THE ONE-LOOP EFFECTS IN THE ELECTROWEAK GLASHOW-WEINBERG-SALAM THEORY*

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In the near future the experiment will reach a great precision and will be able to test the standard electroweak theory. It is important now to put in order calculations of radiative corrections in this theory and to make correct and exact present theoretical predictions for the measured quantities. The survey of some results of group working in the JINR, Dubna, may serve this aim. We discuss here on-mass-shell renormalization scheme in the unitary gauge; the one-loop amplitudes of both charge and neutral currents-induced fermion scatterings; the large constant effects; the dynamical behaviour of the one-loop neutral-current corrections; the calculation of the W- and Z-boson masses; the difference between the various Weinberg parameters $\sin^2\theta_W$.

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1. Introduction

The non-Abelian $SU(2)\times U(1)$ gauge symmetry which unify the weak and electromagnetic interactions was proposed by Glashow, Weinberg and Salam [1]. Their model along with the photon contains three weak intermediate vector bosons $W^{\pm}$ and Z^0 regarded as quanta of gauge fields. The gauge symmetry is spontaneously broken down to Abelian U(1) symmetry of the electromagnetic interactions by introducing the Higgs scalar doublet with four degrees of freedom

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

whose vacuum expectation value is not equal to zero

$$\langle\Phi\rangle_0 = \frac{\lambda}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

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Due to the Higgs mechanism [2] three bosons $W^{\pm}$ and Z^0, swallowing one-each extra degree of freedom acquire masses. The photon remains massless. Correspondingly, one extra degree of freedom remains free and it is the physical Higgs scalar boson χ.

According to 't Hooft such a non-Abelian gauge field theory with spontaneous breakdown of a gauge symmetry is renormalizable [3]. It permits us to obtain manifestly finite predictions for the physical quantities order by order.

The Glashow-Weinberg-Salam (GWS) model is not unique. There was a variety of models for an electroweak theory with the same charge current and various neutral current structures. But GWS theory called the standard theory is just the one which has received a significant experimental confirmation on the tree level. Along with the well-known charge currents this theory contains weak neutral currents which were observed in neutrino scattering experiments [4]. The weak neutral currents in this theory do not conserve the parity and this was seen in the atomic transitions [5] and in deep inelastic scattering of polarized electrons on deuterons [6]. Also a forward-backward asymmetry in the process $e^-e^+ \to \mu^-\mu^+$ [7] and a charge asymmetry in the cross sections of deep inelastic μ^-N and μ^+N scatterings [8] are observed in a good agreement with GWS theory.

A genuine triumph for GWS theory is the discovery of the weak intermediate bosons $W^{\pm}$ and Z^0 [9, 10]. It is remarkable too that theoretical predictions for masses of $W^{\pm}$ and Z^0 bosons are in agreement with the experimental values obtained at CERN. Therefore, we believe there is something very truthful in the standard theory, it is believed to be the correct theory of the electroweak interactions at present.

Now it is important to test the electroweak theory in the field-theoretical aspect. We must check how does the standard theory work at the one-loop level and higher. Therefore, it is necessary to calculate the one-loop radiative corrections and compare the theoretical predictions with corresponding experimental data.

A number of theoretical groups have calculated radiative corrections at the one-loop level in the standard theory. At the end of 1979 several of them discovered independently rather large radiative corrections to vector boson masses M_W and M_Z. It was the group of Marciano and Sirlin [11], our group known as the "Russian group" [12, 13] (though not all of the participants are Russians), the group of Veltman [14], the group of Consoli [15], Japanese group [16] and Oxford group [17]. These groups and other authors carry out a systematic investigation of the one-loop corrections.

Here arises a problem of the choice of a renormalization scheme. One of the difficulties in communication between different groups was that many different choices of a renormalization scheme are employed. It is clear that the physics does not depend on how the theory is renormalized. But perturbative approximations of the physical quantities do depend on the renormalization scheme.

2. The renormalization scheme

It is well-known that in the minimal standard theory dealing with N_f fermion fields there are N_f+4 independent parameters — N_f fermion masses m_f, the Higgs boson mass M_χ and three additional parameters which should be chosen arbitrarily from the following set: the electron charge e, weak charges g and g', vector boson masses M_W and M_Z, the Weinberg parameter $\sin^2 \theta_W$, the vacuum expectation value λ of the Higgs scalar doublet and the coefficient h at the nonlinear term $(\Phi^+\Phi)^2$ in the Higgs potential. Various renormalization schemes differ by a specific choice of these independent parameters, specific generation of counterterms and what is more important, which physical processes are used to fix the renormalized parameters.

The first renormalization scheme proposed by Ross and Taylor [18] uses the electric charge e and vector boson masses M_W and M_Z as independent parameters. This scheme has a suitable choice of the independent parameters but is unattractive in our opinion because the renormalization is done off mass-shell. As a consequence, the external lines of some particles (which correspond to free particles) give finite non-vanishing contributions after renormalization [19]. In order to obtain a consistent free particle interpretation in the theory it is necessary to perform the renormalization on mass-shell.

We can do it very easily in the unitary gauge. The next scheme of Appelquist, Primack and Quinn [20] was developed just in the unitary gauge. But the choice of the weak charge g along with the electric charge e and W boson mass M_W as independent parameters is not suitable in our opinion. As it turned out it is very difficult to define a convenient on-mass-shell renormalization condition for the weak charge g. While the usual renormalization condition known from QED works very well in the standard theory for the electric charge e the analogous condition for the weak charge gives unphysical infrared and mass singularities [21]. To remove them an additional renormalization should be applied using process $W^- \to \mu^- + \bar{\nu}_\mu$ [20–22] or $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ [12]. Therefore, it is not suitable to regard the weak charge g as an independent parameter. On the other hand, in this scheme only W boson mass is an independent parameter and renormalized M_W is the physical W boson mass, while the Z boson mass is dependent parameter and renormalized M_Z is not the physical Z boson mass but a mass which is calculated in the considered perturbation order. It is not convenient to have the pole of the Z boson propagator not at the physical mass. Consequently, both W and Z boson masses must be independent parameters in order to renormalize them equivalently and renormalized masses to be physical masses.

During 1979–1980 the groups of Veltman [23] and Consoli [15, 24] working in the 't Hooft-Feynman gauge tested several renormalization schemes taking e, M_W, M_Z or g, $\sin \theta_W$, M_W as independent parameters and various physical processes to fix them subsequently.

