ROLE OF A CONSISTENCY CONDITION IN MACROSCOPIC--MICROSCOPIC CALCULATIONS OF THE COLLECTIVE POTENTIAL ENERGY*

By K. Böning, Z. Patyk, A. Sobiczewski,

Institute for Nuclear Studies, Warsaw**

B. Nerlo-Pomorska and K. Pomorski

Institute of Physics, The Maria Skłodowska-Curie University, Lublin***

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A relation between the deformation of a single-particle potential of a nucleus and the deformation of the density of matter, generated by this potential, is discussed in detail. Relative difference between the two deformations amounts up to a dozen or so per cent. It is an increasing function of the multipolarity of the deformation. An account of this difference in macroscopic-microscopic calculations of the collective potential energy of a nucleus corrects (increases) its deformation energy.

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1. Introduction

In the standard macroscopic-microscopic description of the dependence of the collective potential energy of a nucleus on deformation, the same deformation (specifically, the deformation of the single-particle potential describing the internal structure of the nucleus) is used for both parts of the energy (cf. e.g. Refs [1–3]). It seems, however, more proper to distinguish between two deformations corresponding to these two parts. The microscopic part, which is a shell correction to the energy, is directly connected with the single-particle potential and can be in a natural way described by the deformation of this potential. The macroscopic part represents the smooth behaviour of the energy and is connected with the density of the nucleus, averaged over shell effects. This part is usually described by such phenomenological models as the liquid-drop (e.g. [4]), droplet (e.g. [5]) or Yukawa-

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** Address: Instytut Problemów Jądrowych, Hoża 69, 00-681 Warszawa, Poland.

*** Address: Instytut Fizyki UMCS, Nowotki 10, 20-031 Lublin, Poland.

-plus-exponential (e.g. [6]) models. It is more natural then to connect the macroscopic part of the energy with the deformation of the density of a nucleus than with the deformation of the single-particle potential.

Such treatment has been proposed around ten years ago [7]. The condition that the density, the deformation of which is ascribed to the macroscopic part of the energy, is the one generated by the single-particle potential describing the microscopic part, may be considered as a simple consistency condition between the two parts of the energy. It may be regarded as a step towards making the non-self-consistent macroscopic-microscopic method self-consistent. In the calculations based on this idea, there was, however, a numerical error [7], which resulted in an overestimation of the effect of the simple consistency condition. The effect of the condition, when the error has been removed, has been discussed in Ref. [8].

The scope of the present paper is to perform a detailed discussion of the consistency condition and its role in the macroscopic-microscopic calculations, especially in the calculations based on the Nilsson single-particle potential.

In Sect. 2, description of the method of the calculations is given. Sect. 3 gives the results and Sect. 4 presents a discussion of various effects. Main conclusions of the study are given in Sect. 5.

2. Description of the calculations

As stated in the Introduction, the idea [7] of a modification of the standard macroscopic-microscopic calculations of the potential energy of a nucleus is to use, for the macroscopic part of the energy, the deformation of the density generated by the single-particle potential and not the deformation of the potential itself. For the phenomenological single-particle potentials (mostly used in practical calculations), as those of Nilsson or of Woods and Saxon, the two deformations, although expected to be close to each other, are not identical.

The deformation of a nucleus can be completely specified by its multipole moments. As we are mainly interested in the deformation of the nuclear surface (i.e. in the dependence of its distance from the center of the nucleus on the direction, $R(\vartheta, \varphi)$), we will limit ourselves to the "surface" moments, with only one (quadratic) power in their dependence on the radius r,

$$q'_\lambda \equiv 2r^2 P_\lambda \quad \text{for} \quad \lambda \neq 1, \tag{2.1}$$

the same for all multipolarities λ ($\lambda \neq 1$).

We assume, in the whole paper, the axial symmetry of a nucleus. For convenience, as we are mainly interested in the calculations with the Nilsson potential [9, 1], we will work in the stretched space, i.e. in the coordinates

$$\xi = x\sqrt{M\omega_\perp/\hbar}, \quad \eta = y\sqrt{M\omega_\perp/\hbar}, \quad \zeta = z\sqrt{M\omega_z/\hbar}, \tag{2.2}$$

by which the potential is defined. Here

$$\omega_z = \omega_0(1-\tfrac{2}{3}\varepsilon) \quad \text{and} \quad \omega_\perp = \omega_0(1+\tfrac{1}{3}\varepsilon) \tag{2.2a}$$

are the frequencies of the harmonic oscillator, appearing in the Nilsson potential, along the symmetry axis Oz and an axis perpendicular to it, respectively. Thus, we will use the multipole operators q_λ defined in that space, i.e.

$$q_\lambda \equiv 2\varrho^2 P_\lambda(\cos\vartheta) \quad \text{for} \quad \lambda \neq 1, \tag{2.3}$$

where $\varrho = \sqrt{\xi^2+\eta^2+\zeta^2}$ and ϑ are the polar coordinates in the stretched space.

Only for $\lambda = 1$, the (dipole) operator is defined by

$$q_1 = 2\varrho P_1(\cos\vartheta) = 2\zeta, \tag{2.3a}$$

as we would like it to be connected with the position of the mass center.

2.1. Microscopic moments

By microscopic moments, we understand the multipole moments calculated microscopically for a given deformation of the single-particle (Nilsson) potential. They describe the distribution of the density generated by this potential.

The deformation is specified by the Nilsson deformation parameters ε_λ. They are defined by the form of the oscillator part of the Nilsson potential, i.e. the only part which depends on the deformation. This form is

$$V_{\text{osc}} = \tfrac{1}{2}\hbar\omega_0(\varepsilon_\mu)\varrho^2 F(\cos\vartheta;\varepsilon_\mu), \tag{2.4}$$

where

$$F(\cos\vartheta;\varepsilon_\mu) = 1+2\varepsilon_1 P_1(\cos\vartheta)-\tfrac{2}{3}\varepsilon P_2(\cos\vartheta)+\sum_{\lambda=3}^{\lambda_{\max}} 2\varepsilon_\lambda P_\lambda(\cos\vartheta). \tag{2.4a}$$

Here, we have denoted traditionally $\varepsilon \equiv \varepsilon_2$.

The total Nilsson hamiltonian, expressed in units of $\hbar\overset{0}{\omega}_0$ is [1]

$$H' = (H_{\text{kin}}+V_{\text{osc}}+V_{\text{corr}})/\hbar\overset{0}{\omega}_0$$

$$= \tfrac{1}{2}\,om(\varepsilon_\mu)\left\{-\Delta+\tfrac{1}{3}\varepsilon\left(2\frac{\partial^2}{\partial\zeta^2}-\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\eta^2}\right)+\varrho^2 F(\cos\vartheta;\varepsilon_\mu)\right\}$$

$$-\kappa[2\boldsymbol{l}\cdot\boldsymbol{s}+\mu(\boldsymbol{l}^2-\langle\boldsymbol{l}^2\rangle_N)], \tag{2.4b}$$

where

$$om(\varepsilon_\mu) \equiv \omega_0(\varepsilon_\mu)/\overset{0}{\omega}_0 \tag{2.4c}$$

with $\overset{0}{\omega}_0 = \omega_0(0)$.

Thus, besides the parameters κ and μ, the only parameters of the hamiltonian H' (which is to be diagonalized) and, consequently, of the wave functions, are the deformation parameters ε_μ.

