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KINETIC COEFFICIENTS FOR QUARK-ANTIQUARK PLASMA*

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The quark-antiquark plasma near equilibrium is studied. The results are based on the Heinz kinetic equations with the Boltzmann collision operator approximated by a relaxation term with the relaxation time, τ, treated as a small parameter. Linear in τ solutions of these equations are used to calculate the transport coefficients: the non-abelian version of Ohm's law and the shear and volume viscosities. We introduce new chemical potentials which determine the color density matrix of quarks (antiquarks). Gradients of these potentials generate color currents.

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1. Introduction

There is a considerable interest in physics of the quark-gluon plasma (QGP) which might be produced in ultrarelativistic collisions of heavy ions; it presumably existed at the early stages of the Universe and might even now exist in some exotic astronomical objects [1, 2]. It is therefore of importance to know the kinetic coefficients of such a plasma. There exist already quite a few attemps to calculate them [3, 4]. The purpose of the present paper is to calculate all kinetic coefficients which can be obtained from the transport equations of the quarks (antiquarks) moving in a classical gauge field. In this approach there are no gluons (as particles) thus, strictly speaking, we are not dealing with a quark-gluon plasma but, rather, with a quark-antiquark plasma ($Q\bar{Q}P$). Although this is still not "the

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real thing" it is — in our judgment — an interesting and useful enough object to deserve a careful and complete analysis.

Our starting point are the drift equations for the quark (antiquark) distribution functions, $f^{\pm}$, in the phase space of space-time x^{μ}, momentum p^{ν} and color Q^{a}

$$L_{\pm}f^{\pm}(x, p, Q) = C_{\pm}, \tag{1.1}$$

$$L_{\pm} = p^{\mu}\partial_{\mu} \pm gQ^{a}F^{a}_{\nu\mu}p^{\mu}\partial^{\nu}_{p} - gf_{abc}A^{b}_{\mu}p^{\mu}Q^{a}\partial^{a}_{Q}, \tag{1.2}$$

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf_{abc}A^{b}_{\mu}A^{c}_{\nu}. \tag{1.3}$$

Here g is the coupling constant, $a = 1, 2 \ldots n$ are the color indices (for color indices we do not distinguish the lower and the upper ones, n is given by the gauge group). Repeated indices are summed. $\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}$, $\partial^{\mu}_{q} \equiv \frac{\partial}{\partial p_{\mu}}$, $\partial^{a}_{Q} \equiv \frac{\partial}{\partial Q^{a}}$. $C_{\pm}$ are the collision terms so far unspecified. Later on we shall approximate them by a relaxation-time expression.

The consecutive terms of the l.h.s. of (1.1) represent the drifts of quark's (antiquark's) positions, momenta and colors and thus are, formally, quite straightforward generalization of the QED expression of the l.h.s. of the Boltzmann equation to the case of QCD. They were to our knowledge, first written down by Heinz [5] and then, together with the field equations, applied to study some properties of $Q\bar{Q}P$ [3, 6] in the Vlasov self consistent description.

In this paper we show how (1.1), treated classically (i.e. with most of quantum effects neglected), leads uniquely to all kinetic coefficients in an internally consistent manner and we obtain analytic expressions for them. Although many of our formulae are correct for SU(3) color group, we do all specific calculations for SU(2) color group.

Since $f^{\pm}$ and $C_{\pm}$ are, in our treatment, invariant functions with respect to rotations in color space, Eq. (1.1) is covariant under such transformation. E.g. one can see it in the case of the infinitesimal SU(2) gauge transformation by substituting in (1.1)

$$A^{a'}_{\mu} = A^{a}_{\mu} + \varepsilon^{amn}\theta^{m}A^{n}_{\mu} - \frac{1}{g}\frac{\partial\theta^{a}}{\partial x^{\mu}},$$

$$Q^{a'} = Q^{a} - \varepsilon^{amn}Q^{m}\theta^{n},$$

$$F^{a'}_{\mu\nu} = F^{a}_{\mu\nu} - \varepsilon^{amn}F^{m}_{\mu\nu}\theta^{n},$$

$$f'(x, p, Q'_{a}) = f(x, p, Q_{a}) = f(x, p, Q'_{a} + \varepsilon_{abc}Q'_{b}\theta_{c}), \tag{1.4}$$

where $\theta_{a} = \theta_{a}(x^{\mu})$ is an arbitrary infinitesimal function of x^{μ}.

Since we are dealing with nonequilibrium processes, the process of production of entropy is of fundamental relevance. We adopt the following expression for the flux of entropy [7]:

$$S^{\mu}(x) = -\int dPdQp^{\mu}[\phi(f^{+}) + \phi(f^{-})], \tag{1.5}$$

where

$$\phi(f^{\pm}) = f^{\pm} \ln f^{\pm} - \varepsilon^{-1}(1+\varepsilon f^{\pm}) \ln (1+\varepsilon f^{\pm}), \tag{1.6}$$

so that the derivative is

$$\phi'(f^{\pm}) \equiv y(f^{\pm}) = \ln \frac{f^{\pm}}{1+\varepsilon f^{\pm}} . \tag{1.7}$$

We work with the units where $c = k_B = 1$ (c — light velocity, k_B — Boltzmann constant). Since we are dealing with fermions $\varepsilon = -1$, but most of our results are for the classical limit $\varepsilon \to 0$. $\int dPdQ$ in (1.5) denotes integration over the momentum and color sector of the phase space and is given explicitly in Appendix 1.

Another basic ingredient of our analysis is the form of the distribution functions in equilibrium. We get

$$f^{\pm}_{(0)} = \frac{1}{e^{-y^{\pm}} - \varepsilon}, \quad y^{\pm} = \frac{1}{T} [\pm \mu \pm \mu^a Q_a - u_\lambda p^\lambda] \tag{1.8}$$

where T is the temperature, μ is the baryon chemical potential, μ^a are the colored chemical potentials and u_λ is the four-velocity of the matter in equilibrium.

One should stress that in a quantum mechanical description of color, Q_a become the generators of the color group. The form $\mu + \mu^a Q_a$ in the exponent (1.8) suggests that μ and the vector μ_a in the color space determine the density matrix of the color quarks (antiquarks) [6]. This means that μ, μ^a not only determine how many quarks have a definite color, but also how many are changing their color from, say, j-th to k-th, $j, k = 1, 2, 3$. The quantum mechanical version of calculations of the transport coefficients is being prepared [12].

The organization of the paper is as follows. In Section 2 we introduce the currents, energy-momentum tensor and their conservation laws. In this Section we also construct the equilibrium distributions. The equilibrium distribution function depends now — besides the temperature, the chemical potential and the local velocity — on a new thermodynamic potential which we shall call the colored chemical potential. In Section 3 some consequences of entropy production are discussed. When entropy is produced, we have to have increasing entropy and this leads to some conditions which the currents must satisfy. Section 4 introduces an explicit expression for the collision terms in the relaxation time approximation and constructs the solutions of the transport equation (1.1) near the equilibrium in the form of an expansion in powers of the relaxation time [8]. All calculations which follow are limited to the lowest correction (linear in the relaxation time) to the equilibrium expressions. This Section gives explicit expressions for the baryonic and color currents in terms of gradients of the standard chemical potential and the colored chemical potentials. These relations lead to the definitions of the kinetic coefficients. Section 5 gives our results for the shear and bulk viscosities. A discussion of the kinetic coefficients is contained in the Section 6. Our conclusions are listed in Section 7. Six Appendices close the paper.

We use the metric $g_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$.

