Vol. B18 (1987) ACTA PHYSICA POLONICA No 10

ON THE Σ HYPERNUCLEAR STATES PRODUCED IN $(K^-, \pi^\pm)$ REACTIONS ON ^{12}C*

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(*Received January 26, 1987*)

Energies and widths of the substitutional $(p3/2, p3/2^{-1})_{\Sigma N}$ states in ${}^{12}_{\Sigma^-}$Be and ${}^{12}_{\Sigma}$C are determined by solving single particle equations with a complex Σ potential, containing the Lane potential v_t and a charge independence breaking part v_{CB}. Magnitude of v_t and v_{CB} compatible with experimental data on ${}^{12}_{\Sigma^-}$Be and ${}^{12}_{\Sigma}$C are discussed. Radial oscillations of the isospin distribution in ${}^{12}_{\Sigma}$C are demonstrated.

PACS numbers: 21.80.+a

1. Introduction

A number of Σ hypernuclear levels have been observed in the $(K^-, \pi^\pm)$ reactions on ^{12}C both by Bertini et al. [1] at CERN with the recolless production method, and by Yamazaki et al. [2] at KEK with the stopped K^- method.

In the case of the (K^-, π^+) reaction, the only possible elementary process, $K^-p \rightarrow \Sigma^-\pi^+$, leads to the formation of the hypernucleus ${}^{12}_{\Sigma^-}$Be, i.e., the $^{11}B+\Sigma^-$ system. In the case of the (K^-, π^-) reaction, two elementary processes are possible: (I) $K^-p \rightarrow \Sigma^-\pi^+$ and (II) $K^-n \rightarrow \Sigma^0\pi^-$. At first sight, one would expect in process (I) the formation of the hypernucleus ${}^{12}_{\Sigma^+}$C, i.e., the $^{11}B+\Sigma^+$ system, and in process (II) the formation of the hypernucleus ${}^{12}_{\Sigma^0}$C, i.e., the $^{11}C+\Sigma^0$ system. In fact, however, because of the presence of the charge mixing Σ's symmetry (Lane) potential, the Σ hyperons do not preserve their charge identity, and the Σ hypernuclear states produced in the (K^-, π^-) reaction are expected to be superpositions of the two charge states ${}^{12}_{\Sigma^+}$C and ${}^{12}_{\Sigma^0}$C, for which we use the symbol ${}^{12}_{\Sigma}$C. Instead of charge, one may use the isospin T in describing the states of ${}^{12}_{\Sigma}$C. However, because of the Coulomb interaction and mass differences, also the isospin is not an exact quantum number of these states.

Thus the first question, investigated in the present paper, is whether the states of ${}^{12}_{\Sigma}$C are better approximated by charge states or by isospin states. Obviously the answer

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* Research supported by the Polish — U.S. Maria Skłodowska-Curie Fund under Grant No P-F7F037P.

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depends on the strength of the Lane potential. It also depends on the isospin invariance of the ΣN interaction, and the second question considered in the present paper is a possible breaking of this invariance. (The need of an isospin symmetry breaking component in the ΣN interaction in the description of the observed levels of ${}^{12}_{\Sigma}$C and ${}^{12}_{\Sigma^-}$Be was noticed in [3] where, however, the Lane coupling was not considered.)

In the present paper, we apply the single particle (s.p.) picture, and restrict ourselves to the substitutional states with the configuration $(p3/2,\ p3/2^{-1})_{\Sigma N}$. The nuclear cores ^{11}B and ^{11}C are considered to be rigid, i.e., undisturbed by Σ. In discussing the isospin T, the nuclear cores are assumed to have isospin $T^c = 1/2$.

All our conclusions are obtained by solving the system of coupled s.p. Schrödinger equations for Σ in the charge basis. This approach gives not only the energies and widths of the levels, but also the corresponding wave functions which reveal interesting features, in particular the isospin oscillations (described briefly in [4]).

The paper is organized as follows. In Sect. 2 the s.p. equations describing the motion of Σ in the substitutional states of ${}^{12}_{\Sigma}$C and ${}^{12}_{\Sigma^-}$Be are derived. The procedure of solving these equations (coupled in the case of ${}^{12}_{\Sigma}$C) with complex eigenvalues is outlined in Sect. 3. In Sect. 4 the charge and isospin distribution in ${}^{12}_{\Sigma}$C, in particular the isospin oscillations, are discussed. The input quantities of the calculations, in particular the Σ absorptive potential, are described in Sect. 5 which then presents the numerical results: general features of the solutions as functions of the strength of the Lane potential (Sect. 5.1); a discussion of a possible breaking of the charge independence of the ΣN interaction and the comparison with the data obtained at CERN (Sect. 5.2) and KEK (Sect. 5.3). Sect. 6 contains a summary and comments on the problem of formation of hypernuclear states.

The main results of the present investigation were presented in [5].

2. *Equations describing ${}^{12}_{\Sigma}C$ and ${}^{12}_{\Sigma^-}Be$*

2.1. Derivation of the equations

We apply the rigid nuclear core model, with the nuclear cores ^{11}B$(T^c = 1/2, T_3^c = -1/2)$ and ^{11}C$(T^c = 1/2, T_3^c = 1/2)$. By T^c and T_3^c, we denote the isospin of the nuclear core and its third component. We use the convention in which the third component of the proton (neutron) isospin is $t_3^p = 1/2$ $(t_3^n = -1/2)$.

The Σ hypernucleus in its CM system is described by the hamiltonian (which includes rest masses)

$$H = H_C + H_\Sigma + H_{C\Sigma}, \tag{2.1}$$

where H_C is the hamiltonian of the nuclear core C, H_Σ is the hamiltonian of an isolated Σ (i.e., its rest mass), and $H_{C\Sigma}$ is the hamiltonian of the relative CΣ motion,

$$H_{C\Sigma} = -(\hbar^2/2\mu_\Sigma)\Delta + \mathscr{V}_{C\Sigma} + V_\Sigma^C, \tag{2.2}$$

where μ_Σ is the CΣ reduced mass, Δ acts on the relative CΣ position vector $\boldsymbol{r}$ (with the origin in the CM of C), and V_Σ^C is the Coulomb and $\mathscr{V}_{C\Sigma}$ the strong interaction between C and Σ.

We approximate $\mathscr{V}_{C\Sigma}$ by a spin independent (centra l)s.p. potential of the form (t^Σ denotes the isospin of Σ):

$$\mathscr{V}_{C\Sigma} \cong \mathscr{V}_\Sigma(r) + v_t(r) T^C t^\Sigma, \tag{2.3}$$

where the second term is the charge mixing Lane potential.

We work in the charge basis. In our rigid nuclear core model, the three charge states, ${}^{12}_{\Sigma^-}\mathrm{Be}$, ${}^{12}_{\Sigma^+}\mathrm{C}$, and ${}^{12}_{\Sigma^0}\mathrm{C}$, are described by the wave functions:

$$\Psi_{\Sigma^-} = R(^{11}\mathrm{B})\chi^{(-)}\psi_{\Sigma^-}(\boldsymbol{r}), \quad \chi^{(-)} = \chi(^{11}\mathrm{B})\chi(\Sigma^-), \tag{2.4}$$

$$\Psi_{\Sigma^+} = R(^{11}\mathrm{B})\chi^{(+)}\psi_{\Sigma^+}(\boldsymbol{r}), \quad \chi^{(+)} = \chi(^{11}\mathrm{B})\chi(\Sigma^+),$$

$$\Psi_{\Sigma^0} = R(^{11}\mathrm{C})\chi^{(0)}\psi_{\Sigma^0}(\boldsymbol{r}), \quad \chi^{(0)} = \chi(^{11}\mathrm{C})\chi(\Sigma^0), \tag{2.5}$$

where $R(^{11}\mathrm{B})$ and $R(^{11}\mathrm{C})$ are normalized wave functions of the nuclear cores $^{11}\mathrm{B}$ and $^{11}\mathrm{C}$, and χ are the isospin wave functions.

To see how the hamiltonian H acts on the functions Ψ_Σ, we notice that

$$T^C t^\Sigma \chi^{(-)} = \tfrac{1}{2}\chi^{(-)}, \quad T^C t^\Sigma \chi^{(0)} = \chi^{(+)}/\sqrt{2},$$

$$T^C t^\Sigma \chi^{(+)} = -\chi^{(+)}/2 + \chi^{(0)}/\sqrt{2}, \tag{2.6}$$

and obtain

$$H\Psi_{\Sigma^-} = \{M(^{11}\mathrm{B}) + M(\Sigma^-) - (\hbar^2/2\mu_{\Sigma^-})\Delta + \mathscr{V}_{\Sigma^-}(r) + v_t(r)/2 + V^C_{\Sigma^-}(r)\}\Psi_{\Sigma^-}, \tag{2.7}$$

$$H\Psi_{\Sigma^+} = \{M(^{11}\mathrm{B}) + M(\Sigma^+) - (\hbar^2/2\mu_{\Sigma^+})\Delta + \mathscr{V}_{\Sigma^+}(r)$$

$$-v_t(r)/2 + V^C_{\Sigma^+}(r)\}\Psi_{\Sigma^+} + (1/\sqrt{2})v_t(r)R(^{11}\mathrm{B})\chi^{(0)}\psi_{\Sigma^+}(\boldsymbol{r});$$

$$H\Psi_{\Sigma^0} = \{M(^{11}\mathrm{C}) + M(\Sigma^0) - (\hbar^2/2\mu_{\Sigma^0})\Delta + \mathscr{V}_{\Sigma^0}(r)\}\Psi_{\Sigma^0}$$

$$+(1/\sqrt{2})v_t(r)R(^{11}\mathrm{C})\chi^{(+)}\psi_{\Sigma^0}(\boldsymbol{r}), \tag{2.8}$$

where M denotes *mass* $\times\, c^2$. Notice that in the case of isospin invariance, we have $\mathscr{V}_{\Sigma^-} = \mathscr{V}_{\Sigma^+} = \mathscr{V}_{\Sigma^0}$ (disregarding effects due to differences in nuclear cores).