The microscopic moment Q^{micr} is usually calculated as

$$Q_\lambda^{\text{micr}}(\varepsilon_\mu) \equiv \langle\text{BCS}|Q_\lambda|\text{BCS}\rangle = \sum_\nu (q_\lambda)_{\nu\nu} 2v_\nu^2, \tag{2.5}$$

where $|\mathrm{BCS}\rangle$ is the Bardeen-Cooper-Schrieffer wave function of a state of a nucleus in which the moment is calculated and $2v_\nu^2$ is the particle-occupation factor for a single-particle Nilsson state $|\nu\rangle$. The moment $Q_\lambda^{\mathrm{micr}}$ is, similarly as the single-particle wave functions $|\nu\rangle$, a function of only the deformation parameters ε_μ and not of the frequency $\overset{0}{\omega}_0$. The multipole operator Q_λ is

$$Q_\lambda = \sum_i q_\lambda(i),$$

where the summation extends over all nucleons. The diagonal matrix element $(q_\lambda)_{\nu\nu}$ in a state $|\nu\rangle$ is

$$(q_\lambda)_{\nu\nu} \equiv \langle\nu|q_\lambda|\nu\rangle = \sum_{\substack{Nl\Lambda\Sigma \\ N'l'\Lambda'\Sigma'}} \langle\nu|Nl\Lambda\Sigma\rangle \langle Nl\Lambda\Sigma|2\varrho^2 P_\lambda|N'l'\Lambda'\Sigma'\rangle \langle N'l'\Lambda'\Sigma'|\nu\rangle$$

$$= 2 \sum_{\substack{Nl\Lambda\Sigma \\ N'l'}} a^\nu_{Nl\Lambda\Sigma} a^\nu_{N'l'\Lambda\Sigma} \langle Nl|\varrho^2|N'l'\rangle \langle l\Lambda|P_\lambda|l'\Lambda\rangle, \tag{2.6}$$

where $|Nl\Lambda\Sigma\rangle$ are the usual spherical (in the stretched space) harmonic-oscillator wave functions [9], multiplied by spin functions, and $a^\nu_{Nl\Lambda\Sigma} \equiv \langle\nu|Nl\Lambda\Sigma\rangle$ are the projections of a Nilsson function $|\nu\rangle$ on the basic functions $|Nl\Lambda\Sigma\rangle$. In Eq. (2.6), there is used the fact that

$$\langle Nl\Lambda\Sigma|\varrho^2 P_\lambda|N'l'\Lambda'\Sigma'\rangle = \langle Nl|\varrho^2|N'l'\rangle \langle l\Lambda|P_\lambda|l'\Lambda\rangle \delta_{\Lambda\Lambda'}\delta_{\Sigma\Sigma'}. \tag{2.6a}$$

Presence of the even-multipolarity deformations $\lambda = 4, 6, \ldots$ in the potential (2.4) results in a coupling of a spherical oscillator shell N with the shells $N' = N \pm 2, \pm 4, \ldots$ For the potential which has only the quadrupole deformation $\lambda = 2$, one avoids these couplings just by the use of the stretched space. This was the reason to introduce it [9]. Presence of any of the odd-multipolarity deformations $\lambda = 3, 5, \ldots$ results in a coupling of a shell N with the shells $N' = N \pm 1, \pm 3, \ldots$

One can see from Eqs (2.5) and (2.6) that to calculate the multipole moments $Q_\lambda^{\mathrm{micr}}$, one needs the same matrix elements, $\langle Nl|\varrho^2|N'l'\rangle$ and $\langle l\Lambda|P_\lambda|l'\Lambda\rangle$, which are needed for the diagonalization of the Nilsson potential in the hamiltonian H' of Eq. (2.4b). It is because of our adjustment of the definition of the moments to the form of the Nilsson potential.

The microscopic moments of Eq. (2.5) contain the effects of the shell structure of a nucleus. The effects are only partly smoothed by the pairing interaction manifested in the formula (2.5) by the occupation factors $2v_\nu^2$. As in our calculations, the microscopic moments are to describe the density deformation which is to be used in the macroscopic, smooth, averaged over shell effects part of the energy, a more consistent procedure is to use moments in which the shell effects are completely smoothed. According to this, instead of the moments (2.5), we use the moments

$$Q_\lambda^{\mathrm{micr}}(\varepsilon_\mu) = \sum_\nu (q_\lambda)_{\nu\nu} 2\tilde{n}_\nu, \tag{2.7}$$

where $\tilde{n}_\nu$ are the occupation factors corresponding to the case when the shell effects are smeared. For the smearing, we use the procedure of Strutinsky [10]. The formula for

$\tilde{n}_\nu$ is then

$$\tilde{n}_\nu = \frac{1}{\sqrt{\pi}\,\gamma} \int_{-\infty}^{\tilde{\lambda}} e^{-u^2_\nu} F(u_\nu) de. \tag{2.7a}$$

where $u_\nu = (e - e_\nu)/\gamma$, e_ν is the energy of the level $|\nu\rangle$, γ is the smearing parameter (taken equal to $1.2\,\hbar\omega_0$ in our calculations), $F(u_\nu)$ is the correction polynomial (see e.g. [1]) and $\tilde{\lambda}$ is the Fermi energy corresponding to the smeared level density

$$\tilde{g}(e) = \frac{1}{\sqrt{\pi}\,\gamma} \sum_\nu e^{-u^2_\nu} F(u_\nu), \tag{2.7b}$$

where the summation extends over all single-particle levels. The relation between the factors $\tilde{n}_\nu$, the number of nucleons l and the smooth density $\tilde{g}(e)$ is

$$l = \sum_\nu 2\tilde{n}_\nu = 2 \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(e) de. \tag{2.7c}$$

2.2. Macroscopic moments

By macroscopic moments, we understand here the moments of a model nucleus (e.g. the liquid drop) for which we already know the radial distribution of the density. We will parametrize the shape of a surface of constant density by ε^d_λ in an analogous way to that of the parametrization of the shape of a surface of constant potential, Eq. (2.4). Thus, we put

$$V_d = \tfrac{1}{2}\hbar\omega_0 \varrho^2 F(\cos\vartheta; \varepsilon^d_\mu), \tag{2.8}$$

where F is the same function as that of Eq. (2.4), but now depending on the density-deformation parameters ε^d_μ instead of the potential-deformation parameters ε_μ, appearing in Eq. (2.4). The idea is to have the equality $\varepsilon^d_\mu = \varepsilon_\mu$ when, and only when, the shape of the equidensity surfaces coincides with the shape of the equipotential surfaces. The equation for an equidensity surface is

$$\tfrac{1}{2}\hbar\overset{0}{\omega}_0 \varrho_0^2 = \tfrac{1}{2}\hbar\omega_0 \varrho^2 F, \tag{2.8a}$$

where $\overset{0}{\omega}_0$ and ϱ_0 are the values of ω_0 and ϱ, respectively, for the spherical shape of the surface. The value ϱ_0 specifies the surface (and thus the density, if there is one-to-one correspondence between the two). Thus, we get for the radius $\varrho(\vartheta)$

$$\varrho(\vartheta) = \varrho_0 om^{-1/2} F^{-1/2}, \tag{2.8b}$$

where $om = \omega_0/\overset{0}{\omega}_0$, according to Eq. (2.4c).

The standard definition of the multipole moment, connected with a given density distribution, together with our definition of the multipole moment operator q_λ, Eq. (2.3),

gives then

$$Q_\lambda^{\text{macr}} \equiv \int_\tau g_t q_\lambda d\tau = 2 \int_\tau g_t \varrho^2(\vartheta) P_\lambda(\cos\vartheta) d\tau \tag{2.9}$$

for $\lambda \neq 1$ and

$$Q_1^{\text{macr}} = 2 \int_\tau g_t \varrho(\vartheta) P_1(\cos\vartheta) d\tau \tag{2.9a}$$

for $\lambda = 1$, where g_t is the density of matter. Assuming, for simplicity, a sharp surface of a nucleus and a uniform distribution of the density inside it, we get

$$Q_\lambda^{\text{macr}} = \tfrac{2}{5} g_t \int \varrho^5(\vartheta) P_\lambda(\cos\vartheta) d\Omega \tag{2.10}$$

for $\lambda \neq 1$. After using Eq. (2.8b) and the assumed axial symmetry of the nucleus, the formula becomes

$$Q_\lambda^{\text{macr}}(\varepsilon_\mu^d) = \tfrac{3}{5} l \varrho_{0l}^2 o m_l^{-5/2} \int_{-1}^{1} F^{-5/2}(x; \varepsilon_\mu^d) P_\lambda(x) dx, \tag{2. 1}$$

where

$$l = \frac{4\pi}{3} \varrho_{0l}^3 g_t$$

is the number of nucleons of one kind ($l = N$ or Z for neutrons or protons, respectively), corresponding to the density g_t, and ϱ_{0l} is the radius of the sphere (in the stretched space) occupied by these nucleons. According to Eq. (2.2),

$$\varrho_{0l} = R_{0l} \sqrt{M \overset{0}{\omega}_{0l}/\hbar}. \tag{2.12}$$

We assume, as usual,

$$R_{0N} = R_{0Z} = R_0 = r_0 A^{1/3}, \tag{2.13}$$

i.e. that the effective radius for neutrons is the same as that for protons. Thus, the expressions (2.9) and (2.9a) become

$$Q_\lambda^{\text{macr}} = \tfrac{3}{5} l R_0^2 \frac{M \overset{0}{\omega}_{0l}}{\hbar} o m_l^{-5/2} \int_{-1}^{1} F^{-5/2}(x; \varepsilon_\mu^d) P_\lambda(x) dx \tag{2.14}$$

for $\lambda \neq 1$ and

$$Q_1^{\text{macr}} = \tfrac{3}{4} l R_0 \left(\frac{M \overset{0}{\omega}_{0l}}{\hbar} \right)^{1/2} o m_l^{-2} \int_{-1}^{1} F^{-2}(x; \varepsilon_\mu^d) P_1(x) dx \tag{2.14a}$$

for $\lambda = 1$, both formulae being valid for one kind of nucleons.