2. Conservation laws. Equilibrium distributions

For the sake of completeness we list in this Section the conservation laws which we will need later. The space-time densities of the macroscopic quantities are the following moments of the distribution functions. The baryon number current

$$b^{\mu}(x) = \int dPdQp^{\mu}(f^{+}-f^{-}), \tag{2.1}$$

the color current

$$j_a^{\mu}(x) = \int dPdQp^{\mu}Q_a(f^{+}-f^{-}) \tag{2.2}$$

and the energy-momentum tensor

$$T^{\lambda\mu}(x) = \int dPdQp^{\lambda}p^{\mu}(f^{+}+f^{-}). \tag{2.3}$$

We require that they satisfy the conservation laws

$$\partial_{\mu}b^{\mu} = 0, \tag{2.4}$$

$$\partial_{\mu}j_a^{\mu}+g\varepsilon_{abc}A_{\mu}^{b}j_c^{\mu} = 0, \tag{2.5}$$

$$\partial_{\mu}T^{\lambda\mu} = gF_a^{\lambda\mu}j_{\mu}^{a}. \tag{2.6}$$

Clearly, (2.4)–(2.6) follow from Eq. (1.1) if $C_{\pm} = 0$. When $C_{\pm} \neq 0$, imposition of (2.4)–(2.6) leads to the following constraints on the collision terms (for calculational details see Appendix 1)

$$\int dPdQ(C_{+}-C_{-}) = \int dPdQQ_a(C_{+}-C_{-}) = \int dPdQp^{\lambda}(C_{+}+C_{-}) = 0. \tag{2.7}$$

These equations find immediate application in construction of the equilibrium distribution. Indeed, in the equilibrium there is no production of entropy thus

$$\partial_{\mu}S^{\mu} = 0. \tag{2.8}$$

From (1.5)–(1.7), assuming that the equilibrium distribution, $f_{(0)}^{\pm}$, satisfies the transport equation (1.1) we obtain (the relevant integrals are given in Appendix 1)

$$\int dPdQ[y(f_{(0)}^{+})C_{+}+y(f_{(0)}^{-})C_{-}] = 0. \tag{2.9}$$

Thus, to have no production of entropy, the l.h.s. of (2.9) must be a linear combination of the relations (2.7), consequently the exponents, $y(f_{(0)}^{\pm})$, of the equilibrium distributions of quarks and antiquarks are

$$y(f_{(0)}^{\pm}) = \pm\xi(x)\mp\chi^{a}(x)Q_a-\beta_{\lambda}(x)p^{\lambda}. \tag{2.10}$$

When ξ, χ^a, β_{λ} are taken as arbitrary functions of space-time constrained only through the conservation laws (2.4)–(2.6) (see Appendix 4), we are dealing with the *local equilibrium*: the entropy is conserved but $f_{(0)}^{\pm}$ do not satisfy (1.1). By demanding that $f_{(0)}^{\pm}$ satisfy (1.1) we introduce constraints on ξ, χ^a, β_λ given by (3.2) (see below), and are dealing with the *global equilibrium.*

Since

$$f^{\pm}_{(0)} = \{\exp[\pm\xi(x)\pm\chi^a(x)Q_a+\beta_\lambda(x)p^\lambda]+1\}^{-1} \tag{2.11}$$

we identify (compare e.g. [9]) $\xi = \frac{\mu(x)}{T}$ and $\beta_\lambda = \frac{u_\lambda(x)}{T}$, where $\mu(x)$ is the standard chemical potential, T — temperature, and $u_\lambda(x)$ is the hydrodynamic four-velocity of the fluid. It is natural therefore to set $\chi^a = -\frac{\mu^a(x)}{T}$ and interpret μ^a as a colored chemical potential: μ^a is related to the color current (2.2) in a completely analogous way μ is related to the baryonic current (2.1). In Appendix 5 we show that the equilibrium distribution (2.11) implies (in the limit $\varepsilon \to 0$) thermodynamics of a mixture of substances whose chemical potentials are $\mu(x)$ and $\mu^a(x)$, $a = 1, 2 \dots n$.

μ^a is, in our interpretation, a vector in the color space therefore $y(f^{\pm}_{(0)})$ is an invariant of rotations in color space (Eq. (1.4)). Thus $f^{\pm}_{(0)}$ is an invariant of the gauge transformations (1.4). Since (ε — energy density, s — entropy density, ϱ — density of baryon current, ϱ_a — density of color current, see Appendix 5 for details)

$$d\varepsilon = Tds+\mu d\varrho+\mu^a d\varrho_a, \tag{2.12}$$

we have

$$\mu^a = \left(\frac{\partial\varepsilon}{\partial\varrho_a}\right)_{\substack{s=\text{const}\\ \varrho,\varrho_b=\text{const},\ b\neq a}}. \tag{2.13}$$

So, indeed, μ^a is the energy gained by the system when the color changes by one unit — in complete analogy with the standard chemical potential μ.

3. *Production of entropy*

Now, let us consider the distribution functions $f^{\pm}$ which satisfy the conservation laws (2.4)–(2.6) and are not far removed from the equilibrium functions $f^{\pm}_{(0)}$. For the production of entropy we get (see Appendix 2) to first order in deviations from an equilibrium

$$\partial_\mu S^\mu = -\partial_\mu\xi\delta b^\mu+\partial_\mu\beta_\lambda\delta T^{\lambda\mu}$$
$$+(\partial_\mu\chi^a+g\varepsilon_{abc}A^b_\mu\chi^c+g\beta_\lambda F^{a\lambda}{}_\mu)\delta j^\mu_a, \tag{3.1}$$

and it must be $\partial_\mu S^\mu \geqslant 0$. Here $\delta b^\mu = b^\mu-b^\mu_{(0)}$, $\delta T^{\lambda\mu} = T^{\lambda\mu}-T^{\lambda\mu}_{(0)}$, $\delta j^\mu_a = j^\mu_a-j^\mu_{(0)a}$ are the differences between the currents and the energy-momentum tensors off- and at the equilibrium. Eq. (3.1) implies that there is no production of entropy when

$$\partial_\mu\xi = \partial_\mu\chi^a+g\varepsilon_{abc}A^b_\mu\chi^c+g\beta_\lambda F^{a\lambda}{}_\mu = \partial_{(\mu}\beta_{\lambda)} = 0, \tag{3.2}$$

where $(\mu\lambda)$ denotes symmetrization. Eqs. (3.2) give the constraints which ξ, χ^a and β_λ must satisfy in order to have the global equilibrium.

It is convenient to collect various components into multicomponent objects

$$G_0^\mu \equiv -\partial^\mu \xi, \qquad G_a^\mu \equiv \partial^\mu \chi_a + g\varepsilon_{abc} A_b^\mu \chi^c + g\beta_\lambda F_{a\mu}^\lambda$$

$$G_A^\mu \equiv (G_0^\mu, G_a^\mu), \qquad A = 0, 1 \ldots n. \tag{3.3}$$

And, similarly,

$$\delta j_0^\mu \equiv \delta b^\mu, \qquad \delta j_A^\mu \equiv (\delta b^\mu, \delta j_a^\mu). \tag{3.4}$$

Now (3.1) takes a more compact form

$$\partial_\mu S^\mu = \delta j_A^\mu G_\mu^A + \delta T^{\lambda\mu} \partial_\mu \beta_\lambda. \tag{3.5}$$

Both terms of the r.h.s. should be non-negative. As will be shown in the following Section (and in Appendix 6) one can express δj_A^μ through the gradients G_A^μ

$$\delta j_A^\mu = \Delta_\alpha^\mu \mathfrak{M}_{AB} G^{\alpha B}, \tag{3.6}$$

where $\Delta_\alpha^\mu = g^\mu{}_\alpha - u^\mu u_\alpha$ is the projection operator. The first term in (3.5) is positive definite when $\mathfrak{M}_{AB}$ is negative definite. Indeed, in the local rest frame we have $u^0 = 1$, $u^k = 0$, and $\Delta_\alpha^\mu = \mathrm{diag}\ (0, 1, 1, 1)$ thus

$$\delta j_A^\mu G_\mu^A = \Delta_\alpha^\mu \mathfrak{M}_{AB} G^{\alpha B} G_\mu^A = -\sum_{k=1}^{3} G_k^A \mathfrak{M}_{AB} G_k^B. \tag{3.7}$$

In the next Section we give a specific realization of $\mathfrak{M}_{AB}$ when the collision terms are given by the relaxation time approximation.