The absence of a conventional renormalization scheme produced essential disadvantages for comparing results and conclusions obtained by different authors. One source of confusion was the definition of the Weinberg parameter $\sin^2 \theta_W$. At the tree level all definitions are equivalent which is not true at the one-loop level. If we take e, M_W, M_Z as independent parameters the Weinberg parameter is dependent and defined in an unique way by the equality

$$\sin^2 \theta_W^M = 1 - M_W^2/M_Z^2, \tag{1}$$

which remains exact at the one-loop level and higher [11]. But if we take e, g, M_W or g, $\sin \theta_W$, M_W as independent parameters various definitions for the Weinberg parameter are possible. For instance, one may define $\sin^2\theta_W$ through the ratio of charged to neutral current neutrino scatterings. At the one-loop level this definition differs from the previous one by terms of order α.

From 1980 our group and other authors began to work in the unique on-mass-shell renormalization scheme with independent parameters e, M_W, M_Z, M_χ and all fermion masses m_f [11, 13, 16, 25]. It was showed by the Japanese group that the on-shell renormalization may be carried out not only in the unitary gauge but in the 't Hooft-Feynman gauge, too [16, 26]. In this scheme, known as Sirlin's scheme, one has only one independent coupling constant, the electric charge e, for which the usual renormalization condition known from QED can be easily extended to the standard theory. The conventional definition at zero momentum transfer, using Thomson formula, can be adopted to fix the electric charge and one gets the fine structure constant

$$\alpha = e^2/4\pi = 1/137. \tag{2}$$

In such a case all expansions in perturbative calculations are done in the only constant α which provides great advantages in higher order calculations. In this scheme the renormalization of all masses is done equivalently by means of the conventional on-shell renormalization conditions, the renormalized masses being physical particle masses. Sirlin's renormalization scheme was recognized as the most convenient at the Workshop in Trieste [27].

Several groups choose the fine structure constant and the physical particle masses as independent parameters. But there are some distinctions in their renormalization schemes. Our group uses the momentum subtraction scheme and the unitary gauge, the Japanese group uses the same subtraction scheme but works in the 't Hooft-Feynman gauge, as well as all other groups, the Oxford group uses the minimal subtraction scheme, the group of Marciano and Sirlin uses both momentum subtraction and minimal subtraction schemes, and gives the coefficient [28] relating the parameter $\sin^2\theta_W^M$ with the parameter $\sin^2\theta_W(M_W)$ defined by modified minimal subtraction with unit of mass μ set equal to M_W. In spite of these distinctions in the gauge and in the subtraction of divergences all these groups have compatible results. By the way, the group of Consoli re-expressed [29] their results in terms of $\sin^2\theta_W^M$ instead of their previous parameter $\sin^2\theta_W$ defined by the ratio of charge and neutral current neutrino scatterings and came to a consent with all groups working with $\sin^2\theta_W^M$. Consequently, ways and means of definition of the parameters are the most important characteristic of the renormalization schemes.

Our group works in the unitary gauge and we insist on the use to work in this gauge. Of course, it is a non-renormalizable gauge but we know that the theory is renormalizable. So, one expects a cancellation of the ultra-violet divergences on the level of amplitudes. However, there are some advantages to work in the unitary gauge. Just in the unitary gauge the on-shell renormalization is quite natural. We have not unphysical scalars and for instance, boson has its three degrees of freedom explicitly, as the propagator shows

$$(\delta_{\alpha\beta} + q_\alpha q_\beta/M_W^2)/(q^2 + M_W^2). \tag{3}$$

In such a case the simple Lorentz condition $q_\alpha W_\alpha = 0$ is available. Therefore, the elementary vertex $WW\gamma$ obeys on-shell current conservation. As a result of it, the Ward identity is valid

$$k_\mu \Gamma_\mu(p, p-k, k) = eQ_f[\Sigma(p)-\Sigma(p-k)] \tag{4}$$

what is not true in the 't Hooft-Feynman gauge. As a consequence, in the unitary gauge the renormalization of the electric charge has a simple form

$$e_0 = Z_A^{-1/2} e, \tag{5}$$

where $Z_A^{1/2}$ is the photon-field renormalization constant.

We renormalize the boson fields in the following way:

$$\begin{pmatrix} A_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} Z_A^{1/2} & Z_M^{1/2} \\ 0 & Z_Z^{1/2} \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix},$$

$$W_0 = Z_W^{1/2} W, \quad \chi_0 = Z_\chi^{1/2} \chi. \tag{6}$$

It is interesting that the renormalization constants Z_W, Z_Z, Z_M and Z_χ are not of great importance because they cancel each other on the level of amplitudes. The photon-field renormalization constant Z_A plays an essential role in the radiative corrections.

$$(Z_A-1)_{\text{finite}} = \frac{\alpha}{4\pi} A(0) = \frac{\alpha}{\pi}\left(\frac{1}{6}-\frac{2}{3}\,\text{Tr}\left(Q_f^2 \ln \frac{M_W}{m_f}\right)\right). \tag{7}$$

The finite transverse photon self-energy function

$$\Pi(q^2)_{\text{fin.}} = \frac{\alpha}{4\pi} A\left(\frac{q^2}{M_W^2}\right) \tag{8}$$

contains [30, 31] the finite one-loop vacuum-polarization function

$$\alpha\Pi_1(q^2) = 2\frac{\alpha}{\pi}\,\text{Tr}\,(Q_f^2 J(q^2, m_f^2, m_f^2)) \tag{9}$$

corresponding to the sum of vacuum-polarization graphs.

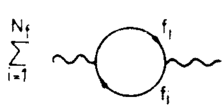

Fig. 1. The sum of photon vacuum-polarization graphs

The function

$$J(q^2, m_f^2, m_f^2) = \int_0^1 dx\,x(1-x) \ln \frac{m_f^2 + x(1-x)q^2}{M_W^2} \tag{10}$$

is equal to

$$(\tfrac{1}{6}) \ln (m_f^2/M_W^2) \quad \text{at} \quad q^2 = 0; \tag{11}$$

$$(\tfrac{1}{6}) (\ln |q^2/M_W^2| - \tfrac{5}{3}) \quad \text{for} \quad |q^2| \gg m_f^2. \tag{12}$$

This function evaluated at $q^2 = 0$ is responsible for large logarithmic corrections of the form

$$\alpha\pi_1(0) = -\tfrac{2}{3} \frac{\alpha}{\pi} \operatorname{Tr}\left(Q_f^2 \ln \frac{M_W}{m_f}\right) \tag{13}$$

not only in the amplitudes but in the cross sections of various processes, too.

We use the Weinberg parameter in the form

$$R = \cos^2 \theta_W = M_W^2/M_Z^2 \tag{14}$$

instead of (1). It is a dependent parameter and its renormalization is a consequence of the W and Z boson mass renormalization

$$R_0 = R + \delta R, \quad \delta R/R = \delta M_W^2/M_W^2 - \delta M_Z^2/M_Z^2. \tag{15}$$

We have

$$(\delta R/R)_{\text{finite}} = (\alpha/4\pi)\,(W(-1) - Z(-1))/(1-R), \tag{16}$$

where the finite functions $W(q^2/M_W^2)$ and $Z(q^2/M_Z^2)$ are connected with self-energy functions of W and Z bosons.