2.2. The ${}^{12}_{\Sigma^-}\mathrm{Be}$ hypernucleus

To determine the $\boldsymbol{r}$ part $\psi_{\Sigma^-}(\boldsymbol{r})$ of the wave function Ψ_{Σ^-} of the ${}^{12}_{\Sigma^-}\mathrm{Be}$ hypernucleus, we insert Ψ_{Σ^-} into the Schrödinger equation

$$H\Psi_{\Sigma^-} = [M({}^{12}_{\Sigma^-}\mathrm{Be}) - i\Gamma_{\Sigma^-}/2]\Psi_{\Sigma^-}. \tag{2.9}$$

The appearance of $M({}^{12}_{\Sigma^-}\mathrm{Be})$ in (2.9) is the consequence of our form of H which includes rest masses. Since $\mathscr{V}_{\Sigma^-}$ is complex (its imaginary part describes the absorption of Σ^- due to the $\Sigma^-\mathrm{p} \to \Lambda\mathrm{n}$ process), the energy eigenvalue is also complex, and its imaginary part is equal to $-\Gamma_{\Sigma^-}/2$ with Γ_{Σ^-} being the width of the state.

For the left hand side of Eq. (2.9), we use expression (2.7), multiply the whole equation by $R^*(^{11}\mathrm{B})\chi^{(-)*}$, integrate over all coordinates except $\boldsymbol{r}$, and obtain the Schrödinger equation for $\psi_{\Sigma^-}(\boldsymbol{r})$:

$$\{-(\hbar^2/2\mu_{\Sigma^-})\Delta+\mathscr{V}_{\Sigma^-}(r)+v_t(r)/2+V^C_{\Sigma^-}(r)\}\psi_{\Sigma^-}(\boldsymbol{r}) = [E_{\Sigma^-}-i\Gamma_{\Sigma^-}/2]\psi_{\Sigma^-}(\boldsymbol{r}), \qquad (2.10)$$

where

$$E_{\Sigma^-} \equiv -B_{\Sigma^-} = M(^{12}_{\Sigma^-}\mathrm{Be})-M(^{11}\mathrm{B})-M(\Sigma^-); \qquad (2.11)$$

where B_{Σ^-} is the binding (or separation) energy of Σ^- in the state of $^{12}_{\Sigma^-}\mathrm{Be}$ considered.

In presenting experimental results obtained in the (K, π) reactions, one usually gives ΔM, the mass difference between the hypernucleus and the target nucleus,

$$\Delta M(^{12}_{\Sigma^-}\mathrm{Be}) = M(^{12}_{\Sigma^-}\mathrm{Be})-M(^{12}\mathrm{C}). \qquad (2.12)$$

To connect $\Delta M(^{12}_{\Sigma^-}\mathrm{Be})$ with E_{Σ^-}, we use the first of the relations

$$\begin{aligned} M(^{12}\mathrm{C}) &= M(^{11}\mathrm{B})+M(\mathrm{p})-B_\mathrm{p} \\ &= M(^{11}\mathrm{C})+M(\mathrm{n})-B_\mathrm{n}, \end{aligned} \qquad (2.13)$$

where $B_\mathrm{p} = 15.96$ MeV and $B_\mathrm{n} = 18.72$ MeV [6] are respectively the proton and neutron separation energies from $^{12}\mathrm{C}$. We get

$$\begin{aligned} \Delta M(^{12}_{\Sigma^-}\mathrm{Be}) &= E_{\Sigma^-}+M(\Sigma^-)-M(\mathrm{p})+B_\mathrm{p} \\ &= E_{\Sigma^-}+275.02\ \mathrm{MeV}. \end{aligned} \qquad (2.14)$$

2.3. The $^{12}_{\Sigma}\mathrm{C}$ hypernucleus

Here, the two charge components, $^{11}\mathrm{B}+\Sigma^+$ represented by Ψ_{Σ^+}, and $^{11}\mathrm{C}+\Sigma^0$ represented by Ψ_{Σ^0}, are mixed by the Lane potential v_t, as is seen in Eqs (2.8). Thus the wave function Ψ of $^{12}_{\Sigma}\mathrm{C}$ is a superposition of the two charge states:

$$\Psi = \Psi_{\Sigma^+}+\Psi_{\Sigma^0}. \qquad (2.15)$$

(Since $\psi_\Sigma(\boldsymbol{r})$ in Eqs (2.5) are not normalized, there is no need of introducing any coefficients on the right hand side of Eq. (2.15)).

To determine the functions $\psi_{\Sigma^+}(\boldsymbol{r})$ and $\psi_{\Sigma^0}(\boldsymbol{r})$, we insert Ψ into the Schrödinger equation

$$H\Psi = [M(^{12}_{\Sigma}\mathrm{C})-i\Gamma/2]\Psi, \qquad (2.16)$$

where Γ is the width of the Ψ state of $^{12}_{\Sigma}\mathrm{C}$. For the left hand side of Eq. (2.16), we use expressions (2.8), multiply the whole equation first by $R^*(^{11}\mathrm{B})\chi^{(-)*}$ and second by $R^*(^{11}\mathrm{C})\chi^{(0)*}$, and in both instances integrate over all coordinates except $\boldsymbol{r}$. Here, we make the approximation:

$$R(^{11}\mathrm{C}) \cong R(^{11}\mathrm{B}), \qquad (2.17)$$

which concerns the coupling term with v_t. Without this approximation, this term would be multiplied by the overlap integral between $R(^{11}C)$ and $R(^{11}B)$, which would slightly diminish the coupling.

The resulting system of coupled Schrödinger equations for $\psi_{\Sigma^+}(r)$ and $\psi_{\Sigma^0}(r)$ is:

$$\{-(\hbar^2/2\mu_{\Sigma^+})\Delta+\mathscr{V}_{\Sigma^+}(r)-v_t(r)/2+V^C_{\Sigma^+}(r)\}\psi_{\Sigma^+}(r)$$
$$+(1/\sqrt{2})v_t(r)\psi_{\Sigma^0}(r) = [E_{\Sigma^+}-i\Gamma/2]\psi_{\Sigma^+}(r),$$
$$\{-(\hbar^2/2\mu_{\Sigma^0})\Delta+\mathscr{V}_{\Sigma^0}(r)\}\psi_{\Sigma^0}(r)$$
$$+(1/\sqrt{2})v_t(r)\psi_{\Sigma^+}(r) = [E_{\Sigma^0}-i\Gamma/2]\psi_{\Sigma^0}(r), \tag{2.18}$$

where

$$E_{\Sigma^+} \equiv -B_{\Sigma^+} = M(^{12}_{\Sigma}C)-M(^{11}B)-M(\Sigma^+),$$
$$E_{\Sigma^0} \equiv -B_{\Sigma^0} = M(^{12}_{\Sigma}C)-M(^{11}C)-M(\Sigma^0), \tag{2.19}$$

where B_{Σ^+} and B_{Σ^0} are respectively the energies needed to remove Σ^+ and Σ^- from the Ψ state of ^{12}C, i.e., B_{Σ^+} and B_{Σ^0} are the respective separation (binding) energies.

Relations (2.13) lead to the following connection between $\Delta M(^{12}_{\Sigma}C) = M(^{12}_{\Sigma}C) - M(^{12}C)$ and $E_{\Sigma^{+(0)}}$:

$$\Delta M(^{12}_{\Sigma}C) = E_{\Sigma^+}+M(\Sigma^+)-M(p)+B_p = E_{\Sigma^+}+267.04 \text{ MeV}$$
$$= E_{\Sigma^0}+M(\Sigma^0)-M(n)+B_n = E_{\Sigma^0}+271.61 \text{ MeV}. \tag{2.20}$$

Notice that

$$E_{\Sigma^+}-E_{\Sigma^0} = B_{\Sigma^0}-B_{\Sigma^+} = \gamma_0, \tag{2.21}$$

where

$$\gamma_0 = M(\Sigma^0)-M(\Sigma^+)+M(^{11}C)-M(^{11}B)$$
$$= M(\Sigma^0)-M(\Sigma^+)-M(n)+M(p)+B_p-B_n = 4.57 \text{ MeV}, \tag{2.22}$$

and we have $E_{\Sigma^+} > E_{\Sigma^0}$, i.e., $B_{\Sigma^0} > B_{\Sigma^+}$, which means that it is easier to remove Σ^+ from $^{12}_{\Sigma}C$ than Σ^0. This conclusion is not quite obvious as it depends on the masses of Σ^0, Σ^+, n, and p.

We expect that the system of two coupled equations (2.18) has two solutions for $E-i\Gamma/2$ and Ψ. When $v_t \to 0$, one of them goes over into a state of $^{12}_{\Sigma^+}C$, and we call it the "Σ^+" state. The other one goes over into a state of $^{12}_{\Sigma^0}C$, and we call it the "Σ^0" state. The two states are not orthogonal because $\mathscr{V}_{\Sigma}$ is complex.

