LETTERS TO THE EDITOR

ALTERNATIVE DESCRIPTION OF SCALAR "NOTIVARG"

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An alternative description of the scalar "notivarg" is proposed. The comparison with the original theory of Deser, Siegel and Townsend is given.

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The theory of the Ogievetsky — Polubarinov "notoph" [1] is the earliest gauge theory of the scalar particle (see Ref. [2] too). Deser, Siegel and Townsend [3] have given another example of a scalar gauge theory. In their approach
(*i*) the field is a twenty component tensor $K^{\mu\nu\alpha\beta}$ with symmetries of the Riemann tensor,
(*ii*) the action can be given in the form

$$I \sim \int dx(\partial_\mu K^{\mu\nu\alpha\beta}\partial_\alpha K_{\nu\kappa\beta}{}^{\kappa} - \tfrac{1}{3}\,\partial_\mu K^{\mu\nu\alpha}{}_{\nu}\partial_\alpha K^{\sigma\lambda}{}_{\sigma\lambda}). \tag{1}$$

We refer to this description of scalar particles as to the theory of scalar "notivarg" [3].

In the present paper we give an alternative formulation of the theory of the scalar "notivarg". We start with the first order action

$$I = \int dx[\sqrt{2}\,\partial_\mu K^{\mu\nu\alpha\beta}S_{\alpha\beta\nu} + \tfrac{1}{2}(S_{\alpha\beta\nu}S^{\alpha\beta\nu} - 2S_\alpha S^\alpha)], \tag{2}$$

where $S^\alpha \equiv S^{\alpha\beta}{}_\beta$. The Lagrange multiplier $S^{\alpha\beta\nu}$ has the following symmetries $S^{\alpha\beta\nu} = -S^{\beta\alpha\nu}$, $\varepsilon_{\mu\nu\alpha\beta}S^{\alpha\beta\nu} = 0$. The potential $K^{\mu\nu\alpha\beta}$ has the symmetries of the Riemann tensor $K^{\mu\nu\alpha\beta} = K^{\alpha\beta\mu\nu} = -K^{\nu\mu\alpha\beta} = -K^{\mu\nu\beta\alpha}$, $\varepsilon_{\mu\nu\alpha\beta}K^{\mu\nu\alpha\beta} = 0$. The action (2) is obtained (by the $m^2 \to 0$ limit) from the action [4] describing a massive spin 2 particle with the help of the 4-t hrank tensor. That description is equivalent to the well known theory of Pauli and Fierz for a massive spin 2 particle.

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The field equations following from the action (2) are

$$S^{\alpha\beta\nu} = -\sqrt{2}\,[\partial_\mu K^{\mu\nu\alpha\beta}+\tfrac{1}{2}(g^{\alpha\nu}K^\beta - g^{\beta\nu}K^\alpha)], \tag{3a}$$

$$\partial^\mu S^{\alpha\beta\nu} - \partial^\nu S^{\alpha\beta\mu} + \partial^\alpha S^{\mu\nu\beta} - \partial^\beta S^{\mu\nu\alpha} = 0, \tag{3b}$$

where $K^\alpha \equiv \partial_\mu K^{\mu\nu\alpha}{}_\nu$. Using Eq. (3a) we can eliminate $S^{\alpha\beta\nu}$ from the action (2). We get

$$I = \int dx[-(\partial_\sigma K^{\sigma\nu\alpha\beta})^2 + (\partial_\sigma K^{\sigma\nu\alpha}{}_\nu)^2]. \tag{4}$$