The weak charge g is coupled with electric charge e by

$$g = e/\sin\theta_W = e(1-R)^{-1/2}. \tag{17}$$

So, the weak charge renormalization is done by means of electric charge renormalization constant $Z_A^{-1/2}$

$$g_0 = Z_A^{-1/2}(1 - \delta R/(1-R))^{-1/2} g. \tag{18}$$

We supplement the considered renormalization scheme by an arbitrary unitary mixing of fermion fields introducing the unitary mixing matrix K with an arbitrary quasidiagonal structure

$$K = \begin{pmatrix} K_1 & 0 & - & - \\ 0 & K_2 & - & - \\ - & - & - & - \end{pmatrix}, \tag{19}$$

where $K_1, K_2, \ldots$ are unitary submatrices mixing fermions in separate subspaces, for example, lepton and quark mixing. All fermions are organized in a big doublet

$$f = \begin{pmatrix} f^u \\ f^d \end{pmatrix}, \tag{20}$$

where f^{u} and f^{d} are columns of up and down fermions. (It does not matter if up or down fermions are mixed.) To cancel Adler anomalies we adopt the standard theory with arbitrary number n of lefthanded lepton doublets and corresponding $3n$ quark doublets which should obey the conventional requirement for up and down quark charges

$$\sum_{i=1}^{3} (Q_i^{\mathrm{u}} + Q_i^{\mathrm{d}}) = 1, \tag{21}$$

where i is the colour index. We adopt also the free quark model and believe that quark masses are physical quantities, well-defined. Following Marciano and Sirlin [32] we regard mixing matrix elements as finite phenomenological parameters which should not be renormalized.

To obtain the fermion mass term in the Lagrangian, it is necessary to consider the interaction of mixing fermions with the scalar doublet Φ. After spontaneous breaking of gauge SU(2) × U(1) symmetry the fermions aquire masses. In the unitary gauge we have

$$-(\bar{f}_{\mathrm{L}} M f_{\mathrm{R}} + \bar{f}_{\mathrm{R}} M^{+} f_{\mathrm{L}}), \tag{22}$$

where M is a fermion mass matrix, in general non-diagonal.

We introduce the fermion field renormalization matrices

$$f_{0\mathrm{L}} = Z_{f_{\mathrm{L}}}^{1/2} f_{\mathrm{L}}, \quad f_{0\mathrm{R}} = Z_{f_{\mathrm{R}}}^{1/2} f_{\mathrm{R}}, \tag{23}$$

and the fermion mass renormalization matrix $Z_{m_{\mathrm{f}}}$ with a dimension of mass

$$M_0 = ((Z_{f_{\mathrm{L}}}^{1/2})^{+})^{-1} Z_{m_{\mathrm{f}}} (Z_{f_{\mathrm{R}}}^{1/2})^{-1}. \tag{24}$$

The renormalized mass term is diagonal

$$-\bar{f} m_{\mathrm{f}} f, \tag{25}$$

m_{f} being the diagonal mass matrix of the physical fermion masses. Hence, the fermion mass counter term is

$$-(\bar{f}_{\mathrm{L}} Z_{m_{\mathrm{f}}} f_{\mathrm{R}} + \bar{f}_{\mathrm{R}} Z_{m_{\mathrm{f}}}^{+} f_{\mathrm{L}} - \bar{f} m_{\mathrm{f}} f). \tag{26}$$

3. *Finite one-loop amplitudes of spin-1/2 fermion scattering*

Because of the precise measurements of the muon's total decay rate, it is interesting to consider in detail the fermion scattering induced by a charge current. In the tree approximation the scattering (or annihilation) of any two fermions through W exchange is described by only one Feynman diagram

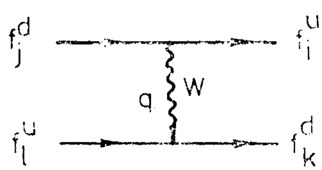

Fig. 2. The Feynman diagram for the scattering of any two fermions through W-boson exchange in the tree approximation

The amplitude is

$$M^{CC}_{0ij,kl} = C_{ij,kl}(\delta_{\alpha\beta}+q_\alpha q_\beta/M_W^2)O_\alpha \times O_\beta, \tag{27}$$

where

$$C_{ij,kl} = -i(2\pi)^4(g^2/8)K_{ij}K^+_{kl}/(q^2+M_W^2), \tag{28}$$

K_{ij} being a mixing matrix element. Here we adopt $O_\alpha = \gamma_\alpha(1+\gamma_5)$ and the direct product $O_\alpha \times O_\beta$ of Dirac matrices stands for the well-known expression $\bar{u}(p_i)O_\alpha u(p_j)\bar{u}(p_k)O_\beta u(p_l)$.

In the one-loop approximation the amplitude has the form

$$M^{CC}_{ij,kl} = M^{CC}_{0ij,kl}Em^{CC}(q^2, S, k^2)+C_{ij,kl}F_1^W(q^2, S, k^2)O_\alpha \times O_\alpha, \tag{29}$$

where $q = p_j - p_i$ is the momentum transfer and $k = p_j - p_k$ is the crossing momentum transfer; $S = -(p_j+p_l)^2$, $t = -q^2$, $u = -k^2$ being the amplitude invariants. Explicit expressions are calculated in our works [13, 33, 34].

The amplitude contains the factorized pure electromagnetic term with genuine infrared divergences which are cancelled subsequently by the bremsstrahlung at the cross section level. In the weak part of the amplitude we neglect the terms of order $O(\alpha m_f^2/M_W^2)$ where m_f is the mass of fermions. So we have only one finite form factor F_1^W.

For simplicity we could propose all amplitude invariants satisfying the inequalities

$$m_f^2 \ll S, |q^2|, |k^2| \ll M_W^2. \tag{30}$$

In such a case we mark them by symbol $\hat{0}$. So, the weak form factor F_1^W turns into

$$F_1^W(\hat{0}) = 1+(\alpha/4\pi)\left\{-A(0)+\frac{R}{(1-R)^2}(W(-1)-Z(-1))+\frac{1}{1-R}\left[W(\hat{0})-W(-1)\right.\right.$$
$$\left.\left.-\tfrac{5}{8}R(R+1)+\tfrac{11}{2}+\tfrac{9}{4}\frac{R}{1-R}\ln R\right]+\left(\tfrac{7}{2}-3\ln\frac{S}{M_Z^2}\right)(|Q_i|\,|Q_k|+|Q_j|\,|Q_f|)\right\}. \tag{31}$$

Here Q_f is the electric charge (in units of e) of incident and outgoing fermions. It is seen from (31), (7) and (13) that one encounters the large logarithmic corrections (13), as a consequence of the weak charge renormalization (18).