In order to exhibit the positive definite nature of the second term of the r.h.s. of (3.5) we write $\delta T^{\lambda\mu}$ in the standard form (see e.g. [7, 11]) exhibiting the shear, η, and the bulk, ζ, viscosity coefficients which are non-negative:

$$\delta T^{\lambda\mu} = \eta \Delta_\alpha^\lambda \Delta_\beta^\mu (\partial^\alpha u^\beta + \partial^\beta u^\alpha - \tfrac{2}{3} \Delta^{\alpha\beta} \Delta^{\varrho\sigma} \partial_\sigma u_\varrho) + \zeta \partial_\nu u^\nu \Delta^{\lambda\mu}. \tag{3.8}$$

In Section 5 we evaluate explicitly η and ζ in terms of the equilibrium parameters also in the relaxation time approximation.

4. Non-equilibrium baryon and color currents in the relaxation time approximation

We are going now to express explicitly the currents through the gradients (3.3) for the following transport equation

$$L_\pm f_\pm = -u_\beta p^\beta \frac{(f'^\pm - f_{(0)}^\pm)}{\tau}, \tag{4.1}$$

where τ is the relaxation time and is a free parameter. As in Ref. [8], we seek the solution in form of a power series in τ:

$$f^\pm = f_{(0)}^\pm + \tau f_{(1)}^\pm + \tau^2 f_{(2)}^\pm + \ldots, \tag{4.2}$$

where $f^{\pm}_{(0)}$ is taken as a local equilibrium distribution function and limit ourselves to just the lowest order corrections:

$$f^{\pm}_{(1)} = -\frac{L_{\pm} f^{\pm}_{(0)}}{u_\beta p^\beta}. \tag{4.3}$$

We also restrict our consideration to the (classical) limit $\varepsilon \to 0$, hence

$$\begin{aligned} f^{\pm}_{(0)}(x, p, Q) &= e^{\pm\xi \mp \chi^a Q_a - p^\mu \beta_\mu} = F_{\pm}(x, Q) G_{\pm}(x, p), \\ F_{\pm}(x, Q) &= e^{\mp \chi^a Q_a}, \quad G_{\pm}(x, p) = e^{\pm\xi - \beta_\lambda p^\lambda}. \end{aligned} \tag{4.4}$$

It is convenient to introduce the following moments of $F_{\pm}(x, Q)$ ($\chi = (\chi_1^2+\chi_2^2+\chi_3^2)^{1/2}$, q — Casimir invariant characterizing the representation, for SU(2) one can take $q^2 = \mathrm{Tr}\, Q_a Q^a = \frac{3}{2}\hbar^2$)

$$\tilde{F}^{\pm}(x) = \int dQ F^{\pm}(x, Q) = \frac{\sinh(\chi q)}{\chi q} \equiv \tilde{F}, \tag{4.5}$$

$$\tilde{F}^{\pm}_a(x) = \int dQ Q_a F^{\pm}(x, Q) = \mp \frac{\chi^a}{\chi^2}\left[\cosh(\chi q) - \frac{1}{\chi q}\sinh(\chi q)\right] \equiv \pm \tilde{F}_a,$$

$$\tilde{F}^{\pm}_{ab}(x) = \int dQ Q_a Q_b F^{\pm}(x, Q) = \delta_{ab}\frac{1}{\chi^2}\left[\cosh(\chi q) - \frac{1}{\chi q}\sinh(\chi q)\right]$$

$$+ \frac{\chi_a \chi_b}{\chi^4}\left\{\chi q \sinh(\chi q) - 3\left[\cosh(\chi q) - \frac{1}{\chi q}\sinh(\chi q)\right]\right\} \equiv \tilde{F}_{ab}.$$

Substituting the approximate form of $f^{\pm}_{(0)}$ (4.4) into (4.3) we get $f^{\pm}_{(1)}$ in terms of $f^{\pm}_{(0)}$ and the gradients (3.3)

$$f^{\pm}_{(1)} = -\frac{f^{\pm}_{(0)}}{u_\beta p^\beta}\left[\pm p^\mu \partial_\mu \xi - p^\mu p^\lambda \partial_\mu \beta_\lambda \mp p^\mu Q_d G^d_\mu\right]. \tag{4.6}$$

Employing (4.6) we get for the correction to the baryon current

$$\delta b^\alpha = \tau b^\alpha_{(1)} = \tau \int dP dQ p^\alpha (f^+_{(1)} - f^-_{(1)})$$

$$= \tau\left[-\int dP dQ p^\alpha \frac{f^+_{(0)}}{u_\beta p^\beta}(p^\mu \partial_\mu \xi - p^\mu p^\lambda \partial_\mu \beta_\lambda - p^\mu Q^d G_{d\mu})\right.$$

$$\left. + \int dP dQ p^\alpha \frac{f^-_{(0)}}{u_\beta p^\beta}(-p^\mu \partial_\mu \xi - p^\mu p^\lambda \partial_\mu \beta_\lambda + p^\mu Q^d G_{d\mu})\right]. \tag{4.7}$$

Now follows a very lengthy but straightforward calculation whose essential ingredients are listed in Appendix 6 and whose many analytical results are taken from Ref. [8]. One also uses the baryon number and energy-momentum conservation (Appendix 4) and the end result is the following

$$\delta b^\alpha = \mathfrak{M}_{00}(-\Delta^{\alpha\mu}\partial_\mu \xi) + \mathfrak{M}_{0a}\Delta^{\alpha\mu} G^a_\mu, \tag{4.8}$$

where

$$\mathfrak{M}_{00} = \tau\tilde{F}m^3 8\pi \cosh\xi \left\{\frac{K_2^2(\gamma)}{\gamma^2 K_3(\gamma)}\tanh^2\xi - \frac{1}{3}\left[\frac{K_2(\gamma)}{\gamma} - K_1(\gamma) + \int_\gamma^\infty dz K_0(z)\right]\right\}$$

$$\mathfrak{M}_{0a} = \tau\tilde{F}_a m^3 8\pi \sinh\xi \left\{\frac{K_2^2(\gamma)}{\gamma^2 K_3(\gamma)} - \frac{1}{3}\left[\frac{K_2(\gamma)}{\gamma} - K_1(\gamma) + \int_\gamma^\infty dz K_0(z)\right]\right\}. \quad (4.9)$$

Similarly, one gets for the correction to the color current

$$\delta j_c^\alpha = \tau \int dPdQQ_c p^\alpha (f_{(1)}^+ - f_{(1)}^-) = \mathfrak{M}_{c0}(-\Delta^{\alpha\mu}\partial_\mu \xi) + \mathfrak{M}_{cd}\Delta^{\alpha\mu}G_\mu^d, \quad (4.10)$$

where $\mathfrak{M}_{c0} = \mathfrak{M}_{0c}$, and

$$\mathfrak{M}_{cd} = \tau m^3 8\pi \cosh\xi \left\{\frac{\tilde{F}_c\tilde{F}_d}{\tilde{F}}\frac{K_2^2(\gamma)}{\gamma^2 K_3(\gamma)} - \tilde{F}_{cd}\frac{1}{3}\left[\frac{K_2(\gamma)}{\gamma} - K_1(\gamma) + \int_\gamma^\infty dz K_0(z)\right]\right\}. \quad (4.11)$$

The $\tilde{F}$'s are given by (4.5) and $\gamma = \frac{m}{T}$ (T — temperature, m — mass of q, $\bar{\text{q}}$), $K_n(\gamma)$ — modified Bessel functions of the second kind. Hence we expressed $\mathfrak{M}_{AB}$ through the equilibrium parameters μ, μ_a, the temperature T and the relaxation time τ.

Note that we do obtain a symmetric matrix

$$\mathfrak{M}_{AB} = \mathfrak{M}_{BA}; \quad A, B = 0, 1 \dots n. \quad (4.12)$$

These are the relativistic Onsager relations [10] generalized to a colored system.