2.3. Consistency condition

Now, we can already write the equations relating the density-deformation parameters ε_μ^d with the potential-deformation parameters ε_μ. The equations are obtained from the condition that the total multipole moments $Q_{\lambda\,\mathrm{tot}}^{\mathrm{macr}}$ of a nucleus, determined by the density-deformation parameters ε_μ^d, are the same as those calculated microscopically (by Eq. (2.7)) with the potential described by the parameters ε_μ, i.e.

$$Q_{\lambda\mathrm{tot}}^{\mathrm{macr}}(\overset{0}{\omega}_{0N}, om_N, \overset{0}{\omega}_{0Z}, om_Z, \varepsilon_\mu^d) = Q_{\lambda\mathrm{tot}}^{\mathrm{micr}}(\varepsilon_\mu) \tag{2.15a}$$

for $\lambda = 1, 2, 3, \ldots, \lambda_{\max}$ and

$$Q_{01}^{\mathrm{macr}}(\overset{0}{\omega}_{0l}, om_l, \varepsilon_\mu^d) = Q_{0l}^{\mathrm{micr}}(\varepsilon_\mu) \tag{2.15c}$$

for $\lambda = 0$, with $l = N, Z$.

For $\lambda = 1$, we additionally require

$$Q_{1\,\mathrm{tot}}^{\mathrm{micr}}(\varepsilon_\mu) = 0, \tag{2.15b}$$

to keep the center of mass of a nucleus at the origin of the coordinate system, when the nucleus is being deformed.

We have used in Eqs (15a) and (15b) the total multipole moments

$$Q_{\lambda\mathrm{tot}} \equiv Q_{\lambda N} + Q_{\lambda Z}, \tag{2.15d}$$

as we are interested in the distribution of the total nuclear matter. The deformations are assumed to be the same for neutrons and protons, both for the potential and the density.

For given deformation ε_ν, $\nu = 2, 3, \ldots, \lambda_{\max}$, Eq. (2.15b) allows us to calculate

$$\varepsilon_1 = \varepsilon_1(\varepsilon_\nu) \tag{2.15b'}$$

and thus, by Eq. (2.7), all $Q_{\lambda\,\mathrm{tot}}^{\mathrm{micr}}(\varepsilon_\mu) = Q_{\lambda\,\mathrm{tot}}^{\mathrm{micr}}[\varepsilon_1(\varepsilon_\nu), \varepsilon_\nu]$, $\lambda \neq 1$. Thus, for given ε_ν, $\nu = 2, 3, \ldots, \lambda_{\max}$, Eqs (2.15a) and (2.15c) constitute a set of $(\lambda_{\max}+2)$ equations for $(\lambda_{\max}+2)$ unknown quantities: ε_μ^d $(\mu = 1, 2, \ldots, \lambda_{\max})$, $\overset{0}{\omega}_{0N}$ or $om_N(\varepsilon_\mu^d)$ and $\overset{0}{\omega}_{0Z}$ or $om_Z(\varepsilon_\mu^d)$. This is because for each deformation ε_ν, only one of the quantities $\overset{0}{\omega}_{0l}$ and om_l is unknown. For the deformation zero, $om_l = 1$ and only $\overset{0}{\omega}_{0l}$ is to be determined. For other deformations, only om_l has to be obtained from Eqs (2.15c).

As the quantities $\overset{0}{\omega}_{0l}$ and om_l enter to $Q_{\lambda\,\mathrm{tot}}^{\mathrm{macr}}$ in a simple (and the same for all λ, $\lambda \neq 1$) way, the solution of the set is simplified if we divide all $Q_{\lambda\,\mathrm{tot}}$, $\lambda \neq 0, 1$, by $Q_{0\,\mathrm{tot}}$. Then Eqs (2.15a) become

$$\frac{Q_{\lambda\mathrm{tot}}^{\mathrm{macs}}}{Q_{0\mathrm{tot}}^{\mathrm{macr}}}(\varepsilon_\mu^d) = \frac{Q_{\lambda\mathrm{tot}}^{\mathrm{micr}}}{Q_{0\mathrm{tot}}^{\mathrm{micr}}}(\varepsilon_\nu), \tag{2.16a}$$

where $\lambda, \mu = 1, 2, \ldots, \lambda_{\max}$, $\nu = 2, 3, \ldots, \lambda_{\max}$. A solution of this set of $\lambda_{\max}$ coupled equations gives all the density-deformation parameters ε_μ^d as functions of the potential-deformation

parameters ε_ν, i.e.

$$\varepsilon_\mu^d = \varepsilon_\mu^d(\varepsilon_\nu). \tag{2.17}$$

With these, the solutions for $\overset{0}{\omega}_{0l}$ and om_l, $l = N$ or Z, are obtained explicitly from Eqs (2.15c)

$$\frac{om_l^{5/2}(\varepsilon_\nu)}{\overset{0}{\omega}_{0l}} = \tfrac{3}{5}\frac{M}{\hbar} lR_0^2 \frac{1}{Q_{0l}^{\mathrm{micr}}(\varepsilon_\nu)} \int_{-1}^{1} F^{-5/2}[x; \varepsilon_\mu^d(\varepsilon_\nu)]dx, \tag{2.16b}$$

where $\mu = 1, 2, \ldots, \lambda_{\max}$ and $\nu = 2, 3, \ldots, \lambda_{\max}$, similarly as in Eqs (2.16a). For the deformation zero, $om_l = 1$ and Eq. (2.16b) directly gives the value of $\overset{0}{\omega}_{0l}$ or $\hbar\overset{0}{\omega}_{0l}$

$$\hbar\overset{0}{\omega}_{0l} = \tfrac{5}{6}\frac{\hbar^2}{MR_0^2}\frac{Q_{0l}^{\mathrm{micr}}(0)}{l}. \tag{2.18}$$

For other deformations, Eq. (2.16b), together with Eq. (2.18), leads to the value of $om_l(\varepsilon_\nu)$

$$om_l(\varepsilon_\nu) = \left\{\frac{Q_{0l}^{\mathrm{micr}}(0)}{Q_{0l}^{\mathrm{micr}}(\varepsilon_\nu)}\tfrac{1}{2}\int_{-1}^{1} F^{-5/2}[x, \varepsilon_\mu^d(\varepsilon_\nu)]dx\right\}^{2/5}. \tag{2.19}$$

The value $\hbar\overset{0}{\omega}_{0l}$ of Eq. (2.18) may be compared with that for a pure harmonic oscillator with closed shells (cf. e.g. Ref. [11])

$$\hbar\overset{0}{\omega}_{0l} = \tfrac{5}{4}\frac{\hbar^2}{MR_0^2}(3l)^{1/3}, \tag{2.20}$$

which, for $r_0 = 1.20$ fm, gives

$$\hbar\overset{0}{\omega}_{0l} = \frac{41.0}{A^{1/3}}(1 \pm I)^{1/3}\ \mathrm{MeV}, \tag{2.20a}$$

or in the first order [1] in the relative neutron excess $I = (N-Z)/A$

$$\hbar\overset{0}{\omega}_{0l} = \frac{41.0}{A^{1/3}}(1 \pm \tfrac{1}{3} I)\ \mathrm{MeV}, \tag{2.20b}$$

where sign plus holds for neutrons ($l = N$) and minus for protons ($l = Z$).

