. Steinmann and D. Buchholz.

APPENDIX

In this appendix we briefly comment on some practical aspects for the evaluation of singlet field expressions. The momentum space Feynman rules for the ghost operator $\mathcal{O}_\mu$ (Eq. (4.9)) in comparison to the composite operator $J^{\Phi}_{\mu Y}$ (Eq. (4.6)) and the Lagrangian vertices, which appear with an additional A_μ or Z_μ field at the vertex depicted are given in Table II.

The computation of S-matrix elements and Green functions of U-gauge fields (in the R-gauge) is not very different from standard R-gauge field calculations. The only difference is the appearance of the composite ghost operators $\mathcal{O}_\mu$ at the *external* vertices. Since these do not exhibit physical one particle poles they do not contribute if the corresponding external line is set on shell. Thus for lines to be put on shell one uses the standard R-gauge fields and wave function renormalization factors determined from them. Contributions from diagrams exhibiting physical lines only (U-gauge diagrams) remain unaltered. On the other hand, the standard ghost diagrams in general are accompanied by partners (generated by $\mathcal{O}_\mu$ vertices) which have the effect to change the standard contribution into a multiple, with a factor depending on the external momentum transfer. We shall illustrate this for the transversal amplitude used in Sect. 5.

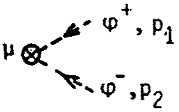
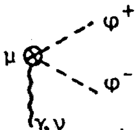
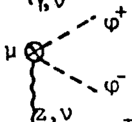
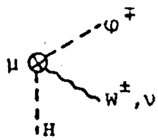
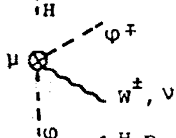
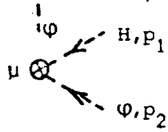
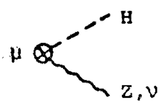
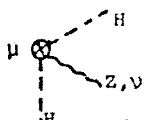
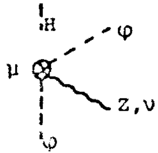
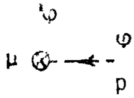
We first consider the γ and Z *selfenergies*. In terms of irreducible diagrams the γ(Z)--propagator is given by

(According to Fig. 1 only A_1 and C_1 contribute to $\Gamma_{\mathcal{O}\mu}$ Eq. (5.2)). For the selfenergy parts this implies

$$\gamma\gamma\colon\ A_1^u = A_1 + 2B_1 q^2 + D_1 q^4; \quad (C_1 = B_1),$$

TABLE II

Feynman rules for composite operators $\left(\partial_\mu = -ip_\mu;\ g_Z = \frac{1}{2}\sqrt{g'^2+g^2} = \frac{g}{2c_\theta}\right)$

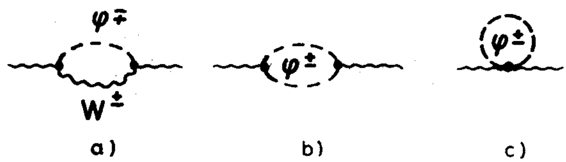
	common	$J_\gamma^{\mu\Phi}$	O^μ	A^μ	Z^μ
	$(p_1 - p_2)^\mu$	1	1	e	$-g_Z(s_\theta^2-c_\theta^2)$
	$2eg^{\mu\nu}$	1	1	e	$-g_Z(s_\theta^2-c_\theta^2)$
	$-2g_Zg^{\mu\nu}$	$(s_\theta^2-c_\theta^2)$	$-2c_\theta^2$	$e(s_\theta^2-c_\theta^2)$	$-g_Z(s_\theta^2-c_\theta^2)^2$
	$\mp igvg^{\mu\nu}$	1	1	$\frac{e}{2}$	$-g_Zs_\theta^2$
	$\mp ig\ g^{\mu\nu}$	1	-1	$\frac{e}{2}$	$-g_Zs_\theta^2$
	$-g\ g^{\mu\nu}$	1	1	$\frac{e}{2}$	$-g_Zs_\theta^2$
	1	$i(p_1-p_2)^\mu$	$i(p_1+p_2)^\mu$	0	$-ig_Z(p_1-p_2)^\mu$
	$-2vg_Zg^{\mu\nu}$	1	0	0	$-g_Z$
	$-2g_Zg^{\mu\nu}$	1	0	0	$-g_Z$
	$-2g_Zg^{\mu\nu}$	1	0	0	$-g_Z$
	$-ivp^\mu$	1	1	0	0

$$\gamma Z:\ A_1^u = A_1 + B_1(q^2 - M_Z^2) + C_1 q^2 + D_1 q^2(q^2 - M_Z^2),$$

$$ZZ:\ A_1^u = A_1 + 2B_1(q^2 - M_Z^2) + D_1(q^2 - M_Z^2)^2; \quad (C_1 = B_1).$$

