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EXCESS AND HOLE 4N-NUCLEI

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In the present paper, the excess and hole 4N-nuclei are defined. For given groups of nuclei, binding energies and relations between quadrupole deformation parameters β are considered theoretically.

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1. *Introduction*

It is very well known that the 4N-nuclei have relatively high stability. In the region of nuclear masses of $A \leqslant 40$, the doubly magic nuclei are only the 4N-ones (^{4}He, ^{16}O, and ^{40}Ca) and 4N-nuclei have also completely filled subshells as $1p_{3/2}$ (^{12}C), $1d_{5/2}$ (^{28}Si), and $2s_{1/2}$ (^{32}S). In the framework of the classical α-particle model [9], it is assumed that the internal α-particles are harmonically bound in a semirigid molecule structure since the binding energy of α-particles in the nucleus per the number of bonds between the α-particles is almost the same for all 4N-nuclei, except of the ^{8}Be nucleus which is unstable. Thus, the 4N-nuclei are in a way distinguished from the others. To investigate the properties of the nuclear matter it would be helpful to classify the nuclei of $A \leqslant 40$ similarly to the 4N-ones, namely, as the excess and hole 4N-nuclei, which are defined as the 4N-ones with one excess nucleon and one nucleon hole, respectively. To justify this classification of nuclei the binding energy as a function of the number of nucleons beyond the magic shells should be investigated. For these groups of nuclei the straightforward dependences of the energy, and half life of the β-decay were theoretically discussed in Ref. [3]. The conclusion can be drawn from Ref. [3] that the specific properties of the nuclear matter can then be found. In this paper, the shapes of given groups of nuclei are theoretically investigated in terms of the quadrupole deformation parameter β. To obtain the relations between quadrupole deformation parameters β for the given group of nuclei the angular frequency for the single particle motion in the deformed potential well and the potential energy of the deformation should be considered.

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2. *Excess and hole 4N-nuclei*

2.1. Binding energy

The straightforward dependences of the binding energy for $4N$-nuclei versus the number of nucleons beyond the magic shells is shown in Fig. 1. In this figure there are also shown the graphs for the $4N$-nuclei with excess proton $(4N)_{+1p}$ and excess neutron $(4N)_{+1n}$, and for the $4N$-nuclei with a proton hole $(4N)_{-1p}$ and a neutron hole $(4N)_{-1n}$. For these nuclei the straightforward dependences are also observed. The binding energy of the nucleus can then be written as follows:

$$B = B_0 + NB_n, \tag{1}$$

where N is the number of nucleons beyond the magic shells and B_n denotes the mean binding energy of 1d-2s shell nucleon. The calculated quantities B_0 and B_n are listed in Table I. As seen from Table I, B_0 is near the binding energy for the ^{16}O nucleus (127.6 MeV [11]). The restriction to the 1d and 2s shells follows from the strong change in the relative binding energy per nucleon for the 1s- and some $1p_{3/2}$-nuclei. For the excess and hole $4N$-nuclei B_0 becomes close to the binding energy for the $4N$-nuclei as ^{20}Ne, ^{24}Mg, ..., if N is lowered by 4, 8, ... Setting B_0 equal to the binding energy B_{4N} for the $4N$-nucleus of the mass which differs from the excess and hole $4N$-nuclear mass correspondingly on ± 1 and B_n as above, one can write the binding energy of the nucleus

$$B = B_{4N} \pm B_n, \tag{2}$$

where plus refers to the excess nucleon and minus refers to the hole. Here, the agreements between experimental data [11] and calculated binding energies appear to be within 0.15% and ~4% for the $(4N)_{-1p}$- and $(4N)_{+1n}$-nuclei, and within 0.6% and 6.6% for the $(4N)_{+1p}$- and $(4N)_{-1n}$-nuclei. The classification of nuclei into the excess and hole $4N$-ones is thus substantiated.

TABLE I

Comparison of calculated quantities B_0 and B_n for the different groups of nuclei

Group of nuclei	B_0 (MeV)	B_n (MeV)
$(4N)_{-1p}$	121.4	9.3
$(4N)_{-1n}$	117.1	9.1
$4N$	126.4	9.1
$(4N)_{+1p}$	116.8	9.4
$(4N)_{+1n}$	122.2	9.3

Experimental data are taken from Ref. [11].