The dependence of om_l on deformation, obtained in Eq. (2.19), may be compared with that used traditionally, when it is deduced from the condition of the conservation of the volume contained inside an equipotential surface. This volume (calculated in the normal, physical space) is

$$\int_{\tau'} d\tau' = \left(\frac{\hbar}{M}\right)^{3/2} \omega_\perp^{-1}\omega_z^{-1/2} \int_{\tau} d\tau,$$

in accordance with Eq. (2.2), where

$$\int_{\tau} d\tau = \frac{2\pi}{3} \int_{-1}^{1} \varrho^3(x) dx,$$

and should be equal to the volume $4\pi R_0^3/3$, contained inside the spherical surface, from which the given surface is obtained by deformation. With the use of Eqs (2.2a) and (2.8b), this gives

$$om(\varepsilon_\mu) = (1+\tfrac{1}{3}\varepsilon)^{-1/3}(1-\tfrac{2}{3}\varepsilon)^{-1/6}\{\tfrac{1}{2}\int_{-1}^{1} F^{-3/2}(x;\varepsilon_\mu)dx\}^{1/3}. \tag{2.21}$$

The result of Eq. (2.19) can be also compared with that obtained from the condition that the volume contained inside an equidensity (and not equipotential) surface should be conserved when the potential is deformed. Such condition leads to the result

$$om(\varepsilon_\nu) = (1+\tfrac{1}{3}\varepsilon)^{-1/3}(1-\tfrac{2}{3}\varepsilon)^{-1/6}\{\tfrac{1}{2}\int_{-1}^{1} F^{-3/2}[x;\varepsilon_\mu^d(\varepsilon_\nu)]dx\}^{1/3}, \tag{2.22}$$

where ε_μ^d are the density-deformation parameters. The numerical results are given in Sect. 3.

2.4. Details of the calculations

The Nilsson potential with the "$A = 225$" parameters [1], i.e.

$$\kappa_p = 0.0590, \quad \mu_p = 0.639 \quad \text{for protons,}$$

$$\kappa_n = 0.0635, \quad \mu_n = 0.346 \quad \text{for neutrons,}$$

is taken to calculate the microscopic moments Q_λ^{micr} of Eq. (2.7). All oscillator shells from $N = 0$ up to $N = N_{max}$ are included. In most calculations $N_{max} = 12$ is taken. The influence of the variation of the value of N_{max} on the results is extensively discussed in Sect. 4. Direct coupling between $\Delta N = 8$ shells is taken into account.

When calculating the microscopic part of the energy, the pairing interaction is treated in the same way as in Ref. [1]. The only difference is that we use here another formula for the strength G of the interaction. The present formula is [11]

$$N^{2/3} \cdot G_N = 0.284\,\hbar\omega_{0N}, \quad Z^{2/3} \cdot G_Z = 0.290\,\hbar\omega_{0Z}. \tag{2.23}$$

It seems that this formula describes better, in a more natural way, the isotopic dependence of G than the formula of Ref. [1], as discussed in Ref. [11]. Numerically, however, the values of G obtained from both formulae are close to each other, for considered nuclei.

3. *Results*

Most of the results presented in this Section are obtained for the case of $Z = 88$ protons of the nucleus $^{224}_{88}$Ra. The results for other numbers of protons or for neutrons are similar, as discussed in subsects. 4.2 and 4.6. The largest multipolarity accounted for,

when solving the consistency equations (2.16a), is $\lambda_{\max} = 8$. The influence of this number of the results is discussed in subsect. 4.4.

Fig. 1 shows the difference

$$\delta\varepsilon_\lambda \equiv \varepsilon_\lambda^d - \varepsilon_\lambda \tag{3.1}$$

between the density deformation, ε_λ^d, and the potential deformation, ε_λ, plotted as a function of ε_λ, for multipolarities $\lambda = 2, 3, 4, 5, 6$. The density deformations ε_λ^d are obtained by solving Eqs. (2.16a). The value of $\lambda_{\max} = 8$ has been taken, as already mentioned above. One can see that the dependence of $\delta\varepsilon_\lambda$ on ε_λ is almost linear. This means that the ratio $\delta\varepsilon_\lambda/\varepsilon_\lambda$ is almost constant, what can be directly seen in Fig. 2, where $\delta\varepsilon_\lambda/\varepsilon_\lambda$ is plotted as a function of ε_λ. One can learn from the figure that the density deformation is, in absolute value, always smaller than the potential deformation and that the effect has a tendency to increase with increasing deformation (see $\lambda = 2, 3, 4, 5$). The effect regularly increases with increasing multipolarity λ. In fact, for $\lambda = 3$, the absolute value of $\delta\varepsilon_\lambda/\varepsilon_\lambda$ is smaller than 1%, it is about 4% for $\lambda = 4$, about 8% for $\lambda = 5$ and about 12% for $\lambda = 6$. The only exception in this regularity is the lowest multipolarity $\lambda = 2$, for which the effect is already relatively large, around 11%.

The oscillator energy $\hbar\overset{0}{\omega}_0$ is illustrated in the upper part of Fig. 3. Its value, obtained for ^{224}Ra from the consistency condition for the monopole moment, Eq. (2.18), is 6.417 MeV

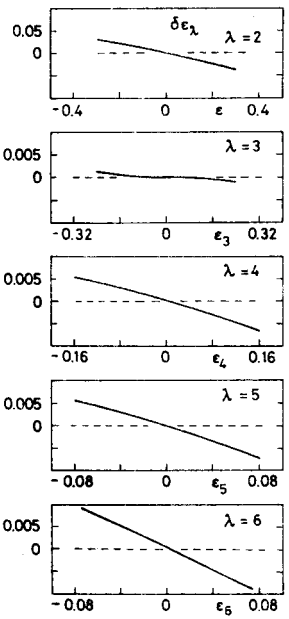

Fig. 1

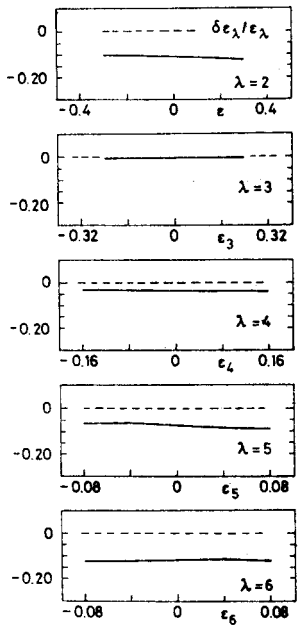

Fig. 2

Fig. 1. Dependence of the difference $\delta\varepsilon_\lambda$, Eq. (3.1), on the potential deformation ε_λ for multipolarities from $\lambda = 2$ up to $\lambda = 6$. The dependence is obtained at deformation $\overset{0}{\varepsilon}_\mu$, $\mu \neq \lambda$, specified in text

Fig. 2. Same as in Fig. 1 but for the ratio $\delta\varepsilon_\lambda/\varepsilon_\lambda$

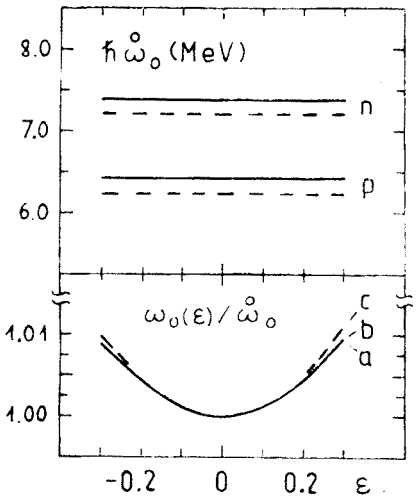

Fig. 3. Upper part: oscillator energy $\hbar\mathring{\omega}_0$ obtained from the consistency condition for the monopole moment (solid line) and from an analytic estimate, usually used (dashed line), both for neutrons and protons. Lower part: dependence of the ratio $\omega_0(\varepsilon)/\mathring{\omega}_0$ on pure quadrupole deformation ε, obtained in three cases (*a*), (*b*) and (*c*), described in text