5. *Non-equilibrium correction to the energy-momentum tensor in the relaxation time approximation*

In the near-equilibrium state the correction to the energy-momentum tensor reads

$$\delta T^{\lambda\mu} = \tau T_{(1)}^{\lambda\mu} = \tau \int dPdQp^\lambda p^\mu (f_{(1)}^+ + f_{(1)}^-). \quad (5.1)$$

Similarly as in the case of currents, very lengthy calculations (Appendix 6) lead to identification (through (3.8)) and evaluation of the two viscosities: The shear viscosity is

$$\eta = \frac{\tau\tilde{F}}{T}\frac{8\pi}{15} m^5 \cosh\frac{\mu}{T}\left\{\frac{T}{m}K_4\left(\frac{m}{T}\right) - 3\frac{T^2}{m^2}K_3\left(\frac{m}{T}\right)\right.$$

$$\left. -2\frac{T}{m}K_2\left(\frac{m}{T}\right) + K_1\left(\frac{m}{T}\right) - \int_{\frac{m}{T}}^\infty dz K_0(z)\right\}. \quad (5.2)$$

The bulk viscosity however is given by a much more complicated implicit expression

$$\zeta = \frac{\tau}{\partial_\mu u^\mu}\frac{8\pi}{3}m^4\left[\frac{T}{m}K_3\left(\frac{m}{T}\right) - \frac{T^2}{m^2}K_2'\left(\frac{m}{T}\right) - \frac{T}{m}K_1\left(\frac{m}{T}\right)\right]$$

$$\times\left[\tilde{F}\sinh\left(\frac{\mu}{T}\right)u_\alpha\partial^\alpha\left(\frac{\mu}{T}\right) - \tilde{F}_a\cosh\left(\frac{\mu}{T}\right)G^a_\alpha u^\alpha\right]$$

$$-\tilde{F}\frac{\tau}{\partial_\mu u^\mu}16\pi m^4\cosh\left(\frac{\mu}{T}\right)\frac{T^2}{m^2}K_3\left(\frac{m}{T}\right)u_\alpha\partial^\alpha\left(\frac{m}{T}\right) + \frac{5}{3}\frac{\tau}{T}\eta, \tag{5.3}$$

where η is given by (5.2) and the five gradients $u_\alpha\partial^\alpha\left(\frac{\mu}{T}\right)$, $u_\alpha\partial^\alpha\left(\frac{1}{T}\right)$ and $G^a_\alpha u^\alpha$ $(a = 1, 2, 3)$ are to be calculated from the five linear equations (see Appendix 4, Eqs. (A4.5), (A4.9) and (A4.14))

$$\tilde{F}\left[u_\alpha\partial^\alpha\left(\frac{\mu}{T}\right)\right] - \tanh\left(\frac{\mu}{T}\right)\left(h - \frac{T}{m}\right)\tilde{F}\left[u_\alpha\partial^\alpha\left(\frac{m}{T}\right)\right]$$

$$-\tanh\left(\frac{\mu}{T}\right)\tilde{F}_a[G^a_\lambda u^\lambda] = \tilde{F}\tanh\left(\frac{\mu}{T}\right)\partial_\mu u^\mu,$$

$$\tanh\left(\frac{\mu}{T}\right)\left(\frac{m}{T}h - 1\right)\tilde{F}\left[u_\alpha\partial^\alpha\left(\frac{\mu}{T}\right)\right] - \left(3h + \frac{m}{T}\right)\tilde{F}\left[u_\alpha\partial^\alpha\left(\frac{m}{T}\right)\right]$$

$$-\left(1 - \frac{m}{T}h\right)\tilde{F}_a[G^a_\lambda u^\lambda] = -\tilde{F}\frac{m}{T}h\partial_\mu u^\mu,$$

$$\tanh\left(\frac{\mu}{T}\right)\tilde{F}_c\left[u_\alpha\partial^\alpha\left(\frac{\mu}{T}\right)\right] - \left(h - \frac{T}{m}\right)\tilde{F}_c\left[u_\alpha\partial^\alpha\left(\frac{m}{T}\right)\right]$$

$$-\tilde{F}_{ac}[G^a_\lambda u^\lambda] = -\tilde{F}_c\partial_\mu u^\mu, \tag{5.4}$$

where $h = \dfrac{K_3\left(\frac{m}{T}\right)}{K_2\left(\frac{m}{T}\right)}$. We can solve (5.4) for all gradients in the brackets [...], which can then be written down as ratios of determinants, and come out to be proportional to $\partial_\mu u^\mu$. Thus all gradients in (5.3) can be eliminated and we end up with ζ expressed through the equilibrium parameters μ, μ_a, T and the relaxation time τ. We shall not write down this final explicit expression for ζ because it is very complicated.

6. Discussion of the transport coefficients

The matrix $\mathfrak{M}_{AB}$ gives the relationships between the all possible "strains" G^{μ}_{A} and the non-equilibrium currents δj^{μ}_{B}. It describes diffusion and conduction of both color neutral and colored matter. As we can see from (4.8) and (4.10) the neutral and colored diffusions are coupled in general: colored gradients may generate neutral currents and neutral gradients may generate colored currents. They are related through the Onsager relations (4.12).

We are not aware of such relations existence in the literature. Perhaps the exceptions are the papers by Heinz [3] who discusses the generalization of Ohm's law for color current induced by an external color electric field. In our case, when the covariant derivative of χ^a vanishes and the baryon chemical potential ξ is constant we obtain from (4.10)

$$\delta j^a_\mu = \mathfrak{M}^a_b g \beta_\lambda F^{b\lambda}{}_{\mu} \tag{6.1}$$

or, equivalently, in the local rest frame

$$\delta j^a_k = -\frac{g}{T}\mathfrak{M}^a_b E^b_k \tag{6.2}$$

where $E^k_b = F^{k0}_b$ is the chromoelectric field. (6.2) is a non-abelian version of the Ohm's law. It differs from the one discussed in Ref. [3] because in our case the off-diagonal elements of $\mathfrak{M}$ are in general different from zero. The expressions for $\mathfrak{M}_{AB}$ (compare (4.9) and (4.11)) simplify in some limiting cases, e.g. when

$$\frac{T}{m} \gg 1, \tag{6.3}$$

i.e. for negligible quark masses (or high temperature). In this limit we can use the asymptotic expressions for the Bessel functions K_n

$$K_n\left(\frac{m}{T}\right) \to \tfrac{1}{2}(n-1)!\left(2\frac{T}{m}\right)^n, \qquad \int\limits_{\frac{m}{T}}^{\infty} dz K_0(z) \to 0, \tag{6.4}$$

and we obtain the following expression for $\mathfrak{M}$ in the limit of vanishing quark masses:

$$\lim_{m\to 0}\mathfrak{M}_{AB} = 4\pi\tau T^3 \begin{bmatrix} \tilde{F}\left(\tanh^2\frac{\mu}{T} - \frac{4}{3}\right)\cosh\frac{\mu}{T} & -\frac{1}{3}\tilde{F}_a \sinh\frac{\mu}{T} \\ -\frac{1}{3}\tilde{F}_a\sinh\frac{\mu}{T} & \left(\frac{\tilde{F}_a\tilde{F}_b}{\tilde{F}} - \frac{4}{3}\tilde{F}_{ab}\right)\cosh\frac{\mu}{T} \end{bmatrix}, \tag{6.5}$$

where the $\tilde{F}$'s are given by (4.5). The Ohm relation now takes the form

$$\lim_{m\to 0}\delta j^k_a = 4\pi\tau T^2 g\left(\tfrac{4}{3}\tilde{F}_{ab} - \frac{\tilde{F}_a\tilde{F}_b}{\tilde{F}}\right)\cosh\left(\frac{\mu}{T}\right)E^k_b. \tag{6.6}$$

Note that the $\tilde{F}$'s have additional temperature dependences through their arguments $\chi q = \frac{(\mu_a\mu^a)^{1/2}}{T} q$ and that (6.5) and (6.6) are good, for $m \to 0$, with finite temperatures. Indeed in the limit of very high temperature $\chi q \to 0$ and

$$\lim_{T\to\infty} \tilde{F} = 1, \qquad \lim_{T\to\infty} \tilde{F}_a = 0, \qquad \lim_{T\to\infty} \tilde{F}_{ab} = \tfrac{1}{3} q^2 \delta_{ab}. \tag{6.7}$$

Also: $\text{th}\,\frac{\mu}{T} \to 0$, $\sinh\frac{\mu}{T} \to 0$, $\cosh\frac{\mu}{T} \to 1$, and we get the leading order contribution to $\mathfrak{M}_{AB}$:

$$\lim_{T\to\infty} \mathfrak{M}_{AB} = 4\pi\tau T^3 \begin{bmatrix} -\frac{4}{3} & 0 \\ 0 & -\frac{4}{9} q^2 \delta_{ab} \end{bmatrix}, \tag{6.8}$$

which is negative definite, as it should. Hence, in the rest system of the fluid, we have

$$\lim_{T\to\infty} \delta j_a^k = \tfrac{16}{9} \pi\tau T^2 g q^2 \delta_a^b E_b^k. \tag{6.9}$$

The off-diagonal matrix elements of $\mathfrak{M}$ are zero only to leading order in T.