2.2. Quadrupole deformation parameter β

For nuclei, the quadrupole deformation parameter β can be taken from:
1) Intrinsic electric quadrupole moment using the relation for the deformed nucleus

$$Q_0 \simeq 0.757ZR^2\beta(1+0.16\beta), \tag{3}$$

where Z refers to the nuclear charge and R is the nuclear radius which is set equal to $r_0 A^{1/3}$.

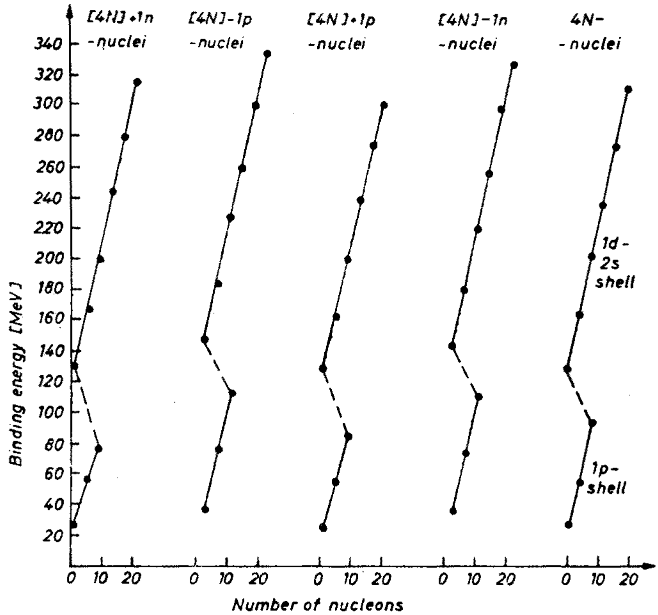

Fig. 1. Dependences of the binding energy of the nucleus as a function of the number of nucleons beyond the magic shells ($Z = N = 2$), lower line and $Z = N = 8$ upper line) for defined groups of nuclei. Solid lines represent calculations. Experimental data are taken from Ref, [11]

The experimental Q_0 can be obtained from:
1a) experimental electric quadrupole moment of the nucleus,
1b) reduced transition probability $B(E2, I_i \to I_f)$ [6] for the rotational states excited in nuclear reactions; for example, the inelastic scattering, and Coulomb excitation.
2) Angular distributions of inelastically scattered particles using the coupled channel analysis and the DWBA analysis, where β is one of the fitting parameters.
3) Nilson's model quadrupole deformation parameter δ [6].
In point 1a), the restriction on the quadrupole deformation parameter β results from the nuclear spin $\frac{1}{2}$ and 0 for which the measured electric quadrupole moments are equal to zero, for example, ^{15}N, ^{19}F, ^{31}P ($(4N)_{-1p}$-nuclei), ^{13}C, and ^{29}Si ($(4N)_{+1n}$-nuclei). Here, it is necessary to note that the experimental evidence of electric quadrupole moments for nuclei of $A \leqslant 40$ is incomplete. Large differences can be observed between the quadrupole deformation parameters β obtained from point 2). This is the reason why only some results can be taken into consideration. In the case of point 3), it is very well known that Nilson's model predictions give a satisfactory agreement with experimental data of nuclear levels in the region of nuclear masses of $A \approx 25$. The obtained quadrupole deformation parameters β for odd mass nuclei are listed in Table II. As seen from Table II, the excess nuclei having one neutron beyond the magic shells (^{17}O) and also those having completely filled 2s-shells (^{33}S) have the negative quadrupole deformation parameter $\beta = -0.11$ (oblate

TABLE II

Comparison of quadrupole deformation parameters β of the excess and hole $4N$-nuclei