As for μ-decay, in particular

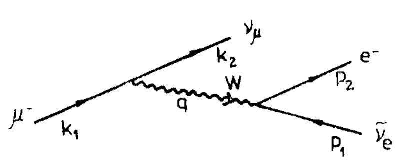

Fig. 3. The Feynman diagram for μ-decay

the amplitude invariants q^2 and k^2 lie in the intervals

$$m_e^2 \leqslant |q^2| \leqslant m_\mu^2, \quad 0 \leqslant |k^2| \leqslant (m_\mu - m_e)^2. \tag{32}$$

So the weak form factor F_1^W in a good approximation is represented by expression (31) taking $Q_i = Q_l = 0$ and $|Q_j| = |Q_k| = 1$.

$$F_1^W = 1+(\alpha/4\pi)X = 1-\alpha\Pi_1(0)+0(\alpha), \tag{33}$$

where

$$X = \tfrac{8}{3}\,\mathrm{Tr}\left(Q_f^2 \ln \frac{M_W}{m_f}\right) - \tfrac{2}{3} + \frac{R}{(1-R)^2}(W(-1)-Z(-1))$$
$$+\frac{1}{1-R}\left[W(\hat{O})-W(-1)-\tfrac{5}{8}R(R+1)+\tfrac{11}{2}+\tfrac{9}{4}\frac{R}{1-R}\ln R\right]. \tag{34}$$

The pure electromagnetic part of the amplitude should be calculated very precisely. Thus in one-loop approximation one obtains [13] for the muon's total decay rate

$$\frac{1}{\tau_\mu} = \frac{P}{32}\frac{g^4}{M_W^4}\left[1+\frac{\alpha}{2\pi}\left(\tfrac{25}{4}-\pi^2+X\right)\right], \tag{35}$$

where

$$P = \frac{m_\mu^5}{192\pi^3}\left(1-8\frac{m_e^2}{m_\mu^2}\right)\left(1+\tfrac{3}{5}\frac{m_\mu^2}{M_W^2}\right). \tag{36}$$

Sirlin in his work [30] discusses in detail the correction to muon's total decay rate of order $O(\alpha^2 \ln(m_f))$, neglecting terms of order $O(\alpha^2)$ and higher. His final result is [30]

$$\frac{1}{\tau_\mu} = \frac{P}{32}\frac{g^4}{M_W^4}\left[1+\frac{\alpha}{2\pi}\left(\tfrac{25}{4}-\pi^2\right)\left(1+\tfrac{2}{3}\frac{\alpha}{\pi}\ln\frac{m_\mu}{m_e}\right)\right]\left(1-\frac{\alpha}{4\pi}X\right)^{-2} \tag{37}$$

where $\alpha\pi_1(0)$ is replaced by $\alpha\Pi_1(0)+\alpha^2\Pi_2(0)$ — quasicomplete vacuum-polarization function evaluated at $q^2 = 0$. By the way, $\alpha^2\Pi_2(0)$ give a very small effect negligible in a numerical evaluation of $\frac{\alpha}{4\pi}X$. Including the weak correction $\frac{\alpha}{4\pi}X$ in the denominator he has summed the leading logarithms $(\alpha \ln(m_f))^n$ to all orders in perturbation theory, as well as the main contributions of order $O(\alpha^2 \ln m_f)$. Thus the large vacuum-polarization effects are already summed and will not appear in higher orders.

One can define G_μ by means of the equation

$$\frac{1}{\tau_\mu} = G_\mu^2 P\left[1+\frac{\alpha}{2\pi}\left(\tfrac{25}{4}-\pi^2\right)\left(1+\tfrac{2}{3}\frac{\alpha}{\pi}\ln\frac{m_\mu}{m_e}\right)\right], \tag{38}$$

where the correction to the muon's total decay rate in the previous V-A theory through terms of order $O\left(\alpha^2 \ln\frac{m_\mu}{m_e}\right)$ is taken into account [35]. Inserting the experimental value of muon's lifetime one obtains:

$$G_\mu = (1.16634 \pm 0.00002)\times 10^{-5}\ \mathrm{GeV}^{-2}. \tag{39}$$

Comparing equations (37) and (38) one finds a very important relation between M_W and $\sin\theta_W$ (having in mind $g = e(\sin\theta_W)^{-1}$)

$$M_W = A(\sin\theta_W^M)^{-1}, \tag{40}$$

where

$$A = A_0(1-(\alpha/4\pi)X)^{-1/2}, \quad A_0 = 37.281 \text{ GeV}. \tag{41}$$

In the last numerical evaluation of $(\alpha/4\pi)X$ Marciano and Sirlin [36] employ $\sin^2\theta_W = 0.217$, $M = M_Z$, $m_t = 36$ GeV, the effective light-quark masses $m_u = m_d = 75$ MeV, $m_s = 250$ MeV obtained by the Wetzel dispersive analyses, using the process $e^-e^+ \to$ hadrons [37], including also QSD corrections. Their result is

$$(\alpha/4\pi)X = 0.0696 \pm 0.0020 \tag{42}$$

where an estimate of uncertainties in the hadronic contributions has been included. We see that the weak correction is significant. Thus we receive for quantity A the very correct evaluation

$$A = 38.65 \pm 0.04 \text{ GeV}. \tag{43}$$

Neutral current induced fermion-fermion scatterings are proper reactions of the standard theory. In the tree approximation they are mediated by neutral bosons γ, Z and χ as it is shown by the Feynman diagrams.

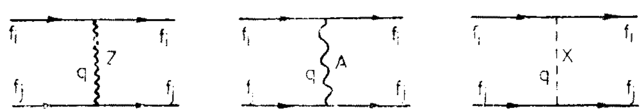

Fig. 4. The Feynman diagrams for the scattering of any two fermions mediated by neutral bosons γ, Z and χ

The amplitude is

$$M_{0ij}^{NC} = C_{ij}^Z(\delta_{\alpha\beta}+q_\alpha q_\beta/M_Z^2)\,[O_\alpha-4(1-R)\,|Q_i|\gamma_\alpha]\times[O_\beta-4(1-R)\,|Q_j|\gamma_\beta]$$

$$+C_{ij}^A\gamma_\alpha\times\gamma_\alpha+C_{ij}I\times I, \tag{44}$$

where

$$C_{ij}^Z = -i(2\pi)^4(g^2/16R)S_iS_j/(q^2+M_Z^2),$$

$$C_{ij}^A = -i(2\pi)^4e^2Q_iQ_j/q^2,$$

$$C_{ij} = i(2\pi)^4(g^2/4)\,(m_im_j/M_W^2)/(q^2+M_\chi^2). \tag{45}$$

Here S_i is a matrix element of the diagonal matrix $S_f = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ equal to twice the third isospin projection of the i^{th} fermion with an electric charge Q_i. We have $S_f|Q_f| = Q_f$.