for protons and 7.370 MeV for neutrons. The corresponding values, obtained analytically for an oscillator with closed shells, Eq. (2.20a), are 6.230 MeV and 7.202 MeV, i.e. by 2.9% and 2.3% smaller, respectively. The values corresponding to the expression (2.20b), linear in I, are 6.269 MeV and 7.233 MeV, respectively, i.e. by about 0.6% and 0.4% larger, respectively, than the latter values. They are not distinguishable, in the figure, from the values obtained from Eq. (2.20a), which is exact in I. The lower part of Fig. 3 illustrates the dependence of the oscillator frequency ω_0 on the deformation. The ratio $\omega_0(\varepsilon)/\mathring{\omega}_0$ is calculated for the case of a pure quadrupole deformation ε from three different conditions: of the consistency condition for the monopole moment (*a*) (Eq. (2.19)), of the volume conservation inside an equidensity surface (*b*) (Eq. (2.22)) and of the volume conservation inside an equipotential surface (*c*) (Eq. (2.21)). The dependence (*c*) is usually used. One can see that all the three dependences are very close to each other. In fact, the curves (*a*) and (*b*) are not distinguishable in the figure, although the scale of it is much enlarged. The curves (*a*) and (*c*) differ by less than about 0.15%. Such difference is practically of no significance for the dependence of the microscopic energy (in which the oscillator frequency ω_0 appears) on deformation. Thus, the simple condition (*c*), usually used, is practically as good as a more exact condition (*a*) for the determination of the dependence of the microscopic energy on deformation, at least for deformations investigated here (i.e. around the equilibrium point). Also the improvement of $\hbar\mathring{\omega}_0$ by the consistency condition does not seem to be of any practical significance, although it is influenced by the condition more than the dependence of ω_0 on deformation. The energy $\hbar\mathring{\omega}_0$ is modified by the consistency condition by about 0.2 MeV, i.e. by about 3%, as can be seen in Fig. 3.

One can also add that the curves (*a*), (*b*) and (*c*) plotted in Fig. 3 for protons differ only negligibly (by less than 0.04%) from the respective curves for neutrons.

Figs. 1, 2 and 3 are plotted for deformations close to their equilibrium values for nuclei in the radium region. Specifically, $\varepsilon_4 = \varepsilon_4^0$ is taken for $\lambda = 2$, $\varepsilon = \varepsilon^0$, $\varepsilon_4 = \varepsilon_4^0$ for $\lambda = 3$, $\varepsilon = \varepsilon^0$ for $\lambda = 4$, $\varepsilon = \varepsilon^0$, $\varepsilon_3 = \varepsilon_3^0$, $\varepsilon_4 = \varepsilon_4^0$ for $\lambda = 5$ and $\varepsilon = \varepsilon^0$, $\varepsilon_4 = \varepsilon_4^0$ for $\lambda = 6$, where $\varepsilon^0 = 0.20$, $\varepsilon_3^0 = 0.12$, $\varepsilon_4^0 = -0.044$. Deformations not specified here are taken zero.

An important question is the effect of the difference $\delta\varepsilon_\lambda$ on the dependence of the energy on deformation. It is interesting to see how much is the dependence of the macroscopic part of the energy changed when the deformation of the density, instead of the deformation of the potential, is taken. In other words, when the energy

$$E_d(\varepsilon_\lambda) \equiv E[\varepsilon_\mu^d(\varepsilon_\lambda)], \tag{3.2a}$$

scaled by the relation $\varepsilon_\mu^d = \varepsilon_\mu^d(\varepsilon_\lambda)$, is considered instead of the non-scaled energy

$$E(\varepsilon_\lambda). \tag{3.2b}$$

We study this problem for three kinds of the macroscopic energy, mostly used at present, i.e. for the liquid-drop, droplet and Yukawa-plus-exponential energies.

Fig. 4 shows the energies E and E_d for the droplet (DT) and Yukawa-plus-exponential (Y) models. The parameters of these models are taken the same as in Refs. [5] and [6], respectively. The relations between ε_λ and ε_λ^d, $\varepsilon_\lambda^d(\varepsilon_\lambda; \varepsilon_\mu^0)$ are those of Fig. 1. As they are calculated at the deformation points $\varepsilon_\mu = \varepsilon_\mu^0 \neq 0$ (for some $\mu \neq \lambda$), the minimal values

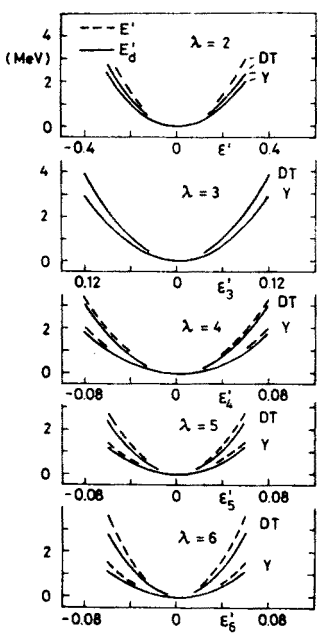

Fig. 4. Dependence of the scaled, E_d', and non-scaled, E', energies (cf. Eqs (3.2) and (3.3)) for the droplet (DT) and Yukawa-plus-exponential (Y) models, as functions of the deformation ε_λ' for $\lambda = 2, 3, 4, 5, 6$

of the energy $E(\varepsilon_\lambda; \varepsilon_\mu^0)$ are not equal to zero and are obtained at $\varepsilon_\lambda = \varepsilon_\lambda^{\min} \neq 0$. The same also concerns the energy $E_d(\varepsilon_\lambda)$. Thus, to make a comparison between the energies easier, we have shifted these minima to a common point: the origin of the coordinate system. In other words, we have plotted the energy

$$E'(\varepsilon'_\lambda) \equiv E(\varepsilon_\lambda - \varepsilon_\lambda^{\min}) - E(\varepsilon_\lambda^{\min}) \tag{3.3}$$

(and the same for $E'_d(\varepsilon'_\lambda)$) instead of the energy $E(\varepsilon_\lambda)$, where

$$\varepsilon'_\lambda \equiv \varepsilon_\lambda - \varepsilon_\lambda^{\min}.$$

The values $\varepsilon_\lambda^{\min}$ are rather small, e.g. $\varepsilon_6^{\min}$ is equal to 0.010 for (DT) and to 0.012 for (Y). One can see in Fig. 4 that the effect of the scaling increases with increasing λ. The only exception to this rule is the multipolarity $\lambda = 2$, for which the effect is remarkably large. This is a consequence of the behaviour of $\delta\varepsilon_\lambda/\varepsilon_\lambda$, as a function of λ, shown in Fig. 2. The scaling effect is naturally stronger for the energy (DT) which is a steep function of ε_λ than for the energy (Y) which is less steep. (In the limit case of a constant energy, $E(\varepsilon_\lambda) = \text{const.}$, there would be no effect at all of the scaling). To be more specific, the scaling reduces the (DT) energy by about 21%, 2%, 8%, 16% and 23% for $\lambda = 2, 3, 4, 5$ and 6, respectively. This is a significant reduction, but is still much smaller than the reduction obtained when the (Y) energy is used instead of the (DT) energy (except for the case of $\lambda = 2$). The latter reduction is by about 14%, 26%, 39%, 48% and 59% for $\lambda = 2, 3, 4, 5$ and 6. Naturally, the largest reduction is obtained when one uses the scaled (Y) energy instead of the non-scaled (DT) energy. This amounts to about 34%, 27%, 44%, 57% and 70% reduction, for $\lambda = 2, 3, 4, 5$ and 6. Thus for $\lambda = 6$, the scaled (Y) energy is about 3 times smaller than the non-scaled (DT) energy.

The above statements and numbers are almost independent of a specific value of ε'_λ, because for not too large ε'_λ (i.e. not too far from the equilibrium point $\varepsilon'_\lambda = 0$), as those considered in Fig. 4, the energies E' (both scaled and non-scaled) are approximately of the form

$$E'(\varepsilon_\lambda) \approx \tfrac{1}{2} C_\lambda (\varepsilon'_\lambda)^2.$$

Thus, the ratio of any two of them (e.g. E'_d/E') is approximately equal to the ratio of the corresponding stiffnesses C_λ and, consequently, almost independent of ε'_λ.

Concerning the effect of using various macroscopic parts of the total energy on the absolute value of this energy at its equilibrium point, one can state the following. If one assumes rather large values of the equilibrium deformation of the total energy ε_λ^0 (which is equal to the shift $\Delta\varepsilon_\lambda$ of the equilibrium point, due to the shell-correction energy): $\varepsilon_\lambda^0 \approx 0.20$ [12], 0.12 [13, 14], 0.08 [12] and 0.04 [15] for $\lambda = 2, 3, 4$ and 6, respectively, one gets from Fig. 4 a lowering of the total energy at this deformation (as $\varepsilon'_\lambda \approx \varepsilon_\lambda$) by about 0.3, 0.1, 0.3 and 0.4 MeV, respectively, by using scaled (DT) instead of non-scaled (DT), and by about 0.2, 1.0, 1.2 and 1.0 MeV, respectively, by taking the non-scaled (Y) energy instead of the non-scaled (DT) energy. Thus, the gain in the deformation energy by scaling (DT) is expected to be smaller than 0.5 MeV, while this gain can be even larger than 1.0 MeV by using the energy (Y) instead of the energy (DT).