Another well defined limit is the limit of very heavy quarks $\gamma = \frac{m}{T} \to \infty$. Employing the expansion

$$K_n(\gamma) = \left(\frac{\pi}{2\gamma}\right)^{1/2} e^{-\gamma}\left(1 + \frac{w-1}{8\gamma} + \frac{(w-1)(w-9)}{2!(8\gamma)^2} + \ldots\right), \qquad w = 4n^2$$

and

$$\int_\gamma^\infty dz K_0(z) = \left(\frac{\pi}{2\gamma}\right)^{1/2} e^{-\gamma}\left(1 - \frac{5}{8\gamma} + \frac{129}{128\gamma^2} - \ldots\right) \tag{6.10}$$

we get from (4.9) and (4.11)

$$\lim_{m\to\infty} \mathfrak{M}_{AB} = 8\pi\tau m^3 \left(\cosh\frac{\mu}{T}\right)\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{T}{m}\right)^{5/2} e^{-\frac{m}{T}} \begin{bmatrix} \tilde{F}\left(\tanh\frac{\mu}{T} - 1\right) & 0 \\ 0 & \frac{\tilde{F}_c\tilde{F}_d}{\tilde{F}} - \tilde{F}_{cd} \end{bmatrix}. \tag{6.11}$$

Again, we decouple the neutral and colored transports in this case. Note that the limit $T \to 0$ is more complicated because it depends on the relative ratios of m, μ and μ^a.

The same limits (i.e. $m \to 0$ and $m \to \infty$) can be found for the shear viscosity

$$\lim_{m\to 0} \eta = 12.8\pi\tau\tilde{F}T^4 \cosh\frac{\mu}{T}, \tag{6.12}$$

$$\lim_{m\to\infty} \eta = 0.135\pi\tau\tilde{F}m^4 \left(\frac{m}{T}\right)^{1/2} e^{-\frac{m}{T}} \cosh\frac{\mu}{T}. \tag{6.13}$$

In the case of the bulk viscosity, the only limit which simplifies (5.3) and (5.4) is the limit of high temperature

$$\lim_{T\to\infty} \zeta = 0.74\tau\pi m^4. \tag{6.14}$$

7. Conclusions

We have constructed an equilibrium distribution function of quarks and antiquarks interacting through classical gauge fields. This distribution is determined completely through the color chemical potentials μ, μ_a which, in turn, define the density matrix of the color for quarks and antiquarks.

For the global equilibrium distribution there exists a global co-moving frame ($u_\lambda = (1, 0, 0, 0)$) in which μ, μ_a and the energy-momentum tensors of $q\bar{q}$ and of the gauge field are all constant.

From near-equilibrium distributions we have computed the kinetic coefficients of the $q\bar{q}$ plasma in terms of the equilibrium distribution characteristics: The chemical potentials μ, μ_a, the temperature T, and the Casimir invariant characterizing the representation of the color group.

The Ohm law has the form of the Ohm law for anisotropic conductors in Electrodynamics: that is to say, the relation between the applied color field and the direction of the color current in the color space is given through a tensor of the color conductivity. We give this tensor explicitly.

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APPENDIX 1

a) Phase space

All calculations are made for SU(2), $n = 3$

$$dQ = \frac{d^3Q}{2\pi q}\,\delta(Q^2-q^2), \tag{A1.1}$$

$Q^2 = Q_aQ^a$ and q is the length of the color vector. It is very useful to derive the following lemma

$$\varepsilon_{abc}\int dQM(Q)Q^a\frac{\partial}{\partial Q^b}N(Q)+\varepsilon_{abc}\int dQN(Q)Q^a\frac{\partial}{\partial Q^b}M(Q) = 0, \tag{A1.2}$$

M and N are arbitrary functions of Q. Indeed

$$0 = \varepsilon_{abc}\int\frac{d^3Q}{2\pi q}\,\frac{\partial}{\partial Q^b}\,[\delta(Q^2-q^2)Q_aM(Q)N(Q)]$$

$$= \varepsilon_{abc} \int \frac{d^3Q}{2\pi q} \delta'(Q^2-q^2)2Q_bQ_aM(Q)N(Q) + \varepsilon_{abc} \int \frac{d^3Q}{2\pi q} \delta(Q^2-q^2)\delta_{ab}M(Q)N(Q)$$

$$+ \varepsilon_{abc} \int dQQ_a \left[\frac{\partial}{\partial Q_b} M(Q) \right] N(Q) + \varepsilon_{abc} \int dQQ_aM(Q) \left[\frac{\partial}{\partial Q^b} N(Q) \right].$$

The first and the second terms vanish in the above equation (they contain products of symmetric and antisymmetric tensors and we get (A1.2)).

$$dP = 2H(p^0)\delta(p^\mu p_\mu - m^2)d^4p, \quad H(p^0) = \begin{cases} 1 & p^0 \geqslant 0, \\ 0 & p^0 < 0, \end{cases} \tag{A1.3}$$

see also Ref. [8] and Appendix 3. The second lemma, analogous to the first one is as follows

$$F^a_{\mu\nu} \int dPM(p)p^\nu \partial^\mu_p N(p) + F^a_{\mu\nu} \int dPN(p)p^\nu \partial^\mu_p M(p) = 0. \tag{A1.4}$$

b) Conservation laws

One can derive (2.4)–(2.6) multiplying (1.1) with 1, p^λ, Q^a integrating over p and Q, taking into consideration (A1.2) and (A1.4), assuming (2.7).

c) Equilibrium distributions

The third lemma which helps us to find equilibrium distributions is

$$\partial_\mu \int dPdQ\phi(f^\pm)p^\mu = \int dPdQ\phi'(f^\pm)C_\pm, \tag{A1.5}$$

where $\phi = \phi(f^\pm)$ and $f^\pm = f^\pm(x, p, Q)$. To show it one uses the transport equations to transform the l.h.s. of (A1.5), and the identities

$$\phi'(f^\pm)p^\alpha \partial^\nu_p f^\pm = p^\alpha \partial^\nu_p \phi(f^\pm),$$
$$\phi'(f^\pm)Q^c \partial^a_Q f^\pm = Q^c \partial^a_Q \phi(f^\pm), \tag{A1.6}$$

and then (A1.2) and (A1.4).

APPENDIX 2

Entropy production

We shall derive Eq. (3.1) which gives production of entropy in an arbitrary state $f^\pm$ to first order in $(f^\pm - f^\pm_{(0)})$, where $f^\pm_{(0)}$ is a local equilibrium distribution.

S^μ is given by (1.5) and (1.6). Using (1.7) we may write [7]

$$\phi(f^\pm) = f^\pm y^\pm - \varepsilon^{-1} \ln(1+\varepsilon f^\pm), \tag{A2.1}$$

so that

$$S^\mu = -\int dPdQp^\mu[-\varepsilon^{-1}\ln(1+\varepsilon f^+_{(0)}) + y^+_{(0)}f^+$$
$$-\varepsilon^{-1}\ln(1+\varepsilon f^-_{(0)}) + y^-_{(0)}f^-], \tag{A2.2}$$

and

$$\partial_\mu S^\mu = -\int dPdQp^\mu[(f^+ - f^+_{(0)})\partial_\mu y^+_{(0)} + (f^- - f^-_{(0)})\partial_\mu y^-_{(0)} + y^+_{(0)}\partial_\mu f^+ + y^-_{(0)}\partial_\mu f^-], \tag{A2.3}$$

where $y^\pm_{(0)}$ are given by (2.10).