3. Procedure of solving equations for ψ_{Σ}

Whereas ψ_{Σ^-} is determined by a single Schrödinger equation, Eq. (2.10), the wave functions ψ_{Σ^+} and ψ_{Σ^0} are determined by system (2.18) of two coupled equations which become uncoupled in the region beyond the range of v_t. Consequently, also in the case of

ψ_{Σ^+} and ψ_{Σ^0} we have to know the asymptotic behaviour of the solution of a single Schrödinger equation. For this reason, we start with a short discussion of a single Schrödinger equation with a complex potential and eigenvalue:

$$\{-(\hbar^2/2\mu)\Delta+\mathscr{V}(r)+V^{\mathrm{C}}(r)\}\psi(\boldsymbol{r}) = \mathscr{E}\psi(\boldsymbol{r}), \tag{3.1}$$

where $\mathscr{V}(r)$ is complex, and

$$\mathscr{E} = E-i\Gamma/2. \tag{3.2}$$

The Coulomb potential V^{C} is assumed to be that of an equivalent uniform charge distribution of radius r_{CH}, which has the same RMS radius as the mass distribution $\varrho(r)$ (normalized to A) of the nuclear core:

$$V^{\mathrm{C}}(r) = \begin{cases}(ZZ'e^2/2r_{\mathrm{CH}})\,[3-(r/r_{\mathrm{CH}})^2] & \text{for} \quad r < r_{\mathrm{CH}},\\ ZZ'e^2/r & \text{for} \quad r > r_{\mathrm{CH}},\end{cases} \tag{3.3}$$

where Z is the number of protons in the nuclear core, and $Z' = t_3^{\Sigma}$.

For a given orbital momentum of Σ, specified by (l, m), we write

$$\psi(\boldsymbol{r}) = [u(r)/r]\,Y_{lm}(\hat{r}), \tag{3.4}$$

and get for $u(r)$ the equation

$$d^2u/dr^2+[(2\mu/\hbar^2)\,(\mathscr{E}-\mathscr{V}-V^{\mathrm{C}})-l(l+1)/r^2]u = 0. \tag{3.5}$$

For $r > \tilde{R}$ = *range of* $\mathscr{V}$, Eq. (3.5) may be written in the form:

$$d^2u/d\varrho^2+[1-2\alpha/\varrho-l(l+1)/\varrho^2]u = 0, \tag{3.6}$$

where

$$\varrho = kr, \qquad \alpha = \mu ZZ'e^2/\hbar k, \tag{3.7}$$

where the complex momentum (in units of $\hbar$) $k = k_{\mathrm{R}}+ik_{\mathrm{I}}$ is defined by $\hbar^2k^2/2\mu = \mathscr{E}$, i.e.,

$$E = \hbar^2(k_{\mathrm{R}}^2-k_{\mathrm{I}}^2)/2\mu, \qquad \Gamma = -2\hbar^2k_{\mathrm{R}}k_{\mathrm{I}}/\mu. \tag{3.8}$$

For exponentially damped solutions (for which we search) $k_{\mathrm{I}} > 0$, and consequently (see the second of Eqs (3.8)) $k_{\mathrm{R}} < 0$. Thus we have

$$k_{\mathrm{R}} = -\sqrt{\mu[E+\sqrt{E^2+(\Gamma/2)^2}]}/\hbar,$$

$$k_{\mathrm{I}} = -\mu\Gamma/2\hbar^2k_{\mathrm{R}} = \sqrt{\mu[-E+\sqrt{E^2+(\Gamma/2)^2}]}/\hbar. \tag{3.9}$$

We follow Fröberg [7], and write u in the form

$$u(\varrho) = \tilde{u}(\varrho)\exp[i(\varrho-\alpha\ln\varrho)] \tag{3.10}$$

and obtain from (3.6)

$$d^2\tilde{u}/d\varrho^2+2i(1-\alpha/\varrho)d\tilde{u}/d\varrho+[i\alpha(1+i\alpha)-l(l+1)]\tilde{u}/\varrho^2 = 0. \tag{3.11}$$

Notice that in the exponent in (3.10), we put $\varrho-\alpha \ln \varrho$ in place of the standard expression $\varrho-\alpha \ln \varrho-l\pi/2+\arg\Gamma(i\alpha+l+1)$. The difference amounts to a r-independent factor which is irrelevant here — it is absorbed by the constants A and B in Eq. (3.16). Notice also that in the present case both ϱ and α are complex.

Expanding $\tilde{u}$ in powers of $1/\varrho$, we get

$$\tilde{u}(\varrho) = 1+[(i\alpha-l)(i\alpha+l+1)/2i]/\varrho$$

$$+[(i\alpha-l)(i\alpha-l+1)(i\alpha+l+1)(i\alpha+l+2)/2!(2i)^2]/\varrho^2+\ldots \tag{3.12}$$

Notice that if $u(\varrho)$, Eq. (3.10), is a solution of Eq. (3.6) then

$$u^*(\varrho) = \tilde{u}^*(\varrho)\exp[-i(\varrho-\alpha\ln\varrho)] \tag{3.13}$$

is also a solution of Eq. (3.6) (the expression for $\tilde{u}^*(\varrho)$ is obtained from the expression for $\tilde{u}(\varrho)$ by changing all factors i on the right hand side of (3.12) into $-i$). Two independent solutions of (3.5) are thus

$$u_1(r) = u(\varrho), \quad u_2(r) = u^*(\varrho). \tag{3.14}$$

They satisfy the Wronskian relation

$$u_1 du_2/dr - u_2 du_1/dr = -2ik. \tag{3.15}$$

For $k_I > 0$, the solution u_1 is exponentially decreasing, and the solution u_2 is exponentially increasing with increasing r.

In the asymptotic region, any solution u of (3.5) is a linear combination of u_1 and u_2,

$$u(r) = Au_1(r)+Bu_2(r), \quad (r > \tilde{R}). \tag{3.16}$$

The relations (obtained with the help of (3.15))

$$A = -(1/2ik)\{u du_2/dr - u_2 du/dr\}_{r>\tilde{R}},$$

$$B = (1/2ik)\{u du_1/dr - u_1 du/dr\}_{r>\tilde{R}}, \tag{3.17}$$

enable us to determine the constants A and B for any solution u.

Let us notice that in the case of $\alpha = 0$ ($V^C = 0$, i.e., $\Sigma = \Sigma^0$) we have

$$u_1 = i^{l+1}h_l^{(1)}(\varrho)\varrho, \quad u_2 = (-i)^{l+1}h_l^{(2)}(\varrho)\varrho. \tag{3.18}$$

Eq. (2.15) for ψ_{Σ^-} is of the form of Eq. (3.1) with $\mathscr{V} = \mathscr{V}_{\Sigma^-}+v_l/2$, $\mu = \mu_{\Sigma^-}$. To determine the complex eigenvalue $\mathscr{E}_{\Sigma^-} = E_{\Sigma^-}-i\Gamma_{\Sigma^-}/2$ and the (regular) eigenfunction ψ_{Σ^-} (or u_{Σ^-}), we assume a value of $\mathscr{E}_{\Sigma^-}$ and solve Eq. (3.5) with the initial condition $u_{\Sigma^-}(r\to 0) \sim r^{l+1}$, and determine the (complex) constant $B = B(\mathscr{E}_{\Sigma^-})$. We repeat it a few times till we get

$$B(\mathscr{E}_{\Sigma^-}) = 0. \tag{3.19}$$

This value of $\mathscr{E}_{\Sigma^-}$, which satisfies the bound state condition, Eq. (3.19), is the desired eigenvalue, with the corresponding u_{Σ^-} being the eigenfunction.

In the case of the coupled functions ψ_{Σ^+}, ψ_{Σ^0}, we write both of them in form (3.4), and for the respective radial functions $u_{\Sigma^+}(r)$, $u_{\Sigma^0}(r)$ we get from Eqs (2.18):

$$d^2u_{\Sigma^+}/dr^2+[(2\mu_{\Sigma^+}/\hbar^2)(\mathscr{E}_{\Sigma^+}-\mathscr{V}_{\Sigma^+}+v_t/2-V^C_{\Sigma^+})-l(l+1)/r^2]u_{\Sigma^+}$$
$$-(\sqrt{2}\,\mu_{\Sigma^+}/\hbar^2)v_t u_{\Sigma^0}=0,$$
$$d^2u_{\Sigma^0}/dr^2+[(2\mu_{\Sigma^0}/\hbar^2)(\mathscr{E}_{\Sigma^0}-\mathscr{V}_{\Sigma^0})-l(l+1)/r^2]u_{\Sigma^0}$$
$$-(\sqrt{2}\,\mu_{\Sigma^0}/\hbar^2)v_t u_{\Sigma^+}=0, \tag{3.20}$$

where $\mathscr{E}_{\Sigma^{+(0)}}=E_{\Sigma^{+(0)}}-i\Gamma/2$. Notice that because of relation (2.21), only one of the two complex energies, e.g. $\mathscr{E}_{\Sigma^+}$, is independent.

To determine the bound state solution of Eqs (3.20) we proceed as follows.