4. Discussion

In this Section, we discuss effects of various factors on the density deformation ε_μ^d and on the smooth part of the collective potential energy.

4.1. Effect of the number of shells included

Fig. 5a illustrates the dependence of the ratio Q_λ/Q_0 on the number of shells included in the calculations. All shells from $N = 0$ up to $N = N_{\max}$ are taken. The calculation is performed for the deformations of only even multipolarities ($\varepsilon^0 = 0.20$, $\varepsilon_4^0 = -0.08$,

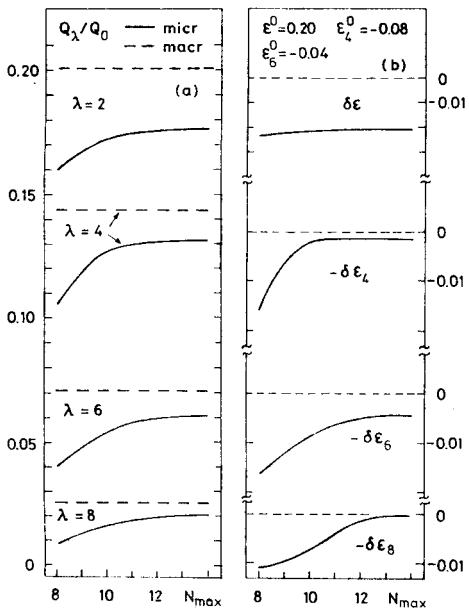

Fig. 5. Microscopic (solid line) and macroscopic (dashed line) values of the ratio Q_λ/Q_0 (*a*) and of the difference $\delta\varepsilon_\lambda$ or $-\delta\varepsilon_\lambda$ (b), Eq. (3.1), as functions of the number of oscillator shells $N_{\max}$ (taken in the diagonalization basis), for the case of only even-multipolarity deformations

$\varepsilon_6^0 = -0.04$). Thus, the odd-multipolarity moments Q_λ, $\lambda = 3, 5, \ldots$, are equal to zero. The values of both microscopic and macroscopic ratios Q_λ/Q_0 for $\lambda = 2, 4, 6, 8$ are given. One can see that sufficiently large basis ($N_{\max}$) should be taken to get proper (independent of $N_{\max}$) microscopic values of Q_λ/Q_0. Higher the multipolarity λ, larger $N_{\max}$ is needed. This is for fixed deformation. Concerning the dependence of $N_{\max}$ on deformation, one can expect that $N_{\max}$ increases with increasing deformation, as the basis is made of the spherical oscillator and higher and higher shells of this oscillator contribute to the levels of the deformed potential, close to the Fermi level, when the deformation increases.

The differences between microscopic and macroscopic values of Q_λ/Q_0, visible in Fig. 5a, determine the differences $\delta\varepsilon_\lambda$, Eq. (3.1). The latter are explicitly shown in Fig. 5b. One can see that although to calculate $\delta\varepsilon_\lambda$ for $\lambda = 2$, it is already enough to take $N_{\max} \approx 8$, one needs $N_{\max} \approx 10$ for $\lambda = 4$ and $N_{\max} \approx 12$ for $\lambda = 6$.

Fig. 6 illustrates the situation when the odd-multipolarity deformations are included ($\varepsilon_3^0 = 0.12$, $\varepsilon_5^0 = -0.022$). One can see that the convergence of both Q_λ/Q_0 and $\delta\varepsilon_\lambda$ to their stable values, when N_{max} is increasing, is slower than in the case of only even-multipolarity deformations. To calculate $\delta\varepsilon_\lambda$ with $\lambda \leqslant 5$, one needs $N_{max} \approx 10$. To get, however, proper values of $\delta\varepsilon_\lambda$ for higher multipolarity, even $N_{max} = 13$ does not seem to be completely enough.

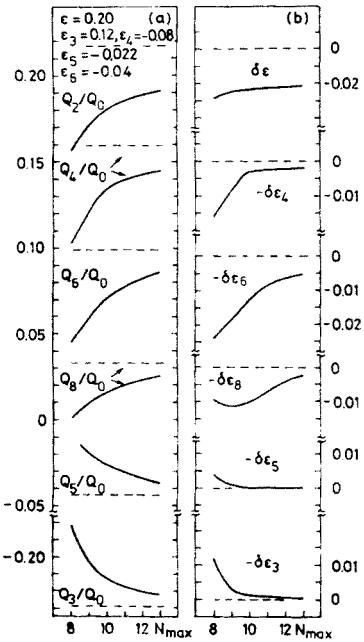

Fig. 6. Same as in Fig. 5, but for the case when odd-multipolarity deformations are also included

4.2. Effect of a change in the single-particle spectrum

To see the effect of the single-particle spectrum on the difference $\delta\varepsilon_\lambda$, we have calculated this quantity for protons and for neutrons, separately. The single-particle spectra differ much in these two cases. The results are given in Fig. 7. The ratio $\delta\varepsilon_\lambda/\varepsilon_\lambda$ is obtained

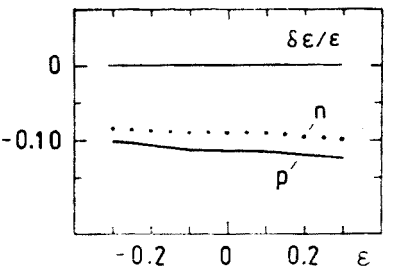

Fig. 7. The ratio $\delta\varepsilon/\varepsilon$ calculated for both protons (p) and neutrons (n) as a function of the potential deformation ε

here for the main (lowest-multipolarity) component of the deformation, $\lambda = 2$. One can see that it differs only a little between the cases of protons and neutrons. The difference is about 5 to 10 times smaller than the value $\delta\varepsilon/\varepsilon$, itself, for any of the two cases. For protons, the density deformation is smaller than that for neutrons, for considered single-particle spectra ("$A = 225$" parameters of the Nilsson potential [1]) and considered nucleus ($Z = 88$, $N = 136$). This small difference between protons and neutrons simultaneously demonstrates that the results obtained and shown in the present paper for protons are also representative for a total nucleus.

It is also interesting to see the role of various parts of the Nilsson potential in generating the difference $\delta\varepsilon/\varepsilon$. According to Eq. (2.4b), the potential has the form

$$V_{\text{Nilss}} = V_{\text{osc}} - \kappa\hbar\overset{0}{\omega}_0[2\boldsymbol{l}\cdot\boldsymbol{s} + \mu(\boldsymbol{l}^2 - \langle\boldsymbol{l}^2\rangle_N)].$$

To see the role of the separate parts, we have calculated $\delta\varepsilon/\varepsilon$ in the following three cases of the potential: (*a*) pure oscillator ($\kappa = 0$), (*b*) oscillator plus spin-orbit term ($\mu = 0$), (*c*) total potential. As in almost the whole paper, the results are obtained for the system of 88 protons. They are given in Fig. 8. One can see that pure oscillator gives less than about half of the value of $\delta\varepsilon/\varepsilon$ obtained with the total potential. More precisely, it gives from about half, for $\varepsilon = -0.3$, down to about one third of that value, for $\varepsilon = 0.3$. The contribution of the spin-orbit term to $\delta\varepsilon/\varepsilon$ is small, so that the curves (*a*) and (*b*) are not distinguishable in Fig. 8.