In the one-loop approximation along with a factorized pure electromagnetic part the amplitude contains a finite weak part which is organized by means of weak form factors [33, 34]:

$$(M_{ij}^{NC})_{weak} = C_{ij}^{Z}(O_\alpha \times O_\alpha F_1^Z - 4(1-R)|Q_i|\gamma_\alpha \times O_\alpha F_2^Z - 4(1-R)|Q_j|O_\alpha \times \gamma_\alpha F_3^Z$$
$$+16(1-R)^2|Q_i|\,|Q_j|\gamma_\alpha \times \gamma_\alpha F_4^Z) + C_{ij}^{A}(\gamma_\alpha \times \gamma_\alpha F_6^A + \gamma_\alpha\gamma_5 \times \gamma_\alpha\gamma_5 F_7^A). \qquad (46)$$

We have $F_3^Z = F_2^Z(i \leftrightarrow j)$. All terms of order $O(\alpha m_f^2/M_W^2)$ are neglected. The form factors are functions of the amplitude invariants q^2 and S, k^2 being equal to $S-q^2$. Moreover, they contain large constant terms, the large logarithms. The vacuum-polarization function evaluated at $q^2 = 0$ (13) is included in form factors F_1^Z, F_2^Z and F_4^Z as a consequence of the weak charge renormalization (18), and in form factor F_A^6 to be renormalized the photon self-energy function $\mathrm{Re}\,\Pi(q^2) = \Pi(q^2) - \Pi(0)$. Form factors F_2^Z and F_4^Z contain the function

$$M(q^2/M_Z^2) = -RA(q^2/M_W^2) + \mathrm{Tr}\,((8Q_f^2 - 2|Q_f|)J(q^2, m_f, m_f)) \qquad (47)$$

connected with the photon-Z boson mixing. We see here the function (10) corresponding to the vacuum polarization. The last function gives a large logarithmic contribution when the amplitude invariants are in the region (30), as the expression (12) shows.

Let us consider low energy values of the form factors. In the region (30) they remain nearly the same. For instance, we take $F_i(\hat{O})$ where the symbol $\hat{O}$ stands for $S = 100$ GeV^2 and $q^2 = 1$ GeV^2. We have numerical evaluated them for $\sin^2\theta_W^M = 0.218$, $M_\chi = 200$ GeV, $M_W = 82.5$ GeV, $M_Z = 93.3$ GeV and the usual constituent quark masses and have obtained:

$$F_1^Z(\hat{O}) = 1.063, \quad F_2^Z(\hat{O}) = 1.088, \quad F_4^Z(\hat{O}) = 1.093, \quad F_6^A(\hat{O}) = 1.025. \qquad (48)$$

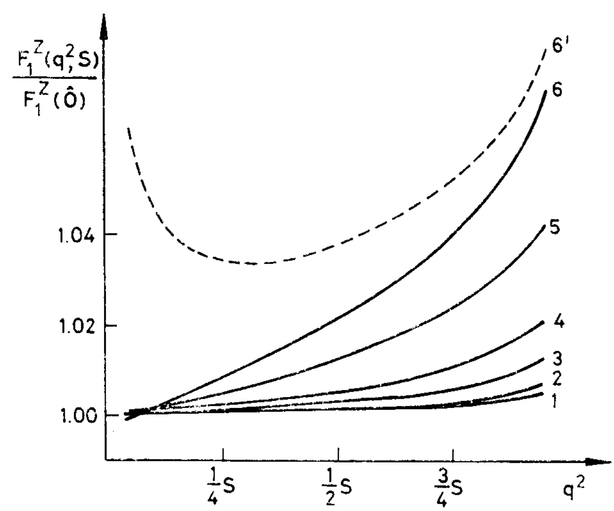

Fig. 5. The ratio $F_1^Z(q^2, S)/F_1^Z(\hat{O})$ versus q^2 for fermions with electric charges $Q_i = Q_j = -1$ and for six values of S in GeV^2: *1)* 10^2; *2)* $2 \cdot 10^3$; *3)* $5 \cdot 10^3$; *4)* 10^4; *5)* $3 \cdot 10^4$ and *6)* 10^5. The curve *6'* corresponds to the asymptotic expression ($|q^2|$, S, $|k^2| \gg M_Z^2$, M_χ^2) for the form factor $F_1^Z(q^2, S)$, taken at $S = 10^5$ GeV^2