We start by assuming a value of $\mathscr{E}_{\Sigma^+}$, and find two independent solutions, $(u^{\rm I}_{\Sigma^+}, u^{\rm I}_{\Sigma^0})$ and $(u^{\rm II}_{\Sigma^+}, u^{\rm II}_{\Sigma^0})$, which are regular at $r=0$. (In the actual calculations we were finding them by solving numerically Eqs. (3.20) with the following initial conditions at a negligibly small distance $r=\varepsilon$:

$$u^{\rm I}_{\Sigma^+}=du^{\rm I}_{\Sigma^+}/dr=0,\quad u^{\rm I}_{\Sigma^0}=\varepsilon^{l+1},\quad du^{\rm I}_{\Sigma^0}/dr=(l+1)\varepsilon^l,\quad \text{and}$$
$$u^{\rm II}=\varepsilon^{l+1},\quad du^{\rm II}_{\Sigma^0}/dr=(l+1)\varepsilon^l,\quad u^{\rm II}=du^{\rm II}/dr=0.)$$

We write the general solution (regular at the origin) as

$$u_{\Sigma^+}=au^{\rm I}_{\Sigma^+}+bu^{\rm II}_{\Sigma^+},\qquad u_{\Sigma^0}=au^{\rm I}_{\Sigma^0}+bu^{\rm II}_{\Sigma^0}. \tag{3.21}$$

Asymptotically ($r>\tilde{R}$) system of Eqs (3.20) decouples into two independent equations for u_{Σ^+} and u_{Σ^0} of the form of Eq. (3.5), and for $r>\tilde{R}$ we have for $J=$ I, II and $\Sigma=\Sigma^+,\Sigma^0$

$$u^J_\Sigma=A^J_\Sigma u_{\Sigma,1}+B^J_\Sigma u_{\Sigma,2}, \tag{3.22}$$

where $u_{\Sigma,1}$ and $u_{\Sigma,2}$ are the functions u_1 and u_2 of Eq. (3.14) with $k=k_\Sigma$ determined in Eqs (3.9) with $E=E_\Sigma$ and $\mu=\mu_\Sigma$, and with $\alpha=\mu_{\Sigma^+}Ze^2/\hbar^2k_\Sigma$ for $\Sigma=\Sigma^+$ and $\alpha=0$ for $\Sigma=\Sigma^0$. Notice that if we know u^J_Σ, we may easily determine the constants A^J_Σ and B^J_Σ by applying Eqs. (3.17).

By combining Eqs (3.22) and (3.21) we see that asymptotically (for $\Sigma=\Sigma^+,\Sigma^0$)

$$u_\Sigma=(aA^{\rm I}_\Sigma+bA^{\rm II}_\Sigma)u_{\Sigma,1}+(aB^{\rm I}_\Sigma+bB^{\rm II}_\Sigma)u_{\Sigma,2}. \tag{3.23}$$

For a bound state, the exponentially increasing part of the wave functions in both channels Σ^+ and Σ^0 must vanish,

$$aB^{\rm I}_\Sigma+bB^{\rm II}_\Sigma=0\quad\text{for}\quad \Sigma=\Sigma^0,\Sigma^+. \tag{3.24}$$

This system of equations has a nonvanishing solution for a and b only if its (complex) determinant vanishes

$$\mathrm{Det} \equiv B^{\mathrm{I}}_{\Sigma^+} B^{\mathrm{II}}_{\Sigma^0} - B^{\mathrm{II}}_{\Sigma^+} B^{\mathrm{I}}_{\Sigma^0} = 0, \tag{3.25}$$

which is the bound state condition.

Notice that Det depends on $\mathscr{E}_{\Sigma^+}$ (and $\mathscr{E}_{\Sigma^0}$ which is determined by $\mathscr{E}_{\Sigma^+}$), $\mathrm{Det} = \mathrm{Det}(\mathscr{E}_{\Sigma^+})$. In general our starting value of $\mathscr{E}_{\Sigma^+}$ leads to a nonvanishing determinant Det. To find the eigenvalue $\mathscr{E}_{\Sigma^+}$ (and $\mathscr{E}_{\Sigma^0}$), we repeat the whole procedure a few times till we get $\mathrm{Det}(\mathscr{E}_{\Sigma^+}) = 0$.

For $\mathscr{E}_{\Sigma^+}$ being the eigenvalue, we may determine a and b (up to a constant factor) from (3.24). E.g., we may put

$$a = 1, \quad b = -B^{\mathrm{I}}_{\Sigma^+}/B^{\mathrm{II}}_{\Sigma^+} = -B^{\mathrm{I}}_{\Sigma^0}/B^{\mathrm{II}}_{\Sigma^0}. \tag{3.26}$$

With these values of a and b, (3.21) give the final expression for the eigenfunctions u_{Σ^+} and u_{Σ^0}. Their asymptotic form is (see Eq. (3.23)):

$$u_\Sigma \sim A_\Sigma u_{\Sigma,1}, \quad \text{for} \quad \Sigma = \Sigma^+, \Sigma^0, \tag{3.27}$$

where

$$A_\Sigma = A^{\mathrm{I}}_\Sigma - A^{\mathrm{II}}_\Sigma B^{\mathrm{I}}_\Sigma / B^{\mathrm{II}}_\Sigma. \tag{3.28}$$

4. *Charge and isospin distribution in* $^{12}_{\Sigma}C$

We consider the state $\Psi = \Psi_{\Sigma^+} + \Psi_{\Sigma^0}$ of $^{12}_{\Sigma}\mathrm{C}$ with Σ orbital momentum l. The two charge components Ψ_Σ are of the form (Eqs (2.5), (2.17))

$$\Psi_{\Sigma^+} = R\chi^{(+)}\psi_{\Sigma^+}(\boldsymbol{r}) = R\chi^{(+)}[u_{\Sigma^+}(r)/r]Y_{lm}(\hat{r}),$$

$$\Psi_{\Sigma^0} = R\chi^{(0)}\psi_{\Sigma^0}(\boldsymbol{r}) = R\chi^{(0)}[u_{\Sigma^0}(r)/r]Y_{lm}(\hat{r}). \tag{4.1}$$

We denote by $p_{\Sigma^+}(r)$ and $p_{\Sigma^0}(r)$ and probabilities of finding respectively Σ^+ and Σ^0 in the unit interval Δr, and $p(r)$ the probability of finding any hyperon (Σ^+ or Σ^0):

$$p_{\Sigma^+}(r) = |u_{\Sigma^+}(r)|^2, \quad p_{\Sigma^0}(r) = |u_{\Sigma^0}(r)|^2,$$

$$p(r) = p_{\Sigma^+}(r) + p_{\Sigma^0}(r). \tag{4.2}$$

The corresponding total probabilities are:

$$P_{\Sigma^+} = \int dr p_{\Sigma^+}(r), \quad P_{\Sigma^0} = \int dr p_{\Sigma^0}(r) = 1 - P_{\Sigma^+}. \tag{4.3}$$

We assume here that Ψ is normalized, $\langle \Psi | \Psi \rangle = 1$, which implies that $\int dr p(r) = 1$.

To make the relative abundance of Σ^+ and Σ^0 more visible, we define the relative probabilities

$$\pi_{\Sigma^+}(r) = p_{\Sigma^+}(r)/p(r), \quad \pi_{\Sigma^0}(r) = p_{\Sigma^0}(r)/p(r) = 1 - \pi_{\Sigma^+}(r). \tag{4.4}$$

To discuss the isospin T of the state Ψ, we decompose the charge states $\chi^{(+)}$ and $\chi^{(0)}$ into eigenstates χ^T of the total isospin (T, T_3). Whereas two values of $T = 1/2$ and $T = 3/2$ are possible for the assumed value of $T^c = 1/2$, the third component $T_3 = 1/2$ is fixed and dropped in our notation. Since $T_3^c = -\frac{1}{2}$ for ^{11}B and $T_3^c = \frac{1}{2}$ for ^{11}C, we have

$$\chi^{(+)} = \sqrt{1/3}\,\chi^{3/2} - \sqrt{2/3}\,\chi^{1/2},$$

$$\chi^{(0)} = \sqrt{2/3}\,\chi^{3/2} + \sqrt{1/3}\,\chi^{1/2}, \tag{4.5}$$

where we add the isospins in the order $\boldsymbol{T} = \boldsymbol{T}^c + \boldsymbol{t}^\Sigma$ (in the case of the reversed order, the coefficients at $\chi^{1/2}$ would have opposite signs).

After substituting decomposition (4.5) into (4.1), we may write

$$\Psi = \Psi_{1/2} + \Psi_{3/2}, \tag{4.6}$$

where the $T = 1/2$ and $T = 3/2$ components of Ψ are

$$\Psi_T = R\chi^T \psi_T(r) = R\chi^T [u_T(r)/r]\, Y_{lm}(\hat{r}), \tag{4.7}$$

where

$$u_{1/2} = \sqrt{1/3}\, u_{\Sigma^0} - \sqrt{2/3}\, u_{\Sigma^+},$$

$$u_{3/2} = \sqrt{2/3}\, u_{\Sigma^0} + \sqrt{1/3}\, u_{\Sigma^+}. \tag{4.8}$$

For the probability $p_T(r) = |u_T(r)|^2$ of finding in the unit interval Δr our system in the isospin T state, we get

$$p_{3/2}(r) = p_{\Sigma^+}(r)/3 + 2p_{\Sigma^0}(r)/3 + 2\sqrt{2p_{\Sigma^+}(r)p_{\Sigma^0}(r)}\cos\phi(r)/3,$$

$$p_{1/2}(r) = 2p_{\Sigma^+}(r)/3 + p_{\Sigma^0}(r)/3 - 2\sqrt{2p_{\Sigma^+}(r)p_{\Sigma^0}(r)}\cos\phi(r)/3, \tag{4.9}$$

where

$$\phi(r) = \arg\{u_{\Sigma^0}(r)/u_{\Sigma^+}(r)\}. \tag{4.10}$$

The corresponding total probabilities $P_T = \int dr p_T(r)$ are

$$P_{3/2} = P_{\Sigma^+}/3 + 2P_{\Sigma^0}/3 + I,$$

$$P_{1/2} = 2P_{\Sigma^+}/3 + P_{\Sigma^0}/3 - I, \tag{4.11}$$

where

$$I = (2\sqrt{2}/3)\int dr\, \sqrt{p_{\Sigma^+}(r)p_{\Sigma^0}(r)}\cos\phi(r). \tag{4.12}$$

For the relative probabilities $\pi_T(r) = p_T(r)/p(r)$, we get expressions analogous to (4.9) with p's replaced by π's.

