STATE DEPENDENT BAG CONSTANT*

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(*Received February 11, 1986*)

Following considerations about large N limit of QCD we propose to introduce a state dependent bag constant. We assume it is proportional to the number of constituents in a hadron. Numerical results for light hadron masses in the bag model support this suggestion.

PACS numbers: 12.40.Aa

In the original formulation of the MIT bag model [1–3] it was assumed that hadrons are bubbles of perturbative vacuum immersed in the physical vacuum. Inside these bubbles valence quarks and gluons are interacting only weakly. The whole complicated structure of many gluon exchanges responsible for confinement was taken into account by considering one universal constant B. This constant B describes the difference in the energy density between perturbative and physical vacuum and does not depend on the number of quarks and gluons that are confined in the bag. On the other hand there has been some interest recently in considering a limit of large number of colors N in Quantum Chromodynamics [4, 5]. It seems that many of the properties of the low energy hadron interactions can be qualitatively explained in the context of large N limit. In this limit the structure and interactions of mesons and baryons are quite different. Mesons in large N limit are free, stable and noninteracting. The number of meson states is infinite and what is important for us meson masses have smooth limits for large N. On the other hand it has been argued by Witten [5] that baryon masses are of order N and only the size and shape of the baryon have smooth limits as $N \to \infty$. Moreover baryons should appear as solitons in the meson

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* This work has been partly supported by the Polish Research Program MRI-7.

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theory. But only recently when the old Skyrme model [6] was reexamined [7, 8] has it become more clear in which sense baryons could be solitons [8, 9].

The assumption of the bag model about one universal state independent constant B is in general not in agreement with what one expects from the large N limit of Quantum Chromodynamics. On the other hand when we limit ourselves to the spectrum of light hadrons looking at the expression for the baryon energy in the bag it is very easy to see how to modify the assumptions of the bag model in order to have the mass of the baryon of order N for large N and the shape having smooth N limit. It is enough to assume that the bag constant is proportional to N. It is not very clear what to take as the bag constant for mesons. In this paper we will assume that the bag constant is proportional to the number of constituents in the hadron. We consider the numerical consequences of this assumption and show that the fit to the light hadron masses of mesons and baryons, taking into account corrections for the center of mass motion, is better than that with one universal constant B. (The number of fitted parameters is not altered.) The proportionality of bag constant B to the number of constituents was obtained before in a different context by Hansson [10] as a consequence of having quark and gluon condensates inside the MIT bag. When we introduce two independent parameters B_M for mesons and B_B for baryons and treat them as free parameters the obtained fit is very similar to our fit with bag constant proportional to the number of constituents ($B_B = \frac{3}{2} B_M$).

In the limit of static spherical cavity the energy of the bag state of a radius R is given by [3]:

$$E = E_k + E_v + E_0 + E_g,$$

where E_k is the quark kinetic energy,

$$E_k = \frac{1}{R} \sum_i^n (x_i^2 + (m_i R)^2)^{1/2},$$

m_i is the quark mass and n is the number of constituents, $n = 2$ for mesons, $n = N = 3$ number of colors for baryons,

$$\operatorname{tg} x_i = \frac{x_i}{1 - m_i R - (x_i^2 + (m_i R)^2)^{1/2}},$$

E_0 and E_g are zero point energy and energy associated with the exchange of a single gluon between two quarks in the bag. The expressions for E_k, E_0 and E_g are taken as in the original MIT fit

$$E_v = \tfrac{4}{3} \pi R^3 n B_0,$$

where n as before is the number of constituents.

The bag radius can be eliminated from the above equations by demanding that

$$\frac{dE}{dR} = 0.$$

The mass of the physical hadron is found by correcting for the center of mass motion [11, 12]

$$M = \left(E^2 - \sum_i^n \frac{x_i^2}{R^2}\right)^{1/2}.$$

We will compare our results with other fits by calculating a quantity χ^2 defined in [13] as

$$\chi^2 = \sum_{\text{hadrons}}^{N} (M_{\text{exp}} - M)^2$$

and an average mass deviation per particle

$$\delta M = \left(\frac{\chi^2}{N}\right)^{1/2},$$

where N is the number of considered hadrons. The parameters to be determined are B_0, α_c, Z_0 and m_s (we set $m_{u,d} = 0$).