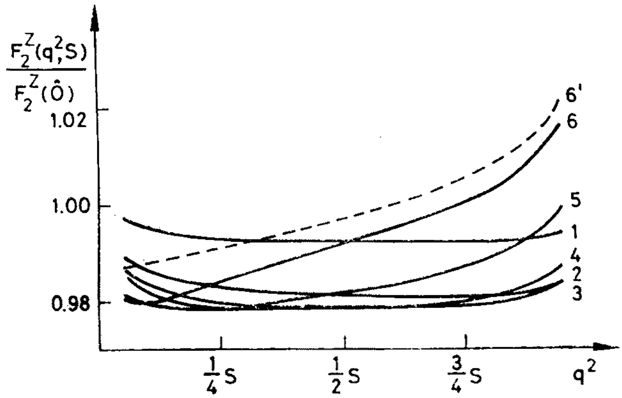

Fig. 6. The same for the form factor F_2^Z

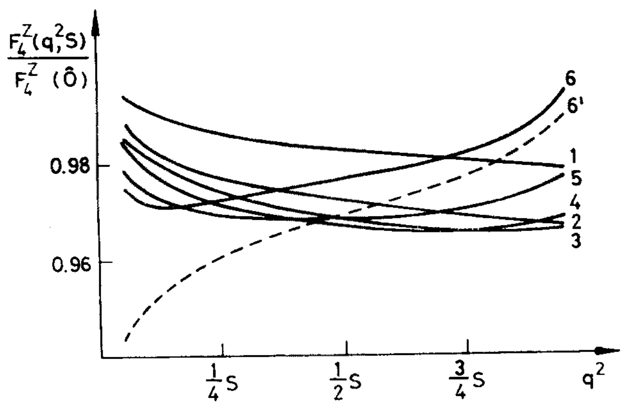

Fig. 7. The same for the form factor F_4^Z

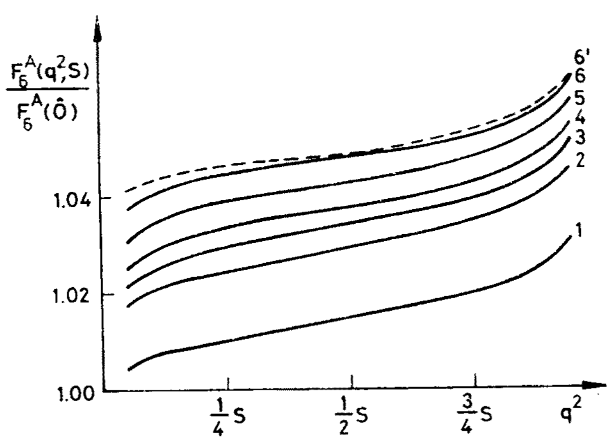

Fig. 8. The same for the form factor F_6^A

(The form factor F_7^A is very close to zero for every q^2 and S.) Obviously the constant effects of the one-loop corrections are significant. They have mainly a vacuum-polarization origin.

The ratios $F_i(q^2, S)/F_i(\hat{0})$ show the purely dynamical behaviour of the form factors because the large constant terms with $\ln(M_W/m_f)$ are cancelled. Their behaviour is determined only by the q^2 and S dependence, and practically do not depend on fermion masses. We take for example the fermions with electric charges $Q_i = Q_j = -1$ and give these ratios versus q^2 in Figs. 5–8 for six values of S in GeV^2: *1)* 10^2; *2)* $2 \cdot 10^3$; *3)* $5 \cdot 10^3$; *4)* 10^4; *5)* $3 \cdot 10^4$; *6)* 10^5. The curve *6'* corresponds to the asymptotic expressions ($|q^2|S$, $|k^2| \gg M_Z^2, M_\chi^2$) for the form factors $F_i(q^2, S)$ taken at $S = 10^5$ GeV^2. One can see from Figs. 5–7 that form factor F_1^Z begins to grow only from $S = 10^4$ GeV^2, i.e. from $E_f = 50$ GeV, and form factors F_2^Z and F_4^Z grow slowly even at higher energies. The comparison between curves *6* and *6'* shows that at $S = 10^5$ GeV^2 form factors F_i^Z are still far from an asymptotic behaviour. Only the form factor F_6^A has an asymptotic behaviour even at such low energies. The genuine electroweak field theory effects due to $(\ln(q^2))^2$ appear already at the $S = 10^5$ GeV^2. But they are very small as compared with the large constant effects. So one could not observe them at such low energies. (Our figures 5–8 are correct everywhere with the exception of the boson resonance points.)

4. *Calculation of the W- and Z-boson masses*

Our calculation of W- and Z-boson masses uses the following three experimental fixing points: the fine structure constant α (2), the Fermi constant G_μ defined from the total muon lifetime (39) and the experimental value for the Weinberg parameter

$$\sin^2 \theta_W^{exp} = 0.224 \pm 0.014 \tag{49}$$

extracted from the data analysis of the SLAC experiment [6] measuring the P-odd asymmetry in the deep-inelastic electron-deuteron scattering.

The theoretical analysis of the one-loop corrections to the P-odd asymmetries has been performed in detail by Bardin, Shumeiko and Fedorenko [12, 38]. It was the first calculation showing the significance of these corrections. We can use for our aim the one--loop corrections to the asymmetry A^-

$$A^-(\lambda) = \frac{1}{\lambda} \frac{d^2\sigma^-(\lambda) - d^2\sigma^-(-\lambda)}{d^2\sigma^-(\lambda) + d^2\sigma^-(-\lambda)}, \tag{50}$$

where λ is the longitudinal polarization of the initial lepton, and $d^2\sigma^-$ is the double differential inclusive cross section of deep-inelastic scattering of polarized leptons on nucleons

$$1^- + N \to 1^- + \text{anything}. \tag{51}$$

In the kinematical region of the SLAC experiment these corrections happen to be constant. Therefore, it is possible to minimize them by redefining the Weinberg parameter

$\sin^2\theta_W$ through the equation

$$\frac{1}{\sqrt{2}} G_\mu \sin^2\theta_W^{\exp} = \frac{g^2}{8M_W^2}\sin^2\theta_W^M\left(1+\frac{\alpha}{4\pi}Y\right). \tag{52}$$

Here we include the large constant part $1+(\alpha/4\pi)Y$ of the low-energy form factor F_2^Z into the definition of the measured quantity $\sin^2\theta_W^{\exp}$, where

$$Y = \frac{1}{1-R}\left[\tfrac{2}{3}\,\mathrm{Tr}\left(|Q_f|\ln\frac{M_W}{m_f}\right) - \tfrac{2}{3} + Z(\hat{O}) - W(-1)\right]. \tag{53}$$

The large logarithm effect comes from combining of two vacuum-polarization functions: of the photon due to the charge renormalization (13), and of the photon-Z boson mixing (47). Having in mind $e^2 = g^2\sin^2\theta_W^M$ we obtain from (52) an equation for the W-boson mass

$$M_W = \frac{37.281\ \mathrm{GeV}}{\sin\theta_W^{\exp}}\left(1+\frac{\alpha}{4\pi}Y\right)^{1/2}. \tag{54}$$

Using Eqs. (40), (41) and (54) we find a relation between the two Weinberg parameters

$$\sin^2\theta_W^M = \sin^2\theta_W^{\exp}\,\frac{1-\dfrac{\alpha}{4\pi}Y}{1-\dfrac{\alpha}{4\pi}X}. \tag{55}$$

The definition (1) of $\sin^2\theta_W^M$ and Eqs. (54) and (55) give for the Z-boson mass

$$M_Z = \frac{37.281\ \mathrm{GeV}}{\sin\theta_W^{\exp}\cos\theta_W^{\exp}}\left(1+\frac{\alpha}{4\pi}Y+\frac{\alpha}{4\pi}(X-Y)\,\mathrm{tg}^2\,\theta_W^{\exp}\right)^{1/2}. \tag{56}$$

The constants X and Y depend on M_W, M_Z and M but, in view of small correction factors, equations (54)–(56) can be solved by iterations with respect to M_W and M_Z. As for M_χ we observe, as the other authors, a weak dependence of solutions on M_χ when varying M_χ within the limits 10 GeV — 1 TeV, as shown in Figs. 9 and 10. In these figures the curves *a*, *b* and *c* correspond to three values for $\sin^2\theta_W^{\exp}$: 0.22, 0.23 and 0.24 respectively, and the numbers *1* and *2* correspond to the current and constituent quark masses. We are in excellent agreement with the results of the other authors.