$$\partial_\mu S^\mu = -\int dPdQp^\mu\{[\partial_\mu\xi - Q^a\partial_\mu\chi^a - p^\lambda\partial_\mu\beta_\lambda](f^+ - f^+_{(0)}) + [-\partial_\mu\xi + Q^a\partial_\mu\chi_a - p^\lambda\partial_\mu\beta_\lambda](f^- - f^-_{(0)}) + (\xi - Q^a\chi_a - p^\lambda\beta_\lambda)\partial_\mu f^+ + (-\xi + Q^a\chi_a - p^\lambda\beta_\lambda)\partial_\mu f^-\}. \tag{A2.4}$$

Taking into consideration (2.4)–(2.6) we obtain

$$\partial_\mu S^\mu = -\partial_\mu\xi\delta b^\mu + (\partial_\mu\chi^d + gf_{dab}A^a_\mu\chi^b + g\beta_\lambda F^{d\lambda}{}_\mu)\delta j^\mu_d + \partial_\mu\beta_\lambda\delta T^{\lambda\mu} + gj^{\mu d}_{(0)}f_{dab}A^a_\mu\chi^b. \tag{A2.5}$$

The last term vanishes so we recover Eq. (3.1). Note that when $f^\pm = f^\pm_{(0)}$, $\delta b^\mu = \delta j^\mu_d = \delta T^{\lambda\mu} = 0$ and $\partial_\mu S = 0$. Thus, to conserve entropy, we do not need the more restrictive global equilibrium, it is enough to have local equilibrium.

APPENDIX 3

List of integrals

K_n is the nth-order modified Bessel function of the second kind. These functions satisfy the relations

$$K_{n+1}(\gamma) = K_{n-1}(\gamma) + \frac{2n}{\gamma}K_n(\gamma),$$

$$\frac{dK_n(\gamma)}{d\gamma} = \frac{nK_n(\gamma)}{\gamma} - K_{n+1}(\gamma).$$

All integrals over p are calculated in Ref. [3].

a) $$\int dPG_\pm = A_\pm(x), \quad A_\pm(x) = 4\pi m^2 e^{\pm\xi}\frac{K_1(\gamma)}{\gamma}.$$

b) $$\int dPG_\pm p^\alpha = n_\pm(x)u^\alpha, \quad n_\pm(x) = 4\pi m^3 e^{\pm\xi}\frac{K_2(\gamma)}{\gamma}.$$

c) $$\int dPG_\pm p^\lambda p^\mu = T^\pm_1(x)u^\lambda u^\mu - T^\pm_2(x)g^{\lambda\mu},$$

$$T^\pm_1(x) = 4\pi m^4 e^{\pm\xi}\frac{K_3(\gamma)}{\gamma}, \quad T^\pm_2(x) = 4\pi m^4 e^{\pm\xi}\frac{K_2(\gamma)}{\gamma^2}.$$

d)
$$\int dPG_{\pm}\frac{p^{\lambda}p^{\mu}}{u_{\beta}p^{\beta}} = \tilde{T}_1^{\pm}(x)u^{\lambda}u^{\mu}-\tilde{T}_2^{\pm}(x)g^{\lambda\mu},$$
$$\tilde{T}_1^{\pm} = \tfrac{1}{3}[4n_{\pm}-4\pi m^3e^{\pm\xi}(K_1-K_{i1})],$$
$$\tilde{T}_2^{\pm} = \tfrac{1}{3}[n_{\pm}-4\pi m^3e^{\pm\xi}(K_1-K_{i1})],$$
$$\tilde{T}_1^{\pm}-\tilde{T}_2^{\pm} = n_{\pm}, \quad K_{i1} = \int_{\gamma}^{\infty}K_0(z)dz.$$

e)
$$\int dPG_{\pm}p^{\lambda}p^{\mu}p^{\alpha} = S_1^{\pm}(x)u^{\lambda}u^{\mu}u^{\alpha}-S_2^{\pm}(x)\,(u^{\lambda}g^{\mu\alpha}+u^{\mu}g^{\alpha\lambda}+u^{\alpha}g^{\lambda\mu}),$$
$$S_1^{\pm} = 4\pi m^5e^{\pm\xi}\frac{K_4(\gamma)}{\gamma}, \quad S_2^{\pm} = 4\pi m^5e^{\pm\xi}\frac{K_3(\gamma)}{\gamma^2},$$
$$S_2^{\pm}\frac{\gamma}{m} = T_1^{\pm}.$$

f)
$$\int dPG_{\pm}\frac{p^{\lambda}p^{\mu}p^{\alpha}}{u_{\beta}p^{\beta}} = \tilde{S}_1^{\pm}(x)u^{\lambda}u^{\mu}u^{\alpha}-\tilde{S}_2^{\pm}(x)\,(u^{\lambda}g^{\mu\alpha}+u^{\mu}g^{\alpha\lambda}+u^{\alpha}g^{\lambda\mu}),$$
$$\tilde{S}_1^{\pm} = 2(T_1^{\pm}-T_2^{\pm})-m^2A_{\pm}, \quad \tilde{S}_2^{\pm} = \tfrac{1}{3}(T_1^{\pm}-T_2^{\pm})-\tfrac{1}{3}m^2A_{\pm},$$
$$3\tilde{S}_2^{\pm}-\tilde{S}_1^{\pm} = -(T_1^{\pm}-T_2^{\pm}), \quad T_2^{\pm} = \tilde{S}_2^{\pm}, \quad n_{\pm} = \frac{\gamma}{m}T_2^{\pm}.$$

g)
$$\int dPG_{\pm}\frac{p^{\lambda}p^{\mu}p^{\alpha}p^{\sigma}}{u_{\beta}p^{\beta}} = \tilde{Q}_1^{\pm}(x)u^{\lambda}u^{\mu}u^{\alpha}u^{\sigma}$$
$$-\tilde{Q}_2^{\pm}(x)\,(g^{\lambda\mu}u^{\alpha}u^{\sigma}+g^{\lambda\alpha}u^{\mu}u^{\sigma}+g^{\lambda\sigma}u^{\mu}u^{\alpha}+g^{\mu\alpha}u^{\lambda}u^{\sigma}$$
$$+g^{\mu\sigma}u^{\lambda}u^{\alpha}+g^{\alpha\sigma}u^{\lambda}u^{\mu})+\tilde{Q}_3^{\pm}(x)\,(g^{\lambda\mu}g^{\alpha\sigma}+g^{\lambda\alpha}g^{\mu\sigma}+g^{\lambda\sigma}+g^{\mu\alpha}),$$
$$\tilde{Q}_1^{\pm} = \tfrac{1}{5}[16S_1^{\pm}-48S_2^{\pm}-12m^2n_{\pm}+4\pi e^{\pm\xi}m^5(K_1-K_{i1})],$$
$$\tilde{Q}_2^{\pm} = \tfrac{1}{15}[6S_1^{\pm}-18S_2^{\pm}-7m^2n_{\pm}+4\pi e^{\pm\xi}m^5(K_1-K_{i1})],$$
$$\tilde{Q}_3^{\pm} = \tfrac{1}{15}[S_1^{\pm}-3S_2^{\pm}-2m^2n_{\pm}+4\pi e^{\pm\xi}m^5(K_1-K_{i1})],$$
$$\tilde{Q}_2^{\pm}-\tilde{Q}_3^{\pm} = S_2^{\pm}.$$

APPENDIX 4

Conservation laws in the lowest order

a)
$$\partial_{\alpha}b^{\alpha}_{(0)} = 0 \tag{A4.1}$$

where

$$b^{\alpha}_{(0)} = \int dPdQp^{\alpha}(f^{+}_{(0)}-f^{-}_{(0)}) \equiv \varrho u^{\alpha}, \tag{A4.2}$$
$$\varrho = \tilde{F}_{+}n_{+}-\tilde{F}_{-}n_{-}. \tag{A4.3}$$

Properties of the Bessel functions give (see Appendix 3)

$$\dot{n}_{\pm} = \pm n_{\pm}\dot{\xi} - \frac{\gamma^2}{m}(3\tilde{S}_2^{\pm} - \tilde{S}_1^{\pm})\left(\frac{1}{\gamma}\right)^{\bullet}. \tag{A4.4}$$