Nucleus	Deformation parameter β obtained from 1a	1b	2	3	Nucleus	Deformation parameter β obtained from 1a	1b	2	3	ΔA_{4N}	Calculated β	N
$(4N)_{-1p}$-nuclei One proton hole in the magic shell					$(4N)_{+1n}$-nuclei One neutron beyond completely filled shells							
^{39}K	0.10[a]	—	0.10±0.01[k] 0.08±0.03[k]	—							0.11	1
					^{17}O	−0.11[d]	—	—	—	0	−0.11	1
					^{33}S	−0.12[a]	—	—	—	10	−0.11	1
					$1p_{3/2}$-shell							
^{7}Li	−0.79±0.13[d]	−0.85[d,h]	—	—	^{9}Be	0.75±0.04[d]	0.88[i]	0.72[f]	—		±0.77[≠]	7
^{11}B	0.60[d]	0.60[i]	—	—							0.55	5
					$1d_{5/2}$-shell							
^{19}F	—	0.40[i]	0.43 ±0.02[d,**]	—	^{21}Ne	0.47[a]	—	—	0.4–0.5[b] ≈0.31[c]		0.44	4
^{23}Na	0.47[g]	0.40[j]	—	0.4[j]	^{25}Mg	0.45±0.01[e]	0.46 ±0.01[e,≠≠]	—	≈0.5[e,i]	4	0.44	4
^{27}Al	0.31±0.01[e]	0.35± ±0.01[e] 0.43± ±0.01[e]	—	0.33[e]							0.33	3
					$1d_{32}$-shell							
^{35}Cl	−0.15[a]	—	—	—							−0.11	1

To obtain the quadrupole deformation parameters β from the intrinsic electric quadrupole moment Q_0 the well-known experimental unit radii for mirror nuclei are used. In the case 2) the averaged values are given except of ^{39}K for which the discrepance in β is appreciable. ΔA_{4N} is evaluated for nuclei of the given group. For the relative nucleus $\Delta A_{4N} = 0$. In the last column, the quadrupole deformation parameter β is evaluated from Eq. (13) for the given integer N. [≠]minus refers to the left-hand side and plus refers to the right-hand side. [≠≠]average value (experimental data are very close to the average value).

[a] Taken from Ref. [11], [b] taken from Ref. [14], [c] taken from Ref. [17], [d] taken from Ref. [2], [e] taken from Ref. [19], [f] taken from Ref. [13], [g] taken from Ref. [18], [h] taken from Ref. [10], [i] taken from Ref. [6], [j] taken from Ref. [8], [k] taken from Ref. [15].

spheroid). For the hole 4N-nucleus (^{39}K) having the proton hole in the magic shells, the quadrupole deformation parameter β is positive and equals 0.1 (prolate spheroid). Moreover, it is also found from Table II that the negative quadrupole deformation parameters β are also observed for nuclei having one proton beyond the completely filled proton shell (^{7}Li and ^{35}Cl, except the ^{19}F nucleus). If the nucleon hole in the completely filled shell is observed, then such nucleus has the positive quadrupole deformation parameter β, as for example, ^{11}B, ^{27}Al and also ^{9}Be, ^{25}Mg. Comparing the results of β for $1d_{5/2}$-shell nuclei (see Table II) with those for the $1d_{5/2}$-shell 4N-nuclei (0.47 ± 0.04, 0.35 ± 0.01 [2] for ^{20}Ne, 0.42 ± 0.04 [16], 0.55 ± 0.06 [1] for ^{24}Mg, 0.40 [11, 12]. 0.48 ± 0.05 [1] for ^{28}Si), it is possible to conclude that these parameters β are rather close to about 0.4. However the quadrupole deformation parameter β for the ^{27}Al nucleus is near to that for the ^{32}S nucleus (0.37 [12] 0.30 ± 0.05 [1]). The results presented in Table II can be theoretically interpreted in the following way. In the case of the single particle motion in the deformed potential well, the angular frequency perpendicular $\omega_\perp$ and parallel $\omega_\parallel$ to the symmetry axis of the nucleus, can be written

$$\omega_\perp \simeq \omega_0(1+\tfrac{1}{3}\beta) \tag{4}$$

and

$$\omega_\parallel \simeq \omega_0(1-\tfrac{2}{3}\beta). \tag{5}$$

The contribution from the deformation to the angular frequency ω_0 for the harmonic oscillator well is then given by

$$\Delta\omega_\perp = \tfrac{1}{3}\beta\omega_0 \tag{6}$$

and

$$\Delta\omega_\parallel = -\tfrac{2}{3}\beta\omega_0. \tag{7}$$

Let us now consider, for example, the relative ratio of $\Delta\omega_\perp$ for the different excess and hole 4N-nuclei:

$$\frac{(\Delta\omega_\perp)_1}{(\Delta\omega_\perp)_2} = \frac{\beta_1}{\beta_2}\left(\frac{\omega_{01}}{\omega_{02}}\right) = \frac{\beta_1}{\beta_2}\left(\frac{A_2}{A_1}\right)^{1/3}, \tag{8}$$

where $\omega_0 = 41\,A^{-1/3}$. As is stated before, it is possible to set

$$A_1 = (A_{4N})_1 \pm 1 \tag{9}$$

and

$$A_2 = (A_{4N})_2 \pm 1. \tag{10}$$

In Eqs (9) and (10) plus refers to the excess 4N-nucleus and minus refers to the hole 4N--nucleus. $(A_{4N})_1$ and $(A_{4N})_2$ refer to nearest masses of 4N-nuclei, as is discussed in Eq. (2). Introducing Eqs (9) and (10) into Eq. (8), we have

$$\frac{(\Delta\omega_\perp)_1}{(\Delta\omega_\perp)_2} = \frac{\beta_1}{\beta_2}\left(1-\frac{\Delta A_{4N}}{(A_{4N})_1\pm 1}\right)^{1/3}, \tag{11}$$

where

$$\Delta A_{4N} = (A_{4N})_1 - (A_{4N})_2. \tag{11.1}$$

Setting

$$\omega' = (\Delta\omega_\perp)_2 \left(1 - \frac{\Delta A_{4N}}{(A_{4N})_1 \pm 1}\right)^{1/3} \tag{12}$$

and

$$\beta_1 = N\beta_2 \quad \text{for} \quad \beta_1 \geqslant \beta_2 \tag{13}$$

and observing that the oscillator has the discrete spectrum of the energy, the oscillator energy

$$\hbar(\Delta\omega_\perp)_1 = N\hbar\omega', \tag{14}$$

where N refers to a given integer in the discrete spectrum in which the minimum energy is equal to $\hbar\omega'$. It is to be noted that similar result may be obtained for $\omega_\parallel$. In the foregoing discussion, it was found that the quadrupole deformation coefficients β for two different nuclei are related by an integer N in Eq. (13). As seen from the columns: 2, 3, 4, 5, 7, 8, 9, 10, and 12 of Table II, this equation is fully justified by experimental data. In the certain group of nuclei, let us now consider two different nuclei for which the quadrupole deformation parameter is β. The potential energy of the quadrupole deformation [5]:

$$V = \tfrac{1}{2} C\beta^2 \tag{15}$$

is directly related to the quadrupole deformation parameter β and the energy of the surface tension in C. For these nuclei the relative ratio of the potential energies of the deformation can then be approximately written

$$\frac{V_1}{V_2} = \left(\frac{A_1}{A_2}\right)^{2/3} \quad \text{for} \quad \beta_1 = \beta_2. \tag{16}$$

Introducing Eqs. (9) and (10) into Eq. (16), the last equation becomes

$$\frac{V_1}{V_2} = \left(1 + \frac{\Delta A_{4N}}{(A_{4N})_2 \pm 1}\right)^{2/3}, \tag{17}$$

where ΔA_{4N} is defined in Eq. (11.1).

As seen from Table II this equation is justified by experimental data for $1d_{5/2}$-shell nuclei and the pair of nuclei of the $1d_{5/2}$-shell (^{17}O) and $1d_{3/2}$-shell (^{33}S). It is also observed for the $1d_{5/2}$-shell $4N$-nuclei for which $\beta \approx 0.4$, as it was mentioned before. Thus, the same shapes predicted by β for the given group of nuclei of the given shell and also of different shells, may be explained in terms of the potential energy of the quadrupole deformation.

3. Conclusion

In conclusion, it can be said that the classification of the given group of nuclei as the excess and hole $4N$-nuclei is justified. The important property of these nuclei is that the quadrupole deformation parameters β are connected by an integer N, which is associated with the discrete spectrum of the oscillator energies. The given quadrupole deformation parameter β for different nuclei of the given group may be explained in terms of the potential energy of the quadrupole deformation.

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