We can redefine the weak charge g, so that the weak corrections to the total muon lifetime in Eq. (37) vanish, as they do in Eq. (38).

$$g^2(1-(\alpha/4\pi)X)^{-1} = g_F^2. \tag{57}$$

The new weak charge g_F is related with the Fermi constant G by the simple equation

$$(2)^{-1/2}G = (g_F)^2/(8M_W^2), \tag{58}$$

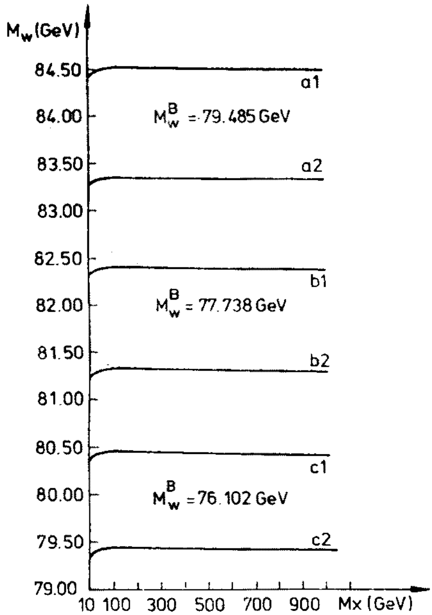

Fig. 9. The mass of W-boson versus M_χ, M_χ varying within the limits 10 GeV–1 TeV. In this figure the curves *a*, *b*, *c* correspond to three values for $\sin^2\theta_W^{exp}$: 0.22, 0.23 and 0.24 respectively, and the numbers 1 and 2 correspond to the current and constituent quark masses

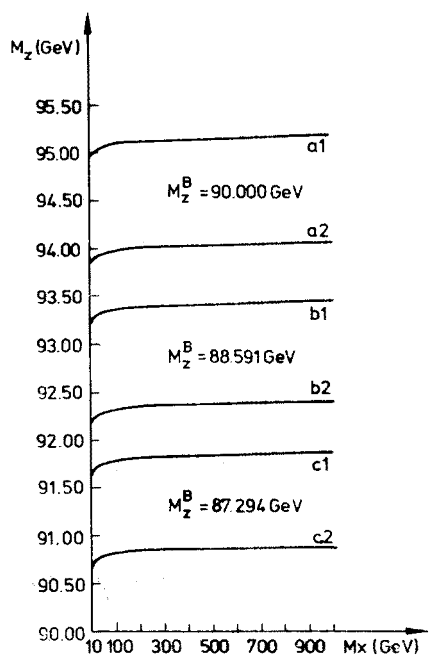

Fig. 10. The same for the mass of Z boson

where M_W is the physical W-boson mass. In such a way we derive the third Weinberg parameter

$$\sin^2 \theta_W^F = e^2/g_F^2 = \sin^2 \theta_W^M \left(1 - \frac{\alpha}{4\pi} X\right) \tag{59}$$

which is a ratio of the two charges e and g_F, each of them defined at momentum transfer close to zero.

The fourth Weinberg parameter is defined in the modified minimal subtraction scheme ($\overline{MS}$). The connection between the parameters $\sin^2 \theta_W(M_W)$ and $\sin^2 \theta_W^M$ is given in (28). Table I illustrates the relationship to order $O(\alpha)$ between the four Weinberg parameters for three values of $\sin^2 \theta_W^{exp}$ both for constituent and current quark masses, M_χ being equal to 200 GeV for definiteness.

TABLE I

The relationship between the four Weinberg parameters $\sin^2 \theta_W^{exp}$, $\sin^2 \theta_W^M$, $\sin^2 \theta_W^F$ and $\sin^2 \theta_W(M_W)$ for three values of $\sin^2 \theta_W^{exp}$: 0.220, 0.230, 0.240 both for constituent and current quark masses, M_χ being equal to 200 GeV for definiteness

$\sin^2 \theta_W^{exp}$	0.220		0.230		0.240	
	curr.	const.	curr.	const.	curr.	const.
$\sin^2 \theta_W^M = 1 - M_W^2/M_Z^2$	0.211	0.214	0.222	0.225	0.232_5	0.235_5
$\sin^2 \theta_W(M_W)$	0.210	0.213	0.220_5	0.223_5	0.231	0.234
$\sin^2 \theta_W^F = e^2/g_F^2$	0.195	0.200	0.205	0.210	0.215	0.220

5. *About high precision experimental tests of the standard $SU(2) \times U(1)$ theory*

There is a variety of Weinberg parameters defined quite differently. If we work with $\sin^2 \theta_W^M$, then Eq. (1) does not have radiative corrections to any order. Using $\sin^2 \theta_W^F$ we avoid the weak corrections to the muon's total decay rate in all orders. The parameter $\sin^2 \theta_W^{exp}$ minimizes the corrections to the P-odd asymmetry in deep-inelastic charged lepton — nucleon scattering. In other processes another parameter $\sin^2 \theta_W^{exp}$ quite different from this will minimize the corresponding corrections. So, parameters $\sin^2 \theta_W^{exp}$ received from various experiments should not be mixed up.

The SLAC experimental value (49) is reduced to

$$\sin^2 \theta_W^M = 0.218 \pm 0.014 \tag{60}$$

by the theoretically predicted shift

$$\delta \sin^2 \theta_W = \sin^2 \theta_W^{exp} - \sin^2 \theta_W^M = 0.006. \tag{61}$$

From the neutrino deep-inelastic scattering data one obtains quite different parameter $\sin^2 \theta_W^{exp}$. Recently Fogly [39] has published the results of a new detailed analysis of all data of this experiment. There are extracted significant radiative corrections

$$\delta \sin^2 \theta_W = 0.0097. \tag{62}$$

It is just equal to our theoretical prediction $\delta \approx 0.01$ [40]. The analysis of the other authors gives the same value [11, 17].

As the other authors (Consoli, Hioki, Sirlin and others) we estimated the dependence of M_W and M_Z on M_χ and m_t (a mass of a heavy up-type quark). Recently Lynn and Stuart have published a careful evaluation of M_W in terms of α, G_μ, M_Z with parameters m_t and M_χ. Our own results obtained from formulae published some time ago [13, 42] are in excellent agreement with Table 2 of these authors for all tabulated W masses in all four published digits. This is an impressive example of the level of reliability being obtained in the calculation of electroweak radiative corrections.

We could receive an important information about masses of the unknown quarks and of this puzzling Higgs boson from Eq. (41) because the quantity A is a function of them. Sirlin's analysis gives for the shift of this quantity $\delta A = A - A_0 = 1.37$ GeV [36]. It is a significant radiative correction. The capabilities of the near future experiments to measure δA and possibility to estimate the Higgs boson mass are considered in [43, 44].