It is instructive to discuss the asymptotic behaviour of the probabilities p and π. In the remaining part of this Section, we shall restrict ourselves to such big values of r that

(for $\Sigma = \Sigma^+, \Sigma^0$)

$$u_\Sigma \sim A_\Sigma \exp(ik_\Sigma r). \tag{4.13}$$

Here, we neglect all terms of order equal and higher than $1/r$, and the slowly varying logarithmic term in the exponent of u_{Σ^+} (see Eq. (3.27)). For the real and imaginary parts of k_Σ, $k_{\Sigma R}$ and $k_{\Sigma I}$, we have expressions (3.9) with $E = E_\Sigma$ and $\mu = \mu_\Sigma$. Notice that $E_{\Sigma^+} > E_{\Sigma^0}$, Eq. (2.21), and thus

$$k_{\Sigma^0 R} > k_{\Sigma^+ R}, \quad k_{\Sigma^0 I} > k_{\Sigma^+ I}. \tag{4.14}$$

Eq. (4.13) leads to the following asymptotic form of ϕ, Eq. (4.10):

$$\phi(r) \sim \Delta k_R r + \delta, \quad \delta = \arg(A_{\Sigma^0}/A_{\Sigma^+}), \tag{4.15}$$

where $\Delta k = k_{\Sigma^0} - k_{\Sigma^+}$. Relations (4.14) imply that both the real and the imaginary part of Δk is positive, $\Delta k_R > 0$ and $\Delta k_I > 0$.

Coming back to expressions (4.9) for p_T we see that in the asymptotic region $p_{3/2}(r)$ and $p_{1/2}(r)$ oscillate with the amplitude $2\sqrt{2p_{\Sigma^+}(r)p_{\Sigma^0}(r)}/3$ and with the wave length

$$\lambda = 2\pi/\Delta k_R \tag{4.16}$$

around the values $p_{\Sigma^+}(r)/3 + 2p_{\Sigma^0}(r)/3$ and $2p_{\Sigma^+}(r)/3 + p_{\Sigma^0}(r)/3$, respectively. Since both $p_{\Sigma^+}(r)$ and $p_{\Sigma^0}(r)$ are decreasing (exponentially), the amplitude is also decreasing. Thus the isospin oscillations, i.e., the oscillations in $p_T(r)$, are damped.

More visible and easier to discuss are the oscillations in the relative probabilities $\pi_T(r)$. Eq. (4.13) leads to the following asymptotic expressions for π_Σ and π_T:

$$\pi_{\Sigma^+} = 1 - \pi_{\Sigma^0} \sim 1/[1 + \beta^2 \exp(-2\Delta k_I r)], \tag{4.17}$$

where

$$\beta = |A_{\Sigma^0}/A_{\Sigma^+}|, \text{ and}$$

$$\pi_{3/2}(r) = \pi_{\Sigma^+}(r)/3 + 2\pi_{\Sigma^0}(r)/3 + \mathscr{A}(r)\cos(2\pi r/\lambda + \delta),$$

$$\pi_{1/2}(r) = 2\pi_{\Sigma^+}(r)/3 + \pi_{\Sigma^0}(r)/3 - \mathscr{A}(r)\cos(2\pi r/\lambda + \delta), \tag{4.18}$$

where the amplitude

$$\mathscr{A}(r) = 2\sqrt{2\pi_{\Sigma^+}(r)\pi_{\Sigma^0}(r)}/3 = (2\sqrt{2}/3)\beta \exp(-\Delta k_I r)/[1 + \beta^2 \exp(-2\Delta k_I r)]. \tag{4.19}$$

Expressions (4.17–19) lead to the following behaviour of the relative probabilities for $r \to \infty$:

$$\pi_{\Sigma^+} \to 1, \quad \pi_{\Sigma^0} \to 0, \quad \pi_{3/2} \to 1/3, \quad \pi_{1/2} \to 2/3. \tag{4.20}$$

This is obviously what one should expect when the separation energy of Σ^0 is bigger than that of Σ^+ (Eq. (2.21). Because of the tighter binding of Σ^0 than that of Σ^+, the wave function u_{Σ^+} spreads much further than u_{Σ^0}, and at sufficiently big distances r the state is practically a pure Σ^+ state with the corresponding probabilities $\pi_{3/2} = 1/3$ and $\pi_{1/2} = 2/3$.

Before reaching the limiting values, Eq. (4.20), the relative probabilities π_T oscillate with the wave length λ, Eq. (4.16), and the amplitude $\mathscr{A}$, Eq. (4.18). Since $\mathscr{A}(r)$ decreases (exponentially) with r, the oscillations are damped. For $\pi_{\Sigma^+}(r) = \pi_{\Sigma^0}(r) = 1/2$ (i.e., for $\beta \exp(-\Delta k_I r) = 1$), the amplitude reaches its maximum value $\mathscr{A}_{max} = \sqrt{2}/3 = 0.47$, for which the probabilities π_T oscillate between 0.03 and 0.97. For $(\pi_{\Sigma^+}, \pi_{\Sigma^0}) = (\frac{1}{3}, \frac{2}{3})$ $[(\frac{2}{3}, \frac{1}{3})]$ we have $\mathscr{A} = 4/9$ and $\pi_{3/2}$ oscillates between 1/9 and 1 [0 and 8/9], and $\pi_{1/2}$ oscillates between 0 and 8/9 [1/9 and 1]. These numbers are of course approximate because within the wavelength λ, the probabilities π_Σ change their values (especially for a strong damping, i.e., for a big value of Δk_I).

Instead of working in the charge basis, we could work from the beginning in the isospin basis, Eqs (4.6–7). Instead of Eqs (2.18) we would have the following system of equations for $\psi_{3/2}$ and $\psi_{1/2}$:

$$\{-(\hbar^2/2\mu)\Delta + \mathscr{V} + v_t/2 + V^C_{\Sigma^+}/3\}\psi_{3/2} + (\sqrt{2}/3)(\gamma_0 - V^C_{\Sigma^+})\psi_{1/2}$$
$$= (E_{3/2} - i\Gamma/2)\psi_{3/2},$$
$$\{-(\hbar^2/2\mu)\Delta + \mathscr{V} - v_t + 2V^C_{\Sigma^+}/3\}\psi_{1/2} + (\sqrt{2}/3)(\gamma_0 - V^C_{\Sigma^+})\psi_{3/2}$$
$$= (E_{1/2} - i\Gamma/2)\psi_{1/2}, \tag{4.21}$$

where

$$E_{1/2} = \tfrac{2}{3}E_{\Sigma^+} + \tfrac{1}{3}E_{\Sigma^0}, \qquad E_{3/2} = \tfrac{1}{3}E_{\Sigma^+} + \tfrac{2}{3}E_{\Sigma^0}. \tag{4.22}$$

Notice that $E_{1/2} - E_{3/2} = \gamma_0/3$.

Eqs (4.21) may be easily obtained from Eqs (2.18) by expressing in them ψ_{Σ^+} and ψ_{Σ^0} through $\psi_{3/2}$ and $\psi_{1/2}$, and by properly combining the resulting equations. For the sake of simplicity, we assume here that $\mathscr{V}_{\Sigma^+} = \mathscr{V}_{\Sigma^0} = \mathscr{V}$ and make the approximation $\mu_{\Sigma^+} = \mu_{\Sigma^0} = \mu$.

Although in the isospin basis the coupling due to the Lane potential v_t has disappeared, but in place of it the coupling due to mass differences (the γ_0 term) and the Coulomb interaction appears in Eqs (4.21). In contradistinction to Eqs (2.18) for ψ_{Σ^+} and ψ_{Σ^0}, which decouple into separate equations for ψ_{Σ^+} and ψ_{Σ^0} when $r \to \infty$, Eqs (4.21) for $\psi_{3/2}$ and $\psi_{1/2}$ always remain coupled because of the constant (in space) coupling due to mass differences. The asymptotic form of Eqs (4.21) for $r \to \infty$ is:

$$\{-(\hbar^2/2\mu)\Delta - (E_{3/2} - i\Gamma/2)\}\psi_{3/2} + (\sqrt{2}/3)\gamma_0\psi_{1/2} = 0,$$
$$\{-(\hbar^2/2\mu) - (E_{1/2} - i\Gamma/2)\}\psi_{1/2} + (\sqrt{2}/3)\gamma_0\psi_{1/2} = 0. \tag{4.23}$$

The exponentially decaying solutions $(\psi_{1/2}, \psi_{3/2})$ of Eqs (4.23) are just the linear combinations of $[\exp(ik_{\Sigma^+}r)/r]Y_{lm}$ and $[\exp(ik_{\Sigma^0}r)/r]Y_{lm}$ consistent with expressions (4.8) and (4.13). Thus the coupling due to mass differences, present in the Schrödinger equations for ψ_T (and absent in the equations for ψ_Σ), is responsible for the oscillations of π_T.

Less formally, we may explain the oscillations of π_T by noticing that in the asymptotic region ψ_{Σ^+} and ψ_{Σ^0}, as solutions of the Schrödinger equation, describe the physical states of Σ^+ and Σ^0 (bound to the nuclear core). On the other hand, neither $\psi_{3/2}$ nor $\psi_{1/2}$ are separately even in the asymptotic regions solution of the Schrödinger equation. Thus they do not describe single physical states, but are superpositions of two physical states, ψ_{Σ^+} and ψ_{Σ^0}. Consequently, the probability $|\psi_T|^2$ reveals oscillations caused by the interference between the two components, ψ_{Σ^+} and ψ_{Σ^0}.

The isospin oscillations discussed here are of the same nature as the oscillations of the neutral K meson, known since a long time. A special feature of the isospin oscillations in Σ hypernuclei is that they appear in a bound state. Necessary for their appearance is a nonvanishing imaginary potential (which in Σ hypernuclei describes the absorption due to the $\Sigma N \to \Lambda N$ process).

5. *Numerical results*

In all calculations, we consider only the substitutional states with the configuration $(p_{3/2}, p3/2^{-1})_{\Sigma N}$.