To determine the spin content of the action (4) we analyse Eq. (3b). It can be regarded as a constraint on the field $S^{\alpha\beta\nu}$ determining its general form:

$$S^{\alpha\beta\nu} = \frac{1}{\sqrt{3}}(\partial^\nu A^{\alpha\beta} + \partial^\alpha B^{\beta\nu} - \partial^\beta B^{\alpha\nu}),$$

where $A^{\alpha\beta} = -A^{\beta\alpha}$ and $B^{\alpha\beta} = -B^{\beta\alpha}$. The factor $1/\sqrt{3}$ is chosen for convenience. The fields $A^{\alpha\beta}$ and $B^{\alpha\beta}$ are not independent for $\varepsilon_{\mu\nu\alpha\beta}S^{\alpha\beta\nu} = 0$. In terms of $A^{\alpha\beta}$ the action (4) can be rewritten in the form

$$I = -\tfrac{1}{2}\int dx[(\partial^\nu A^{\alpha\beta})^2 - 2(\partial_\alpha A^{\alpha\beta})^2]. \tag{5}$$

This is the action for the Ogievetsky — Polubarinov "notoph" [1]. So, the action [4] describes a scalar particle.

Let us investigate the gauge invariance of the action (4). It is invariant under the following gauge transformation

$$\delta K^{\mu\nu\alpha\beta} = \varepsilon^{\mu\nu\sigma\lambda}\partial_\sigma\omega^{\alpha\beta}{}_\lambda + \varepsilon^{\alpha\beta\sigma\lambda}\partial_\sigma\omega^{\mu\nu}{}_\lambda$$
$$+g^{\mu\alpha}(\partial^\nu\eta^\beta + \partial^\beta\eta^\nu) + g^{\nu\beta}(\partial^\mu\eta^\alpha + \partial^\alpha\eta^\mu) - g^{\mu\beta}(\partial^\nu\eta^\alpha + \partial^\alpha\eta^\nu)$$
$$-g^{\nu\alpha}(\partial^\mu\eta^\beta + \partial^\beta\eta^\mu) - 2(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha})\partial_\sigma\eta^\sigma, \tag{6}$$

where $\omega^{\alpha\beta\nu}$ has the symmetries $\omega^{\alpha\beta\nu} = -\omega^{\beta\alpha\nu}$, $\varepsilon_{\mu\nu\alpha\beta}\omega^{\alpha\beta\nu} = 0$ and obeys the condition $\partial_\alpha\omega^{\alpha\beta\nu} = 0$. We adopt

$$K^{\mu\beta\alpha}{}_\beta = 0, \tag{7a}$$

$$\partial_\mu\partial_\alpha K^{\mu\nu\alpha\beta} = 0 \tag{7b}$$

as the gauge conditions. It can be verified that taking into account these conditions (*i*) one gets the field equation

$$\Box K^{\mu\nu\alpha\beta} = 0,$$

(*ii*) in the momentum space in the frame $p^\mu = (p, 0, 0, p)$ the tensor $K^{\mu\nu\alpha\beta}$ has five independent components

$$S_{ij} \equiv -\tfrac{1}{4}(\varepsilon_{imk}K_{0j}{}^{mk} + \varepsilon_{jmk}K_{0i}{}^{mk}), \tag{8}$$

$(S_{ij} = S_{ji}, S^i_i = 0)$. These components belong to $(2,0)\oplus(0,2)$ Lorentz part of $K^{\mu\nu\alpha\beta}$, (*iii*) the conditions (7) do not remove a gauge freedom completely. Only the component S^{33} is invariant under the remained gauge transformation. It describes the helicity 0.

The canonical analysis (it will be given elsewhere) confirms the **0** spin content of the theory described by the action (4).

We finish with some remarks regarding comparison of both approaches:

1. The actions (1) and (4) are not connected by the point transformation of $K^{\mu\nu\alpha\beta}$.
2. The action (1) results from the first order action (Eq. (2.6) of Ref. [3]) containing two auxiliary symmetric fields, while we use one field $S^{\alpha\beta\nu}$.
3. As it results from the canonical analysis, the scalar "notivarg" is described by

— the longitudinal part of the six component tensor K^{0i0j} in the case of the action (1) (see Ref. [3]),

— the longitudinal part of the traceless tensor (8) in the case of the action (4).

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