The dot denotes operator $u^{\mu}\partial_{\mu}$. This operator represents the change of any quantity in the co-moving frame, i.e. in the frame where the local velocity of the plasma vanishes. Using (A4.4) we get the following form of the baryon current conservation

$$\tilde{F}_{+}\left[n_{+}\dot{\xi} - \frac{\gamma^2}{m}(3\tilde{S}_2^{+} - \tilde{S}_1^{+})\left(\frac{1}{\gamma}\right)^{\bullet} + n_{+}\theta\right]$$

$$-\tilde{F}_{-}\left[-n_{-}\dot{\xi} - \frac{\gamma^2}{m}(3\tilde{S}_2^{-} - \tilde{S}_1^{-})\left(\frac{1}{\gamma}\right)^{\bullet} + n_{-}\theta\right] - \dot{\chi}^{a}(\tilde{F}_a^{+}n_{+} + \tilde{F}_a^{-}n_{-}) = 0, \tag{A4.5}$$

where θ is the divergence of the four-velocity u^{μ}.

b) For the color current we obtain similar relations:

$$\partial_{\alpha}j^{\alpha}_{(0)c} + g f_{cab} A^{a}_{\alpha} j^{\alpha b}_{(0)} = 0, \tag{A4.6}$$

$$j^{\alpha}_{(0)c} = \int dP dQ p^{\alpha} Q_c (f^{+}_{(0)} - f^{-}_{(0)}) \equiv \varrho_c u^{\alpha}, \tag{A4.7}$$

$$\varrho_c = \tilde{F}_c^{+} n_{+} - \tilde{F}_c^{-} n_{-}, \tag{A4.8}$$

and finally the conservation of the color current results in the following relation

$$\tilde{F}_c^{+}\left[n_{+}\dot{\xi} - \frac{\gamma^2}{m}(3\tilde{S}_2^{+} - \tilde{S}_1^{+})\left(\frac{1}{\gamma}\right)^{\bullet} + n_{+}\theta\right]$$

$$-\tilde{F}_c^{-}\left[-n_{-}\dot{\xi} - \frac{\gamma^2}{m}(3\tilde{S}_2^{-} - \tilde{S}_1^{-})\left(\frac{1}{\gamma}\right)^{\bullet} + n_{-}\theta\right]$$

$$-\dot{\chi}^{a}(\tilde{F}_{ac}^{+}n_{+} + \tilde{F}_{ac}^{-}n_{-}) + g f_{cab} A^{a}_{\lambda} u^{\lambda}(\tilde{F}_b^{+} n_{+} - \tilde{F}_b^{-} n_{-}) = 0. \tag{A4.9}$$

c)

$$\partial_{\beta} T^{\alpha\beta}_{(0)} = g F^{\alpha\nu}_{a} j^{a}_{(0)\nu}, \tag{A4.10}$$

where

$$T^{\alpha\beta}_{(0)} = \int dP dQ p^{\alpha} p^{\beta} (f^{+}_{(0)} + f^{-}_{(0)})$$

$$\equiv (\varepsilon_{+} + P_{+}) u^{\alpha} u^{\beta} - P_{+} g^{\alpha\beta} + (\varepsilon_{-} + P_{-}) u^{\alpha} u^{\beta} - P_{-} g^{\alpha\beta}$$

$$\equiv (\varepsilon + P) u^{\alpha} u^{\beta} - P g^{\alpha\beta}, \tag{A4.11}$$

and

$$\varepsilon_{\pm} + P_{\pm} = \tilde{F}_{\pm} T_1^{\pm} \qquad P_{\pm} = \tilde{F}_{\pm} T_2^{\pm}, \tag{A4.12}$$

$$\partial_\alpha(\varepsilon_\pm + P_\pm) = \mp \tilde{F}_a^\pm T_1^\pm \partial_\alpha \chi^a + \tilde{F}^\pm \left[\pm T_1^\pm \partial_\alpha \xi - \frac{\gamma^2}{m} (2S_2^\pm - S_1^\pm) \partial_\alpha \left(\frac{1}{\gamma} \right) \right],$$

$$\partial_\alpha P_\pm = \mp \tilde{F}_a^\pm T_2^\pm \partial_\alpha \chi^a + \tilde{F}^\pm \left[\pm T_2^\pm \partial_\alpha \xi + \frac{\gamma^2}{m} S_2^\pm \partial_\alpha \left(\frac{1}{\gamma} \right) \right]. \tag{A4.13}$$

From Eq. (A4.10) we get

$$u_\alpha \partial_\beta T_{(0)}^{\alpha\beta} = 0. \tag{A4.14}$$

Substituting (A4.11) into (A4.10), using (A4.13), (A4.14) and (A4.7) we obtain the following relation from the energy-momentum conservation

$$\frac{\gamma^2}{m} \Delta_\mu^\alpha \partial^\mu \left(\frac{1}{\gamma} \right) - \frac{\gamma}{m} \dot{u}^\alpha = \frac{-\Delta^{\alpha\mu} \partial_\mu \xi (\tilde{F}^+ T_2^+ - \tilde{F}^- T_2^-) + \Delta^{\alpha\mu} G_\mu^a (\tilde{F}_a^+ T_2^+ - \tilde{F}_a^- T_2^-)}{\tilde{F}^+ S_2^+ + \tilde{F}^- S_2^-}. \tag{A4.15}$$

APPENDIX 5

Colored chemical potentials

Integration of the equilibrium distribution functions (4.4) gives (see (A4.2), (A4.7), (A4.11) and Appendix 3)

$$b_{(0)}^\mu = \varrho u^\mu, \quad j_{(0)a}^\mu = \varrho_a u^\mu,$$
$$T_{(0)}^{\lambda\mu} = (\varepsilon + P) u^\lambda u^\mu - P g^{\lambda\mu}. \tag{A5.1}$$

The above equations identify ε, P, ϱ and ϱ_a as the energy density, pressure, baryon density and color density, measured in the rest frame.

Eq. (1.5) yields ($\varepsilon \to 0$)

$$S_{(0)}^\mu = \left(\frac{\varepsilon + P}{T} - \xi\varrho + \chi^a \varrho_a \right) u^\mu \equiv s u^\mu \tag{A5.2}$$

Here s is the entropy density and

$$T^{-2} = \beta_\lambda \beta^\lambda, \quad T \equiv \frac{1}{\beta}. \tag{A5.3}$$

Taking into consideration (A4.12) one gets

$$T^{-1} dP + (\varepsilon + P) d(T^{-1}) = \varrho d\xi - \varrho_a d\chi^a. \tag{A5.4}$$

Eqs. (A5.2) and (A5.4) lead to the first law of thermodynamics in the following form

$$d\varepsilon = T ds + T\xi d\varrho - T\chi^a d\varrho_a. \tag{A5.5}$$

Substituting $\xi = \frac{\mu}{T}$, $\chi^a = -\frac{\mu^a}{T}$ we recover Eq. (2.12).

APPENDIX 6

a) Non-equilibrium currents

Let us consider Eq. (4.7). Integration over the color and momentum sections of the phase space gives

$$\begin{aligned} b^{\alpha}_{(1)} &= (-\tilde{F}^{+}\partial_{\mu}\xi+\tilde{F}^{+}_{d}G^{d}_{\mu})(\tilde{T}^{+}_{1}u^{\alpha}u^{\mu}-\tilde{T}^{+}_{2}g^{\alpha\mu}) \\ &+(-\tilde{F}^{-}\partial_{\mu}\xi+\tilde{F}^{-}_{d}G^{d}_{\mu})(\tilde{T}^{-}_{1}u^{\alpha}u^{\mu}-\tilde{T}^{-}_{2}g^{\alpha\mu}) \\ &+\tilde{F}^{+}\partial_{\mu}\beta_{\lambda}[\tilde{S}^{+}_{1}u^{\lambda}u^{\mu}u^{\alpha}-\tilde{S}^{+}_{2}(u^{\alpha}g^{\mu\lambda}+u^{\mu}g^{\lambda\alpha}+u^{\lambda}g^{\alpha\mu})] \\ &-\tilde{F}^{-}\partial_{\mu}\beta_{\lambda}[\tilde{S}^{-}_{1}u^{\lambda}u^{\mu}u^{\alpha}-\tilde{S}^{-}_{2}(u^{\alpha}g^{\mu\lambda}+u^{\mu}g^{\lambda\alpha}+u^{\lambda}g^{\alpha\mu})]. \end{aligned} \tag{A6.1}$$