For the density of the nuclear core, the modified harmonic oscillator density is used,

$$\varrho(r) = \varrho(0)\,[1+a(r/c)^2]\exp[-(r/c)^2], \tag{5.1}$$

with $c = 1.69$ fm and $a = 0.811$, determined by electron scattering on ^{11}B [8]. The central density $\varrho(0) = 0.185\ \mathrm{fm}^{-3}$ is determined from the normalization

$$A = \int d\boldsymbol{r}\varrho(r) = \varrho(0)\pi^{3/2}c^3(1+\tfrac{3}{2}a). \tag{5.2}$$

The root mean square radius of distribution (5.1) is

$$\langle r^2\rangle^{1/2} = [c^2(6+15a)/(4+6a)]^{1/2} = 2.42\ \mathrm{fm}. \tag{5.3}$$

To calculate the Coulomb potential, Eq (3.3), we use an equivalent uniform charge distribution of radius

$$r_{\mathrm{CH}} = \sqrt{5/3}\,\langle r^2\rangle^{1/2} = 3.12\ \mathrm{fm}. \tag{5.4}$$

The coupling (Lane) potential $v_t(r)$ is assumed in the form

$$v_t(r) = V_t\varrho(r)/\varrho_0, \tag{5.5}$$

where $\varrho_0 = 0.166\ \mathrm{fm}^{-3}$ is the equilibrium density of nuclear matter. The depth V_t (estimated by Dover and Gal [9] with Model D of the Nijmegen baryon-baryon interaction to be equal 5 MeV) is treated as a free parameter.

The real part of

$$\mathscr{V}_\Sigma = v_\Sigma + iw_\Sigma \tag{5.6}$$

is assumed in the form

$$v_\Sigma(r) = V_\Sigma\varrho(r)/\varrho_0, \tag{5.7}$$

with the depth V_Σ (expected to be about $-(20$–$30)$ MeV) treated as a free parameter.

To calculate w_Σ, we consider a Σ hyperon moving with momentum $\tilde{k}_\Sigma$ in nuclear matter of density ϱ, for which the absorptive potential $w_{NM}(\varrho, \tilde{k}_\Sigma)$ was calculated in [10] (see also [11] and [12]) with the result

$$w_{NM}(\varrho, \tilde{k}_\Sigma) = -\tfrac{1}{2}\varrho(\hbar^2/2\mu_{\Sigma N})\nu\langle Qk_{\Sigma N}\bar{\sigma}\rangle, \tag{5.8}$$

where $\mu_{\Sigma N}$ is the ΣN reduced mass, $k_{\Sigma N}$ is the relative ΣN momentum, $\langle\rangle$ indicates averaging over nucleon Fermi motion, Q denotes the exclusion principle operator for final nucleon states in the ΛN channel, $\nu = \nu(\varrho)$ (with $\nu(\varrho_0) = 0.7$) is the ratio of the effective to the real mass (assumed to be the same for nucleons and the Λ particle), and

$$\bar{\sigma} = \tfrac{1}{2}\{[1+(N-Z)/A]\sigma(\Sigma n)+[1-(N-Z)/A]\sigma(\Sigma p)\} \tag{5.9}$$

is the average total $\Sigma N \to \Lambda N$ conversion cross-section. We have:

$$\bar{\sigma} = \begin{cases} \tfrac{1}{2}[1-(N-Z)/A]\sigma(\Sigma^- p \to \Lambda n) & \text{for} \quad \Sigma^-, \\ \tfrac{1}{2}[1+(N-Z)/A]\sigma(\Sigma^0 n \to \Lambda n)+\tfrac{1}{2}[1-(N-Z)/A]\sigma(\Sigma^0 p \to \Lambda p) & \text{for} \quad \Sigma^0, \\ \tfrac{1}{2}[1+(N-Z)/A]\sigma(\Sigma^+ n \to \Lambda p) & \text{for} \quad \Sigma^+. \end{cases} \tag{5.10}$$

Assuming isospin invariance of the $\Sigma N \to \Lambda N$ transition matrix, and neglecting mass differences, we have

$$\sigma(\Sigma^0 n \to \Lambda n) = \sigma(\Sigma^0 p \to \Lambda p) = \tfrac{1}{2}\sigma(\Sigma^- p \to \Lambda n) = \tfrac{1}{2}\sigma(\Sigma^+ n \to \Lambda p), \tag{5.11}$$

and $\bar{\sigma}$ may be written as

$$\bar{\sigma} = \tfrac{1}{2}[1-2T_3^{NM}t_3^\Sigma/A]\sigma(\Sigma^- p \to \Lambda n), \tag{5.12}$$

where $T_3^{NM} = (Z-N)/2$ is the third component of the total isospin of nuclear matter. For $\sigma(\Sigma^- p \to \Lambda N)$, the parametrization given by Gal et al. [13] is used.

Expression (5.8) differs from the semiclassical one by the appearance of the Q operator and the factor ν, which take care of the most important many-body effects: the exclusion principle and binding effects. The Q operator is a function of the final nucleon momentum (in the conversion process), determined by the energy conservation which involves the s.p. energies of N, Λ, and Σ in nuclear matter. The effective mass approximation is applied to all the s.p. energies as in [10], except for two differences: (a) we drop the unnecessary approximation of the s.p. nucleon potential in the Fermi sea by its average value; (b) we assume that $M^*(\Sigma) = M(\Sigma)$. Point (a) is motivated by the analysis of the absorptive nuclear optical model potential [14]. Point (b) concerns the least known of the three s.p. energies and is introduced to avoid possible complications in finite systems (r — dependent effective mass). The two modifications, (a) and (b), affect only slightly the resulting w_{NM}. For the width of the ground state of Σ in nuclear matter $\Gamma_{NM} = -2w_{NM}(\varrho_0, \tilde{k}_\Sigma = 0)$, we get 5.4 MeV, compared to 5.9 MeV obtained in [10].

The present calculation of w_Σ follows exactly that of [15] in which the binding energies of Σ hypernuclei with $A \lesssim 20$ (without the Lane potential) were calculated in the local density approximation for the absorptive potential, $w_\Sigma = w_{NM}(\varrho, \tilde{k}_\Sigma)$. A detailed comparison with the experimental data was not completely satisfactory. In particular, for the energy of Σ^- in ${}^{12}_{\Sigma^-}$Be, we obtained in [15] $E_{\Sigma^-} \sim 1$ MeV, compared with the experimental results $E_{\Sigma^-} \sim 2$ MeV [1], [2]. Higher positive results for E_Σ could be obtained easily with a stronger absorption w_Σ.

Some points of [15] require a more careful analysis: (*i*) Surface effects, neglected in the local density approximation, may be important. (*ii*) To account for the finite range of the ΣN interaction one should probably fold in the ΣN interaction into nucleon density which would lead to a spatial extension of $\mathscr{V}_\Sigma$ exceeding $\varrho(r)$, and to an increase in E_Σ [16]. (*iii*) The calculation of w_{NM} requires input data (σ, effective masses) which are not accurately known. Furthermore, taking into account a realistic momentum distribution in nuclear matter would also affect w_{NM} (as in the case of the nuclear optical model potential [17]).

Instead of analyze quantitatively all these difficult problems, we introduce a correction factor x (to be fitted to the experimental data), and write w_Σ in the form

$$w_\Sigma = w_\Sigma(r, E_\Sigma) = x w_{NM}(\varrho(r), \tilde{k}_\Sigma(r)), \tag{5.13}$$

where the local momentum

$$\tilde{k}_\Sigma(r) = (M_\Sigma/\mu_\Sigma)\,[2\mu_\Sigma(E_\Sigma - \tilde{V}_\Sigma(r))]^{1/2}, \tag{5.14}$$

where

$$\tilde{V}_\Sigma(r) = v_\Sigma(r) + V_\Sigma^C(r) + v_t(r) T_3^c t_3^\Sigma. \tag{5.15}$$

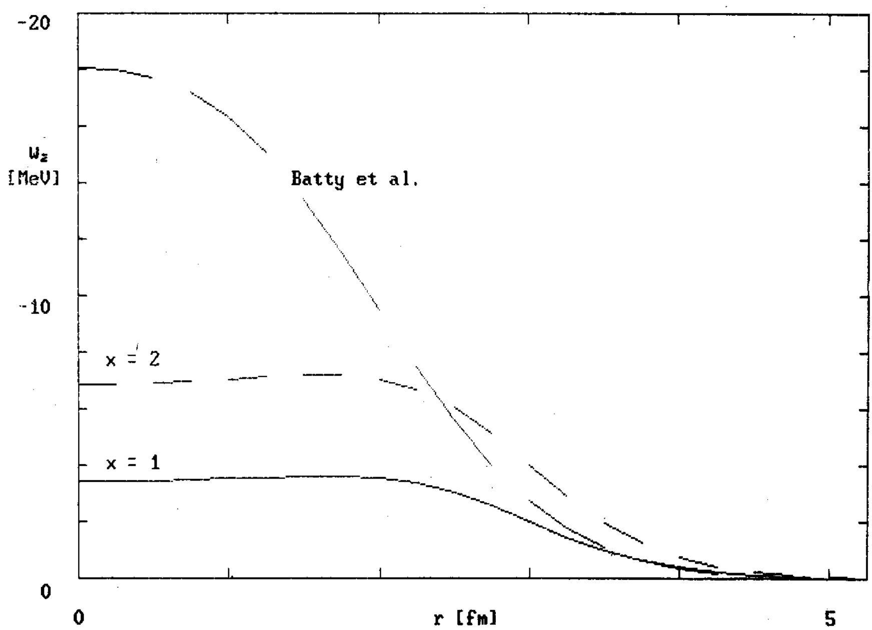

Fig. 1. The absorptive potential w_Σ of Σ^- for $E_{\Sigma^-} = 2$ MeV, $V_{\Sigma^-} = -30$ MeV, $V_t = 0$, compared with the potential of Batty et al. [16] (with $a_I = 0.2$ fm, $R_I = R_R = 0.8$ fm)

The factor M_Σ/μ_Σ takes care of the difference between the relative ΣC momentum and the Σ momentum $\tilde{k}_\Sigma$ in the rest frame of nuclear matter. For points r where $E_\Sigma - \tilde{V}_\Sigma(r) < 0$, we put $\tilde{k}_\Sigma(r) = 0$.