We will not follow the procedure of the MIT group [3] to calculate the parameters from masses of N, Δ, ω and Ω^- but make an overall fit to the masses of light hadrons (pion excluded). Our results are presented in Table I. Because we make the overall fit to the masses of light hadrons and take into account center of mass corrections our results cannot be directly compared with the original fit of the MIT group. Fit with the center of mass corrections and the standard way of calculating bag parameters is given in [14]. Our results are compared with the fit of Bartelski et al. [13], last column of Table I, obtained

TABLE I

Results of the fit with $B_B = 3B_0$, $B_M = 2B_0$, $\alpha_c = 2.05$, $m_{u,d} = 0$, $m_s = 0.279$ GeV, $z_0 = 0.34$, $B_0 = 0.101$ GeV, $\chi^2 = 2769$ (MeV)^2, $\delta M = 14.6$ MeV. In the last column for comparison the results of the fit $B_B = B_M$ from [13]

Particle	R (GeV^{-1})	M_{exp} (MeV)	M (MeV)	ΔM (MeV) $B_B = \frac{3}{2} B_M$	ΔM (MeV) $B_B = B_M$ [13]
N	6.00	938.9	959.9	−21	−35
Λ	5.94	115.6	1119.9	−4.3	−22
Σ	5.94	1193.1	1162.7	30.4	21
Ξ	5.89	1318.1	1305.3	12.8	−3
Δ	6.39	1232.0	1243.1	−11.1	14
Σ*	6.34	1385.0	1383.2	1.8	19
Ξ*	6.29	1530.0	1527.4	2.6	18
Ω^-	6.24	1672.2	1676.2	−4	5
ρ	6.35	776.0	764.5	11.5	−7
ω	6.35	782.0	764.5	17.5	8
K*	6.27	892.0	899.3	−7.3	−17
φ	6.19	1019.6	1043.6	−24	−35
K	5.36	496.0	498.2	−2.2	15

TABLE II

Magnetic moments of the baryon octet calculated using the parameters of Table I, center of mass correction [16]. The experimental values were taken from [17]

Particle	$\mu_{\text{bag}}\left(\frac{e}{2M_p}\right)$	$\mu_{\text{exp}}\left(\frac{e}{2M_p}\right)$
p	2.72	2.793
N	−1.78	−1.913
Λ	−0.63	-0.613 ± 0.004
Σ^+	2.5	2.379 ± 0.020
Σ^0	0.77	—
Σ^-	−0.96	-1.10 ± 0.05
Ξ^0	−1.39	-1.250 ± 0.014
Ξ^-	−0.57	-1.85 ± 0.75

under the same assumptions but with a universal bag constant B. The values given in [13] are $\chi^2 = 4917\ (\text{MeV})^2$ and $\delta M = 19.5$ MeV. We see that there is a decrease in χ^2 and δM. We can say that numerical results favor the relation

$$B_B = \tfrac{3}{2} B_M.$$

Even if we accept a more pessimistic point of view, i.e. that the crudeness of the MIT bag model prevents us from taking the quantitative results too seriously, the fact that modified hamiltonian gives predictions qualitatively the same as the original one seems to us to be interesting. Because in general a large N limit suggests that the bag constant for mesons and baryons could be different (B_M/B_B does not equal 2/3 for $N = 3$) we made another fit treating B_M and B_B as independent parameters. With the one additional parameter no essential improvement of the fit was obtained. We were surprised to see that the results are not very different from those obtained for $B_B = 3/2\ B_M$ showing that our assumption of $B = nB_0$ was reasonable. It may support the view that physics with $N = 3$ is not very far from that with $N \to \infty$. We want to stress that because the bag constant for mesons is relatively smaller than for baryons the radii of mesons are bigger and for example the radius for meson ϱ is bigger than for proton. The results of the recent lattice QCD analyses [15] show the same tendency. Having radii of hadrons we can calculate electroweak parameters. As an example the magnetic moments of the baryon octet with the center of mass correction taken into account according to the formulas of [16] are presented in Table II.

As previously the conclusion is that the agreement with experiment is not worse than in the framework with one universal constant B. In summary: we have modified the assumptions of the bag model in such a way that the masses of light mesons and baryons behave for large N as expected from QCD and introduced the state dependent bag constant proportional to the number of constituents in the hadron. The obtained results are slightly better than with the universal constant B.

One of the authors, S. T., would like to thank Prof. H. Genz and Prof. H. Pilkuhn for the hospitality extended to him in Karlsruhe.

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