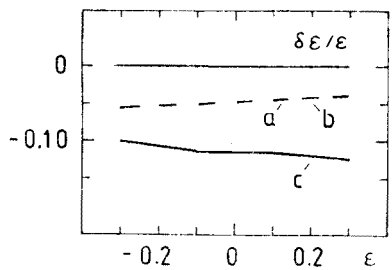

Fig. 8. The ratio $\delta\varepsilon/\varepsilon$ calculated in three cases of the Nilsson potential, described in text

4.3. Effect of a change in the multipole moments subjected to the consistency condition

In our whole paper, the consistency condition is imposed on the "stretched" multipole moments Q_λ, i.e. on the moments corresponding to the operators q_λ, Eq. (2.3), in the stretched space. One can ask for the effect of imposing the condition on the "normal" moments Q'_λ, corresponding to the operators q'_λ, Eq. (2.1), defined in the normal space, instead of the moments Q_λ. A study of the effect is easy in the case of a pure quadrupole deformation ε, i.e. for the case of $\varepsilon_\lambda = 0$ for $\lambda \neq 2$. In this case, the density distribution of a nucleus is characterized by two microscopic moments Q'_0 and Q'_2 which can be expressed by the "stretched" moments Q_0 and Q_2. For given deformation ε of the

potential, we have

$$Q_0' = h(\varepsilon)\{(1-\tfrac{1}{3}\varepsilon)Q_0+\tfrac{2}{3}\varepsilon Q_2\},$$
$$Q_2' = h(\varepsilon)\{\tfrac{1}{3}\varepsilon Q_0+Q_2\}, \tag{4.1}$$

where

$$h(\varepsilon) = (1+\tfrac{1}{3}\varepsilon)^{-1}(1-\tfrac{2}{3}\varepsilon)^{-1}\hbar/M\omega_0(\varepsilon).$$

Thus, having calculated microscopic moments Q_0 and Q_2, we also have at our disposal microscopic moments Q_0' and Q_2'. From Eqs (4.1), one gets

$$\frac{Q_2'}{Q_0'} = \frac{\varepsilon+3(Q_2/Q_0)}{(3-\varepsilon)+2\varepsilon(Q_2/Q_0)}. \tag{4.1a}$$

In the pure quadrupole-deformation case, the "normal" macroscopic moments,

$$Q_\lambda'^{\text{macr}} = \int_{\tau'} g(r')q_\lambda(r')d\tau', \quad \lambda = 0, 2, \tag{4.2}$$

are simple functions of the deformation parameter ε^d of the macroscopic density distribution. In particular,

$$\frac{Q_2'^{\text{macr}}}{Q_0'^{\text{macr}}} = \frac{(1+\frac{1}{3}\varepsilon^d)^2-(1-\frac{2}{3}\varepsilon^d)^2}{(1+\frac{1}{3}\varepsilon^d)^2+2(1-\frac{2}{3}\varepsilon^d)^2}. \tag{4.3}$$

Thus, the solution of the consistency condition

$$Q_2'^{\text{macr}}/Q_0'^{\text{macr}} = Q_2'^{\text{micr}}/Q_0'^{\text{micr}} \tag{4.4}$$

is explicitly given by

$$\varepsilon^d = (m+\tfrac{1}{3})^{-1}[(m+1)\pm(1+m-2m^2)^{1/2}], \tag{4.5}$$

where $m = Q_2'^{\text{micr}}/Q_0'^{\text{micr}}$. (Only one of the two solutions (4.5) exists in the physical region.) Fig. 9 shows the result for $\delta\varepsilon/\varepsilon$ in the case when the consistency condition, Eq. (4.4), is imposed on the ratio Q_2'/Q_0' of the "normal" moments (nor). The result, when the condi-

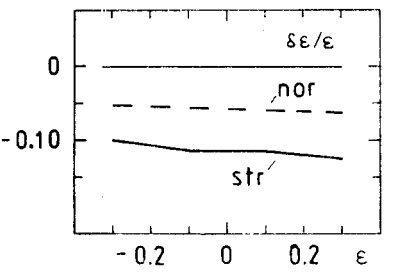

Fig. 9. The ratio $\delta\varepsilon/\varepsilon$ calculated in two cases: when the consistency condition is imposed on the "normal" multipole moments (nor) and on the "stretched" ones (str)

tion is imposed on the ratio Q_2/Q_0 of the "stretched" moments (str), is also shown, for comparison. The latter result is the same as that of p in Fig. 7 and c in Fig. 8. One can see that the ratio $\delta\varepsilon/\varepsilon$ (and thus also the difference $\delta\varepsilon$) is about two times smaller in the "nor" case than in the "str" case. This stresses the importance of a proper choice of the moments to be subjected to the consistency condition.

The fact that $\delta\varepsilon/\varepsilon$ is about two times smaller in the "nor" case than in the "str" case is mostly due to the form, itself, of the multipole operators q'_λ and q_λ, and thus has a rather general character. Really, using the expression for $Q_2'^{\mathrm{macr}}/Q_0'^{\mathrm{macr}}$ in terms of ε^d, Eq. (4.3), and the similar expression for $Q_2^{\mathrm{macr}}/Q_0^{\mathrm{macr}}$,

$$Q_2^{\mathrm{macr}}/Q_0^{\mathrm{macr}} = \varepsilon^d/(3-\varepsilon^d), \tag{4.6}$$

one can relate the solution $(\varepsilon^d)^{\mathrm{nor}}$ of the consistency condition in the "nor" case, Eq. (4.4), with the solution $(\varepsilon^d)^{\mathrm{str}}$ of the condition in the "str" case,

$$Q_2^{\mathrm{macr}}/Q_0^{\mathrm{macr}} = Q_2^{\mathrm{micr}}/Q_0^{\mathrm{micr}}, \tag{4.7}$$

taking additionally into account the expression for the ratio Q'_2/Q'_0 in terms of the ratio Q_2/Q_0, Eq. (4.1a). For not too large deformations ε, the relation obtained this way is

$$(\delta\varepsilon)^{\mathrm{nor}} \approx \tfrac{1}{2}(\delta\varepsilon)^{\mathrm{str}}. \tag{4.8}$$

The relation is almost independent of a particular value of Q_2/Q_0, i.e. of a particular deformation, single-particle potential, etc. To check this, one can perform direct calculations of $(\delta\varepsilon)^{\mathrm{nor}}$ in e.g. the three cases of the potential a, b and c, for which the values of $(\delta\varepsilon)^{\mathrm{str}}$ have been obtained (Fig. 8). The calculations, performed by us, show that the relation (4.8) really holds in all these cases and in the whole deformation interval, $-0.3 \leqslant \varepsilon \leqslant 0.3$, considered.

4.4. Effect of the number of multipole moments subjected to the consistency condition

As the potential with even no deformations of multipolarities μ, $\varepsilon_\mu = 0$ $(\mu \geqslant \lambda_0)$, generates the microscopic moments $Q_\mu^{\mathrm{micr}}(\varepsilon_\nu) \neq 0$, on one hand, and any of $\lambda \leqslant \lambda_{\max}$ consistency equations (2.16a) for the density deformations ε_μ^d is coupled with each other by the functional dependence of $Q_\lambda^{\mathrm{macr}}(\varepsilon_\mu^d)$ on ε_μ^d, on the other hand, the results for ε_μ^d generally depend on the number of equations $\lambda_{\max} \geqslant \lambda_0$.

To study this, we have taken the deformations of the potential: $\varepsilon^0 = 0.20$, $\varepsilon_4^0 = -0.08$, $\varepsilon_6^0 = -0.04$ and $\varepsilon_\mu^0 = 0$ for $\mu \neq 2, 4, 6$ (in particular, $\varepsilon_8^0 = 0$). We have solved the equations (2.16a) in two cases: with $\lambda_{\max} = 6$ and with $\lambda_{\max} = 8$. It appears that the solutions for ε_μ^d, $\mu \leqslant \lambda_0 = 6$, obtained in the two cases are practically the same. It is important for this, however, to take sufficiently large number of the oscillator shells $N_{\max}$. For example, with $N_{\max} = 8$, the difference between the values of ε_6^d obtained in the two cases amounts to about 6%, while it reduces to only about 0.1% for $N_{\max} = 14$. The effect on the deformations of a lower multipolarity is even smaller.

4.5. Effect of the number of coupled oscillator shells

The Nilsson potential couples generally all shells of the spherical oscillator, taken as a basis. Naturally, the coupling between the nearest shells is most important [16]. Coupling between the neighbouring $\Delta N = 8$ shells is usually taken into account in calculations.

To see the effect of a change in the number ΔN on the values of the density deformations ε_μ^d, we take the same deformation point as in the preceding subsect. 4.4: $\varepsilon^0 = 0.20$, $\varepsilon_4^0 = -0.08$, $\varepsilon_6^0 = -0.04$, and find ε_μ^d in two cases: $\Delta N = 8$ and $\Delta N = N_{\max} = 13$ (i.e. when all shells are coupled). It appears that the difference between the values of ε_6^d obtained in the two cases is negligible: it amounts to only about 0.1 %. For ε_μ^d of lower multipolarities, the difference is even smaller.