If the sensitivity to A of the forthcoming experiments could be about 0.05 GeV the test of the genuine electroweak dynamical corrections becomes possible. It will be a decisive verification of the standard theory as a field theory of the electroweak interactions.

Editorial note. This article was proofread by the editors only, not be the author.

REFERENCES

[1] S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961); St. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, Proc. 8$^{\text{th}}$ Nobel symp., Stockholm 1968, p. 367.

[2] P. W. Higgs, *Phys. Lett.* **12**, 132 (1964); *Phys. Rev.* **145**, 1156 (1966).

[3] G. 't Hooft, *Nucl. Phys.* **B33**, 173 (1971).

[4] F. J. Hasert et al., *Phys. Lett.* **46B**, 121, 138 (1973).

[5] L. M. Barkov, M. S. Zolotarev, *Zh. Eksp. Teor. Fiz. Pisma* **27**, 379 (1978).

[6] C. Y. Prescott et al., *Phys. Lett.* **77B**, 347 (1978).

[7] J. G. Branson, Preprint DESY, 81-073 (1981).

[8] A. Argento et al., *Phys. Lett.* **120B**, 245 (1983).

[9] G. Arnison et al. (UA-1), *Phys. Lett.* **122B**, 103 (1983); M. Banner et al. (UA-2), *Phys. Lett.* **122B**, 476 (1983).

[10] G. Arnison et al. (UA-1), *Phys. Lett.* **126B**, 398 (1983); P. Bagnaia et al. (UA-2), *Phys. Lett.* **129B**, 130 (1983).

[11] W. J. Marciano, *Phys. Rev.* **D20**, 274 (1979); A. Sirlin, *Phys. Rev.* **D22**, 971 (1980); W. J. Marciano, A. Sirlin, *Phys. Rev.* **D22**, 2695 (1980); *Nucl. Phys.* **B189**, 442 (1981).

[12] D. Yu. Bardin, O. M. Fedorenko, *Yad. Fiz.* **30**, 811 (1979); D. Yu. Bardin, O. M. Fedorenko, N. M. Shumeiko, *Yad. Fiz.* **32**, 782 (1980); *J. Phys. G* **7**, 1331 (1981).

[13] D. Yu. Bardin, P. Ch. Christova, O. M. Fedorenko, *Nucl. Phys.* **B197**, 1 (1982); *Yad. Fiz.* **35**, 1220 (1982).

[14] M. Veltman, *Phys. Lett.* **91B**, 95 (1980).

[15] F. Antonelli, G. Corbo, M. Consoli, *Phys. Lett.* **91B**, 90 (1980); F. Antonelli, G. Corbo, M. Consoli, Pellegrino, *Nucl. Phys.* **B183**, 195 (1981).
[16] K.-I. Aoki, Z. Hioki, R. Kawabe, M. Konuma, T. Muta, *Prog. Theor. Phys.* **65**, 1001 (1981).
[17] C. H. Llewellyn Smith, J. F. Wheater, *Phys. Lett.* **105B**, 486 (1981); *Nucl. Phys.* **B208**, 27 (1982).
[18] D. A. Ross, J. C. Taylor, *Nucl. Phys.* **B51**, 125 (1973).
[19] S. Sakakibara, *Phys. Rev.* **D24**, 1149 (1981).
[20] T. W. Appelquist, J. R. Primack, H. R. Quinn, *Phys. Rev.* **D7**, 2998 (1973).
[21] W. J. Marciano, A. Sirlin, *Phys. Rev.* **D8**, 3612 (1973).
[22] P. Salomonson, Y. Ueda, *Phys. Rev.* **D11**, 2606 (1975).
[23] G. Passarino, M. Veltman, *Nucl. Phys.* **B160**, 151 (1979); M. Lemoine, M. Veltman, *Nucl. Phys.* **B164**, 445 (1980); M. Green, M. Veltman, *Nucl. Phys.* **B169**, 137 (1980).
[24] M. Consoli, *Nucl. Phys.* **B160**, 208 (1979); F. Antonelli, M. Consoli, G. Corbo, *Phys. Lett.* **99B**, 475 (1981).
[25] K. Inoue et al., *Prog. Theor. Phys.* **64**, 1008 (1980); J. Fleischer, F. Jegerlehner, *Phys. Rev.* **D23**, 2001 (1981).
[26] K. I. Aoki, Z. Hioki, R. Kawabe, M. Konuma, T. Muta, *Prog. Theor. Phys. Suppl.* **73**, 1 (1982); Z. Hioki, *Nucl. Phys.* **B229**, 284 (1983).
[27] *Workshop on Radiative Corrections in* $SU(2)\times U(1)$, eds. B. W. Lynn and J. F. Wheater, Trieste, June 1983.
[28] S. Sarantakos, A. Sirlin, W. J. Marciano, *Nucl. Phys.* **B217**, 84 (1983).
[29] M. Consoli, S. Lo Presti, L. Maiani, *Nucl. Phys.* **B223**, 474 (1983).
[30] A. Sirlin, *Phys. Rev.* **D29**, 89 (1984).
[31] D. Yu. Bardin, Preprint JINR, Dubna, P2-84-112 (1984).
[32] W. J. Marciano, A. Sirlin, *Nucl. Phys.* **B93**, 303 (1975).
[33] D. Yu. Bardin, O. M. Fedorenko, Comm. JINR, Dubna, P2-11413 and P2-11414 (1978).
[34] D. Yu. Bardin, P. Ch. Christova, O. M. Fedorenko, *Nucl. Phys.* **B175**, 435 (1980).
[35] M. Roos, A. Sirlin, *Nucl. Phys.* **B29**, 296 (1971).
[36] W. J. Marciano, A. Sirlin, *Phys. Rev.* **D29**, 945 (1984).
[37] W. J. Marciano, A. Sirlin, *Phys. Rev.* **D29**, 75 (1984).
[38] D. Yu. Bardin, O. M. Fedorenko, N. M. Shumeiko, Preprint JINR, Dubna, E2-12564 (1979).
[39] G. L. Fogli, *Phys. Lett.* **158B**, 66 (1985).
[40] D. Yu. Bardin, V. A. Dokuchaeva, *Sov. Yad. Fiz.* **36**, 482 (1982).
[41] B. W. Lynn, R. G. Stuart, *Nucl. Phys.* **B253**, 216 (1985).
[42] D. Yu. Bardin, V. A. Dokuchaeva, *Nucl. Phys.* **B246**, 221 (1984).
[43] D. Yu. Bardin, V. Khovansky, Preprint ITEP-61, Moscow 1984.
[44] A. A. Akhundov, D. Yu. Bardin, T. Riemann, Preprint JINR, Dubna, E2-85-454 (1985).