Functions $\tilde{T}^{\pm}_{1}$, $\tilde{T}^{\pm}_{2}$, $\tilde{S}^{\pm}_{1}$, $\tilde{S}^{\pm}_{2}$ are defined in Appendix 3 (following Ref. [8]). Multiplying all the terms in (A6.1) one obtains

$$\begin{aligned} b^{\alpha}_{(1)} &= (\tilde{F}^{+}\tilde{T}^{+}_{2}+\tilde{F}^{-}\tilde{T}^{-}_{2})\partial^{\alpha}\xi-(\tilde{F}^{+}_{d}\tilde{T}^{+}_{2}+\tilde{F}^{-}_{d}\tilde{T}^{-}_{2})G^{d\alpha} \\ &+u^{\alpha}[-(\tilde{F}^{+}\tilde{T}^{+}_{1}+\tilde{F}^{-}\tilde{T}^{-}_{1})\dot{\xi}+(\tilde{F}^{+}_{d}\tilde{T}^{+}_{1}+\tilde{F}^{-}_{d}\tilde{T}^{-}_{1})G^{d}_{\mu}u^{\mu} \\ &+\beta\theta(\tilde{F}^{-}\tilde{S}^{-}_{2}-\tilde{F}^{+}\tilde{S}^{+}_{2})]-(\tilde{F}^{+}\tilde{S}^{-}_{2}-\tilde{F}^{-}\tilde{S}^{-}_{2})\partial^{\alpha}\beta-(\tilde{F}^{+}\tilde{S}^{+}_{2}-\tilde{F}^{-}\tilde{S}^{-}_{2})\dot{u}^{\alpha}\beta \\ &+u^{\alpha}[\tilde{F}^{+}\dot{\beta}(\tilde{S}^{+}_{1}-2\tilde{S}^{+}_{2})-\tilde{F}^{-}\dot{\beta}(\tilde{S}^{-}_{1}-2\tilde{S}^{-}_{2})], \end{aligned} \tag{A6.2}$$

$\left(\beta = \dfrac{1}{T},\ \theta = \partial_{\mu}u^{\mu},\ \dot{\beta} = u^{\mu}\partial_{\mu}\beta\right)$. Using the baryon number conservation (Appendix 4) the last term in (A6.2) may be replaced and consequently Eq. (A6.2) reduces to

$$\begin{aligned} b^{\alpha}_{(1)} &= \left(\frac{\gamma^{2}}{m}\Delta^{\alpha}_{\mu}\partial^{\mu}\left(\frac{1}{\gamma}\right)-\frac{\gamma}{m}\dot{u}^{\alpha}\right)(\tilde{F}^{+}\tilde{S}^{+}_{2}-\tilde{F}^{-}\tilde{S}^{-}_{2}) \\ &+\Delta^{\alpha\mu}\partial_{\mu}\xi(\tilde{F}^{+}\tilde{T}^{+}_{2}+\tilde{F}^{-}\tilde{T}^{-}_{2})-\Delta^{\alpha\mu}G^{d}_{\mu}(\tilde{F}^{+}_{d}\tilde{T}^{+}_{2}+\tilde{F}^{-}_{d}\tilde{T}^{-}_{2}) \\ &+g(\tilde{F}^{+}_{d}n_{+}+\tilde{F}^{-}_{d}n_{-})u^{\alpha}f_{dab}A^{a}_{\mu}u^{\mu}\chi^{b}. \end{aligned} \tag{A6.3}$$

The last term in (A6.3) vanishes so that taking into consideration the energy conservation (Appendix 4) one gets Eq. (4.8). Similarly we obtain Eq. (4.10).

b) Non-equilibrium energy-momentum tensor

Using (5.1) and integrating over p and Q one gets

$$\begin{aligned} T^{\lambda\mu}_{(1)} &= u^{\lambda}u^{\mu}\{[\tilde{F}^{+}(2\tilde{S}^{+}_{2}-\tilde{S}^{+}_{1})-\tilde{F}^{-}(2\tilde{S}^{-}_{2}-\tilde{S}^{-}_{1})]\dot{\xi} \\ &+[\tilde{F}^{+}_{a}(\tilde{S}^{+}_{1}-2\tilde{S}^{+}_{2})-\tilde{F}^{-}_{a}(\tilde{S}^{-}_{1}-2\tilde{S}^{-}_{2})]G^{a}_{\alpha}u^{\alpha}+\tilde{F}^{+}[\dot{\beta}(\tilde{Q}^{+}_{1}-5\tilde{Q}^{+}_{2}+2\tilde{Q}^{+}_{3}) \\ &-\beta\theta(\tilde{Q}^{+}_{2}+\tfrac{2}{3}\tilde{Q}^{+}_{3})]+\tilde{F}^{-}[\dot{\beta}(\tilde{Q}^{-}_{1}-5\tilde{Q}^{-}_{2}+2\tilde{Q}^{-}_{3})-\beta\theta(\tilde{Q}^{-}_{2}+\tfrac{2}{3}\tilde{Q}^{-}_{3})]\} \\ &+g^{\lambda\mu}\{(\tilde{F}^{+}\tilde{S}^{+}_{2}-\tilde{F}^{-}\tilde{S}^{-}_{2})\dot{\xi}-(\tilde{F}^{+}_{a}\tilde{S}^{+}_{2}-\tilde{F}^{-}_{a}\tilde{S}^{-}_{2})G^{a}_{\alpha}u^{\alpha} \\ &+\tilde{F}^{+}[\dot{\beta}(\tilde{Q}^{+}_{2}-\tilde{Q}^{+}_{3})-\tfrac{5}{3}\beta\theta\tilde{Q}^{+}_{3}]-\tilde{F}^{-}[\dot{\beta}(\tilde{Q}^{-}_{2}-\tilde{Q}^{-}_{3})-\tfrac{5}{3}\beta\theta\tilde{Q}^{-}_{3}]\} \\ &+(\tilde{F}^{+}\tilde{Q}^{+}_{3}+\tilde{F}^{-}\tilde{Q}^{-}_{3})\beta\Delta^{\lambda}_{\alpha}\Delta^{\mu}_{\beta}(\partial^{\alpha}u^{\beta}+\partial^{\beta}u^{\alpha}-\tfrac{2}{3}\Delta^{\alpha\beta}\Delta^{\varrho\sigma}\partial_{\sigma}u_{\varrho}). \end{aligned} \tag{A6.4}$$

Eqs. (4.3) and (A4.10) lead to the condition

$$T^{\lambda\mu}_{(1)}u_\mu = 0, \tag{A6.5}$$

so that terms standing beside the tensors $u^\lambda u^\mu$ and $g^{\lambda\mu}$ must differ only in sign. Therefore we conclude that

$$\begin{aligned} T^{\lambda\mu}_{(1)} = \Delta^{\lambda\mu}\{(\tilde{F}^+\tilde{S}_2^+ - \tilde{F}^-\tilde{S}_2^-)\dot{\xi} - (\tilde{F}_a^+\tilde{S}_2^+ - \tilde{F}_a^-\tilde{S}_2^-)G^a_\alpha u^\alpha \\ -\tilde{F}^+[\dot{\beta}(\tilde{Q}_2^+ - \tilde{Q}_3^+) - \tfrac{5}{3}\beta\theta\tilde{Q}_3^+] - \tilde{F}^-[\dot{\beta}(\tilde{Q}_2^- - \tilde{Q}_3^-) - \tfrac{5}{3}\beta\theta\tilde{Q}_3^-]\} \\ +(\tilde{F}^+\tilde{Q}_3^+ + \tilde{F}^-\tilde{Q}_3^-)\beta\Delta^\lambda_\alpha\Delta^\mu_\beta(\partial^\alpha u^\beta - \partial^\beta u^\alpha - \tfrac{2}{3}\Delta^{\alpha\beta}\Delta^{\varrho\sigma}\partial_\sigma u_\varrho). \end{aligned} \tag{A6.6}$$

We see that $\delta T^{\lambda\mu}$ has such a decomposition as was suggested in Section 3.

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