4.6. Dependence on the proton and neutron numbers

As the microscopic moments Q_λ^{micr}, used in our calculations, are averaged over the shell effects, they are rather smooth functions of the proton Z and neutron N numbers. Additionally, the moments $Q_\lambda^{\text{micr}}(\lambda \neq 0)$ and Q_0^{micr} depend in about the same way on Z and N. Thus, the ratios $Q_\lambda^{\text{micr}}/Q_0^{\text{micr}}$, which determine the density deformations ε_μ^d (through Eqs (2.16a)), are almost independent of the numbers Z and N. Due to this, also only a very weak dependence of the deformations ε_μ^d on these numbers may be expected.

To check this, we have taken the potential deformations: $\varepsilon^0 = 0.20$, $\varepsilon_3^0 = 0.12$, $\varepsilon_4^0 = -0.04$, $\varepsilon_5^0 = -0.08$ (and zero for other multipolarities) and calculated ε_μ^d for the proton number Z varying from $Z = 84$ up to $Z = 94$. The neutron number N has been kept constant, $N = 136$. It appears that the deformation ε_5^d changes by only less than 1 % with a change of Z inside the above limits. The changes of the deformations ε_μ^d of lower multipolarity are even smaller.

A similar dependence of ε_μ^d on the neutron number N is expected.

4.7. The condition for the center of mass of a nucleus

As stated in subsect. 2.3, to keep the center of mass of a nucleus at the origin of the coordinate system, the condition (2.15b),

$$Q_{1\text{tot}}^{\text{micr}}(\varepsilon_\mu) = 0, \tag{2.15b}$$

is required to be fulfilled for each deformation of the potential. In real calculations, however, it is much easier to fulfil the condition

$$Q_{1\text{tot}}^{\text{macr}}(\varepsilon_\mu) = 0, \tag{4.9}$$

as Q_1^{macr} is a much simpler function of deformations than the function Q_1^{micr}. Due to this, for given potential deformations ε_ν, $\nu = 2, 3, \ldots, \lambda_{\max}$, instead of Eq. (2.15b), the equation (4.9) is solved to find

$$\bar{\varepsilon}_1 = \bar{\varepsilon}_1(\varepsilon_\nu).$$

With this value, we find all multipole moments $Q_\lambda^{\text{micr}}[\bar{\varepsilon}_1, \varepsilon_\nu]$, $\lambda \neq 1$ and the further procedure is the same as described in subsect. 2.3.

To see how good approximation is the use of $\bar{\varepsilon}_1$ instead of ε_1, we study the difference $\bar{\varepsilon}_1 - \varepsilon_1$ and also the effect of this difference on the density deformations ε_μ^d, $\mu > 1$, and on the energy of a nucleus. As the smoothed multipole moments are rather insensitive to the single-particle structure (cf. subsect. 4.2), it seems sufficient for the study to perform it for only one kind of nucleons, e.g. for protons.

Fig. 10 shows ε_1, which is a solution of Eq. (2.15b) for the deformation of the potential: $\varepsilon^0 = 0.20$, $\varepsilon_3^0 = 0.12$, $\varepsilon_4^0 = -0.08$, $\varepsilon_5^0 = -0.022$, $\varepsilon_6^0 = -0.04$ and zero for other multi-

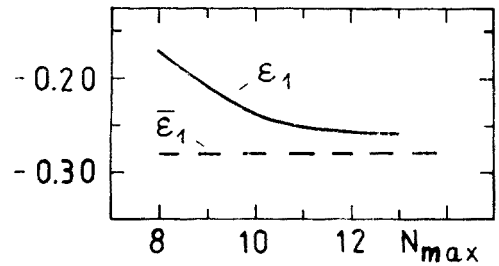

Fig. 10. Solution of the microscopic Eq. (2.15b), ε_1, as a function of the number of the oscillator shells included $N_{\max}$. Solution of the macroscopic Eq. (4.9), $\bar{\varepsilon}_1$, is shown for comparison

polarities, as a function of the number of the oscillator shells included $N_{\max}$. One can see that the solution depends significantly on $N_{\max}$, especially for lower values of $N_{\max}$. The value of ε_1 approaches the value $\bar{\varepsilon}_1$ when $N_{\max}$ increases. This stresses once more the importance of including a sufficient number of shells in the calculation of the microscopic moments Q_λ^{micr}. For $N_{\max} = 12$, ε_1 differs from $\bar{\varepsilon}_1$ by about 9%. This difference results in a difference in the density deformations ε_μ^d. The deformations, obtained with ε_1 differ from those obtained with $\bar{\varepsilon}_1$ by less than about 5%. The difference is larger for higher multipolarities (it is about 5% for $\lambda = 5$) and smaller for lower multipolarities (it is smaller than 1% for $\lambda = 2$). Due to this, the effect on the energy is rather small. It is smaller than 0.13 MeV for all three kinds of the smooth part of the energy considered by us (i.e. liquid drop, droplet and Yukawa-plus-exponential energies). As the deformation ε_1 also enters the microscopic part of the energy (shell correction), we can also speak about the effect of using $\bar{\varepsilon}_1$ instead of ε_1 on this energy. The effect is, however, around only 0.02 MeV (for protons) and may be completely disregarded. Thus, the use of $\bar{\varepsilon}_1$ instead of ε_1 may be considered as a sufficiently good approximation.

5. *Conclusions*

Our study has been performed for particular (Nilsson) single-particle potential with particular ("$A = 225$") parameters and for particular nuclei (in the radium region). Still, due to an extensive discussion of a sensitivity of the results to various changes (changes of the potential, of the number and kind of nucleons, etc.), a number of conclusions of the study are probably of a general character. The conclusions are:

(1) Deformation ε_λ^d of the density distribution, generated by the single-particle potential

is always smaller, in absolute value, than the deformation of the potential itself. At least in the deformation region, around equilibrium values, investigated here.

(2) For not too large deformation ε_λ, the difference $\delta\varepsilon_\lambda \equiv \varepsilon_\lambda^d - \varepsilon_\lambda$ is approximately proportional to ε_λ. Thus, the ratio $\delta\varepsilon_\lambda/\varepsilon_\lambda$ is approximately constant.

(3) The ratio $\delta\varepsilon_\lambda/\varepsilon_\lambda$ systematically increases with increasing multipolarity λ, from about 1% for $\lambda = 3$ up to about 12% for $\lambda = 6$. An exception of this regularity is the lowest-multipolarity (quadrupole) deformation, $\lambda = 2$, for which the effect is already large, around 11%.

(4) The difference $\delta\varepsilon_\lambda$ obtained for protons is close to that for neutrons.

(5) A large contribution to $\delta\varepsilon$ comes from the correction term $-\mu(l^2 - \langle l^2\rangle_N)$ in the Nilsson potential. An omission of this term reduces the difference $\delta\varepsilon$ (i.e. for $\lambda = 2$) by about (50–70)%.

(6) Scaling of the macroscopic part of the total energy by using the density deformation ε_λ^d instead of the potential deformation ε_λ makes the total energy surface $E(\varepsilon_\lambda) = E^{\text{macr}}[\varepsilon_\mu^d(\varepsilon_\lambda)] + \delta E^{\text{shell}}(\varepsilon_\lambda)$ more flat, i.e. more soft with respect to the potential deformation ε_λ. In particular, it increases the deformation energy $E_{\text{def}} = E(0) - E(\varepsilon_\lambda^{\min})$ of a deformed nucleus in the radium region by up to about (0.3–0.4) MeV.

(7) The difference $\delta\varepsilon_\lambda$ depends on the kind (on the exact form) of the multipole moments, on which the consistency condition is imposed. For example, a use of the "normal" surface moments $Q'_\lambda = \langle 2r^2 P_\lambda(\cos\vartheta')\rangle$ instead of the "stretched" moments $Q_\lambda = \langle 2\varrho^2 P_\lambda(\cos\vartheta)\rangle$, for $\lambda = 0, 2$, reduces $\delta\varepsilon$ by a factor of 2.

(8) An important point in the calculation of the microscopic moments Q_λ, and thus also of the differences $\delta\varepsilon_\lambda$, is to use sufficiently large basis for the diagonalization of the single-particle potential.

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