The potential w_Σ in the case of Σ^- with $E_{\Sigma^-} = 2$ MeV, $V_{\Sigma^-} = -30$ MeV, $V_t = 0$, is shown in Fig. 1 for two values of $x = 1, 2$, together with the potential w_Σ of Baty, Gal and Toker [16], adjusted to K^- atomic data. Since these authors assume that $w_\Sigma \sim \varrho$ their absorptive potential inside of the nuclear core is much stronger than our potential in which Pauli blocking and binding effects are taken into account.

5.1. General features of the solutions

To see the dependence of the solutions on the coupling (Lane) potential, in this subsection we fix $V_{\Sigma^-} = V_{\Sigma^0} = V_{\Sigma^+} = -30$ MeV, $x = 1.5$, and show our results as a function of V_t. Following the estimates of [9] and [18], we concentrate our attention on positive values of V_t.

Fig. 2 shows the calculated masses (more precisely mass differences) ΔM, Eqs. (2.14) and (2.20), and the widths Γ of the substitutional p states in ${}^{12}_{\Sigma^-}$Be (which we shall refer to as the Σ^- state) and in ${}^{12}_{\Sigma}$C (here we have two states, "Σ^+" and "Σ^0", defined at the end of Section 2).

Predictably (see Eq. (2.10)), $\Delta M(\Sigma^-)$ increases linearly with V_t. Similarly, for small values of V_t, ΔM("Σ^+") decreases linearly with V_t, and ΔM("Σ^0") remains approximately

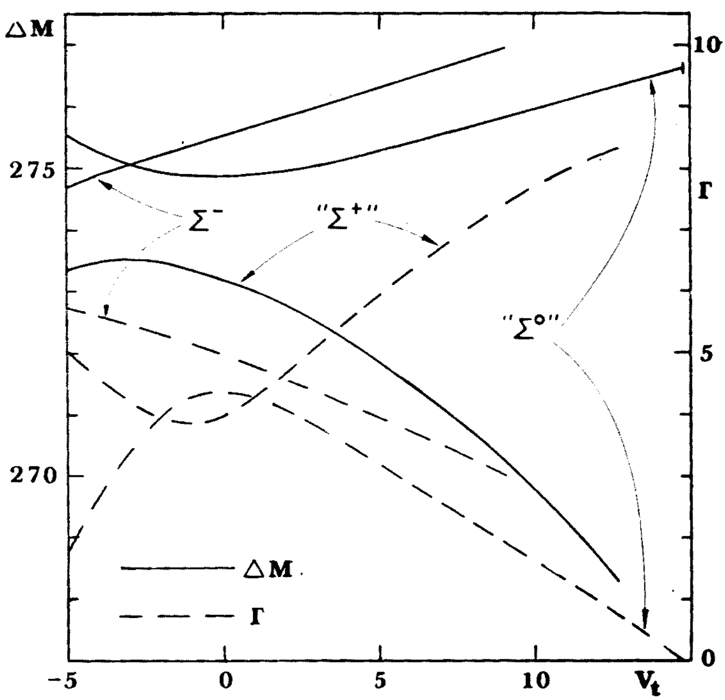

Fig. 2. ΔM and Γ as functions of V_t for $V_\Sigma = -30$ MeV, $x = 1.5$. All numbers are in MeV

unchanged. For bigger values of V_t, ΔM("Σ^+") starts decreasing faster than linearly, and ΔM("Σ^0") starts increasing. The reason for this behaviour of ΔM("Σ^+") and ΔM("Σ^0") is that in the equation for ψ_{Σ^0} (the second of Eqs (2.18)) v_t enters only as the coupling with ψ_{Σ^+}, and thus it affects the energy only in the second order. On the other hand in the equation for ψ_{Σ^+} (the first of Eqs (2.18)) v_t appears not only in the coupling with ψ_{Σ^0} but also as a potential in the Σ^+ channel and thus affects the energy already in the first order.

With the increase (decrease) of the mass, i.e., of the energy, we see in Fig. 2 a corresponding decrease (increase) of the width Γ of the respective state. This behaviour

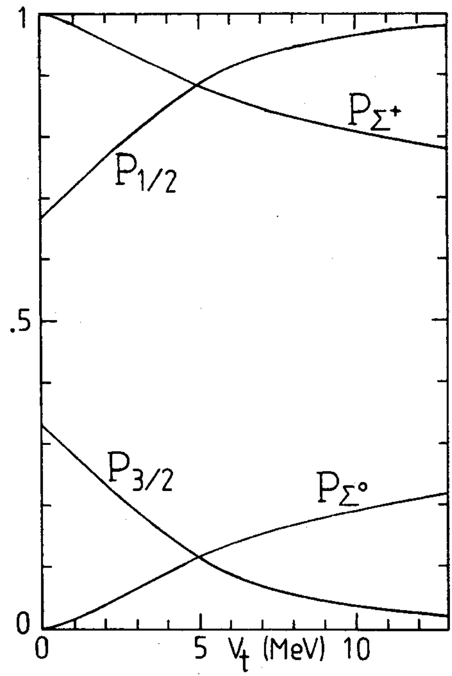

Fig. 3. The probabilities P_Σ and P_T of the "Σ^+" state for $V_\Sigma = -30$ MeV, $x = 1.5$

is easily understood. As the energy of a Σ hypernuclear state increases, the wave function of Σ spreads more and more outside the nuclear core, and because of the small overlap of Σ with nucleons, the decay probability ($\sim\Gamma$) of the state decreases.

In discussing the isospin structure of ${}^{12}_{\Sigma}C$, we start with the "Σ^+" state. The dependence of the probabilities P_Σ and P_T, Eqs (4.3) and (4.11), on V_t is shown in Fig. 3. For $V_t = 0$ the "Σ^+" is a pure ${}^{11}B+\Sigma^+$ state: $P_{\Sigma^+} = 1, P_{\Sigma^0} = 0, P_{1/2} = 2/3, P_{2/3} = 1/3$. With increasing V_t, the admixture of the ${}^{11}C+\Sigma^0$ state increases, i.e., P_{Σ^0} increases and correspondingly P_{Σ^+} decreases. At the same time $P_{1/2}$ increases and $P_{3/2}$ decreases. In the limit of a very strong coupling V_t, we would expect that $P_{1/2} \to 1$, $P_{3/2} \to 0$ and consequently $P_{\Sigma^+} \to 2/3$ and $P_{\Sigma^0} \to 1/3$. For a sufficiently strong coupling ($V_t \gtrsim 5$ MeV) $P_{1/2} > P_{\Sigma^+}$, and the state may be called a "$T = 1/2$" state rather than a "Σ^+" state.

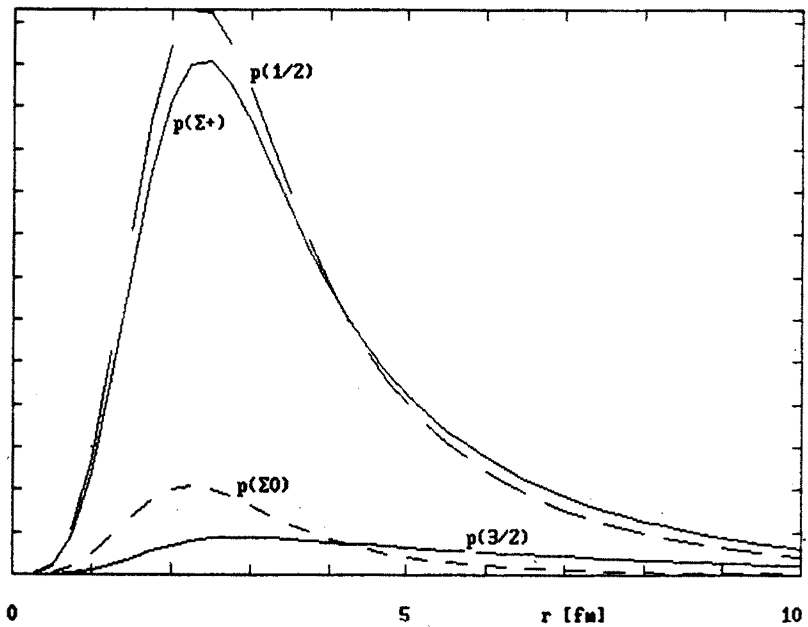

Fig. 4. The distribution of charge and isospin in the "Σ^+" state for $V_\Sigma = -30$ MeV, $x = 1.5$, $V_t = 5$ MeV