Obviously the non standard terms (B_1, C_1 and D_1) do not contribute on shell.

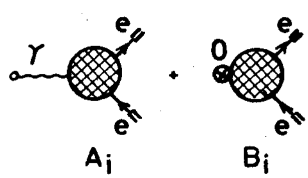
By inspection of the diagrams one finds that the extra terms may be taken into account by multiplying the standard charged ghost loop diagrams



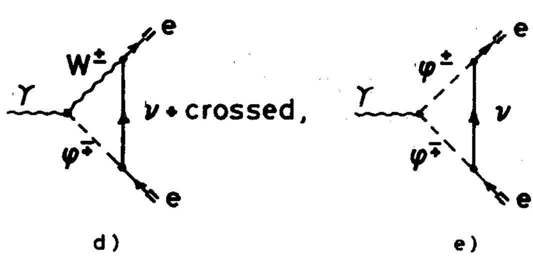
by the following simple factors:

	(a)	(b)	(c)
γγ:	$\left(1-\frac{q^2}{M_W^2}\right)^2$	$\left(1-\frac{q^2}{2M_W^2}\right)^2$	$\left(1-\frac{q^2}{M_W^2}\right)$
γZ:	$\left(1-\frac{q^2}{M_W^2}\right)\left(1-\frac{q^2-M_Z^2}{M_W^2-M_Z^2}\right)$	$\left(1-\frac{q^2}{2M_W^2}\right)\left(1-\frac{q^2-M_Z^2}{2M_W^2-M_Z^2}\right)$	$\left(1-\frac{2q^2-M_Z^2}{2M_W^2-M_Z^2}\right)$
ZZ:	$\left(1-\frac{q^2-M_Z^2}{M_W^2-M_Z^2}\right)^2$	$\left(1-\frac{q^2-M_Z^2}{2M_W^2-M_Z^2}\right)^2$	$\left(1-\frac{4M_W^2(q^2-M_Z^2)}{(2M_W^2-M_Z^2)^2}\right)$

Similarly for the γee-vertex the singlet photon diagrams



yield the form factors $A_i^u = A_i + B_i q^2$ and the ghost loop contributions B_i are included upon multiplication of the standard charged ghost loops



by the following factors

	(d)	(e)
γee:	$\left(1-\frac{q^2}{M_W^2}\right)$	$\left(1-\frac{q^2}{2M_W^2}\right)$

It is remarkable to notice that there are no new diagrams (and hence no new Feynman integrals) to be considered and that the highly non trivial gauge dependence of the standard off shell amplitudes completely drops out by a change of the weight factors of ghost diagrams. Since the singlet field amplitudes differ from the standard amplitudes only by contributions from external ghost vertices this kind of cancellation mechanism also works in higher orders of perturbation theory.

We add some technical remarks. Our results could be obtained as well by a straightforward U-gauge calculation [26]. However, working in the U-gauge, the high energy behavior is more difficult to control due to the bad high energy behavior of the vector boson propagators and furthermore one has no checks on gauge invariance. In the singlet field formulation one takes advantage of the following points: (*i*) propagators and internal vertices are given by the standard R-gauge expressions; (*ii*) the ghost operator terms appearing in the singlet fields do not contribute on shell and thus can be ignored for on shell fields; (*iii*) considering off shell fields, the bad high energy behavior is isolated in the *external* composite ghost vertices and hence can be controlled in an optimal way; (*iv*) gauge invariance of singlet fields (off shell) can be checked explicitly (independence on the gauge parameter α); (*v*) the pathological Lee-Yang terms [26] proportional to $\delta^4(0)$, are absent.

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