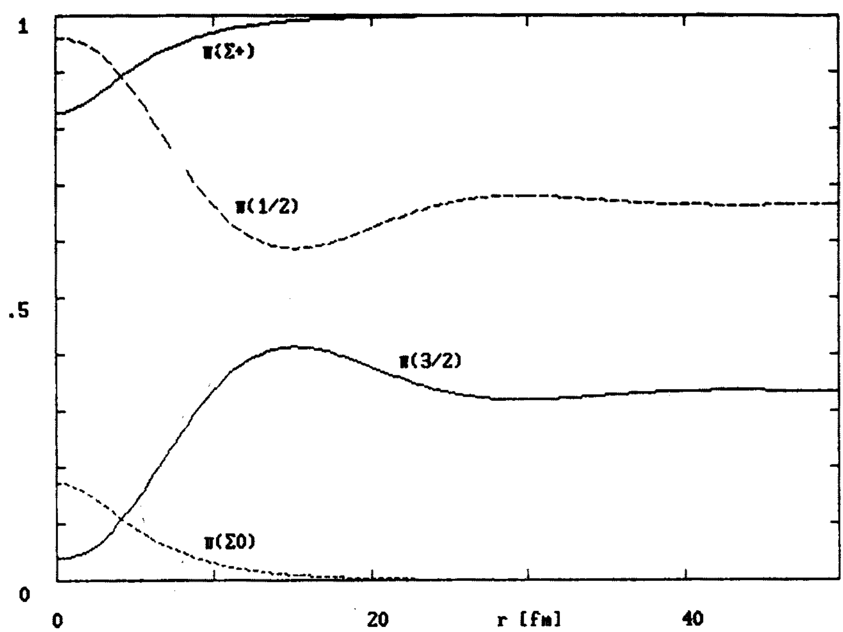

Fig. 5. The relative probabilities π_Σ and π_T for the "Σ^+" state for $V_\Sigma = -30$ MeV, $x = 1.5$, $V_t = 5$ MeV

The radial distribution of the probabilities p_Σ, p_T and π_Σ, π_T are shown in Figs 4 and 5 for the "Σ^+" state at $V_t = 5$ MeV. Within the nuclear core (whose RMS radius is 2.4 fm) the coupling diminishes π_{Σ^+} from its original value 1 at $V_t = 0$ to $\pi_{\Sigma^+} \gtrsim 0.8$. As r increases, π_{Σ^+} approaches the limiting value $\pi_{\Sigma^+}(r \to \infty) = 1$, Eq. (4.20). Of course, $\pi_{\Sigma^0} = 1-\pi_{\Sigma^+}$ decreases from the value $\lesssim 0.2$ at $r = 0$ to the value $\pi_{\Sigma^0}(r \to \infty) = 0$. The relative probability $\pi_{1/2}(r) = 0.97$ at $r = 0$, and with increasing r it decreases to $\pi_{1/2}(r \to \infty) = 2/3$. This decrease is not monotonic but is accompanied by damped oscillations, in agreement

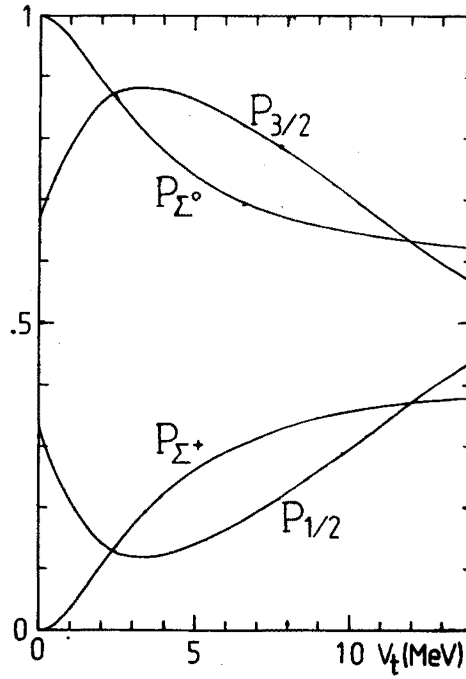

Fig. 6. The same as Fig. 3 but for the "Σ^0" state

with Eq. (4.18). Similarly $\pi_{3/2} = 1-\pi_{1/2}$ is increasing (with oscillations) from $\pi_{3/2}(r = 0) = 0.03$ to $\pi_{3/2}(r \to \infty) = 1/3$. At the value of $V_t = 5$ MeV considered, the resulting probability $P_{\Sigma^+} = 0.88$ and $P_{1/2} = 0.89$.

Now, let us discuss the "Σ^0" state. The dependence of the probabilities P_Σ and P_T on V_t is shown in Fig. 6. Up to the strength of $V_t \sim$ 3–4 MeV, the situation is analogous to that of the "Σ^+" state. For $V_t = 0$ the "Σ^0" state is a pure $^{11}C+\Sigma^0$ state: $P_{\Sigma^0} = 1$, $P_{\Sigma^+} = 0$, $P_{3/2} = 2/3$, $P_{1/2} = 1/3$. When V_t increases, the admixture of the $^{11}B+\Sigma^+$ state increases, i.e., P_{Σ^+} increases and P_{Σ^0} decreases. At the same time $P_{3/2}$ increases and $P_{1/2}$ decreases. However, for $V_t \gtrsim 4$ MeV the probability $P_{3/2}$ starts decreasing again.

One difference between the "Σ^0" state and the "Σ^+" state is that in the "Σ^0" case we cannot increase V_t indefinitely. As V_t increases, the energy (mass) of the "Σ^0" state increases (see Fig. 1). For V_t bigger than a certain critical value (14.8 MeV in the present case)

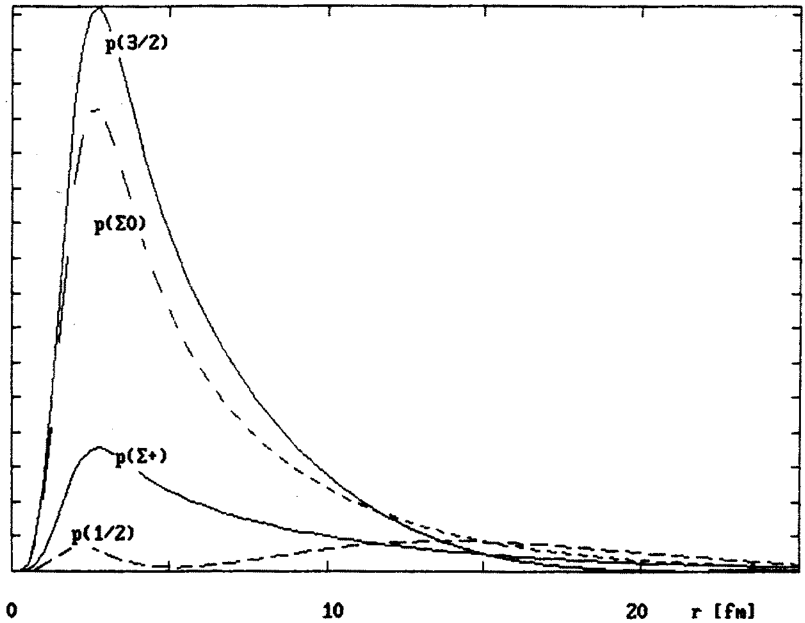

Fig. 7. The same as Fig. 4 but for the "Σ^0" state

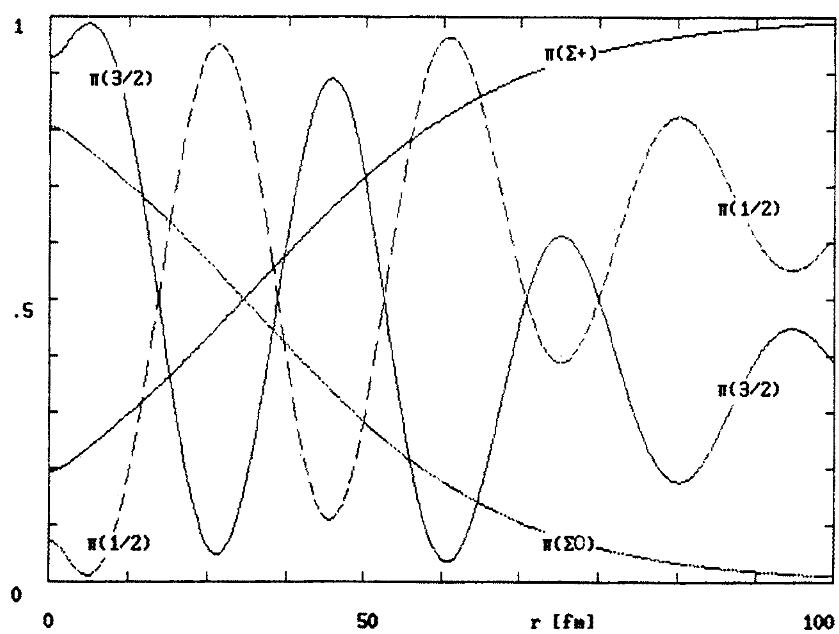

Fig. 8. The same as Fig. 5 but for the "Σ^0" state

no bound, i.e., exponentially decaying state exists. For a strong coupling V_t (but not exceeding the critical value) the system in the "Σ^0" state becomes only loosely bound (in contradistinction to the "Σ^+" state whose binding increases with V_t), and the tail of the wave function spreading far away from the nuclear core becomes important. One may say that in the limit of zero binding, the hyperon spends all the time outside of the nuclear core, and does not feel the coupling potential v_t.

For $V_t = 5$ MeV, i.e., for an intermediate strength of the coupling, the radial distribution of the probabilities p_Σ, p_T, and π_Σ, π_T are shown in Figs 7 and 8. At $r = 0$, the coupling diminishes π_{Σ^0} from its original value 1 for $V_t = 0$ to $\pi_{\Sigma^0}(r = 0) = 0.8$. As r increases π_{Σ^0} decreases towards the limiting value of $\pi_{\Sigma^0}(r \to \infty) = 0$. Of course, $\pi_{\Sigma^+}(r) = 1 - \pi_{\Sigma^0}(r)$ increases from $\pi_{\Sigma^+}(r = 0) = 0.2$ to $\pi_{\Sigma^+}(r \to \infty) = 1$. Consequently, π_{Σ^0} and π_{Σ^+} are of a comparable magnitude in an appreciable range of r's (this does not occur in the "Σ^+"

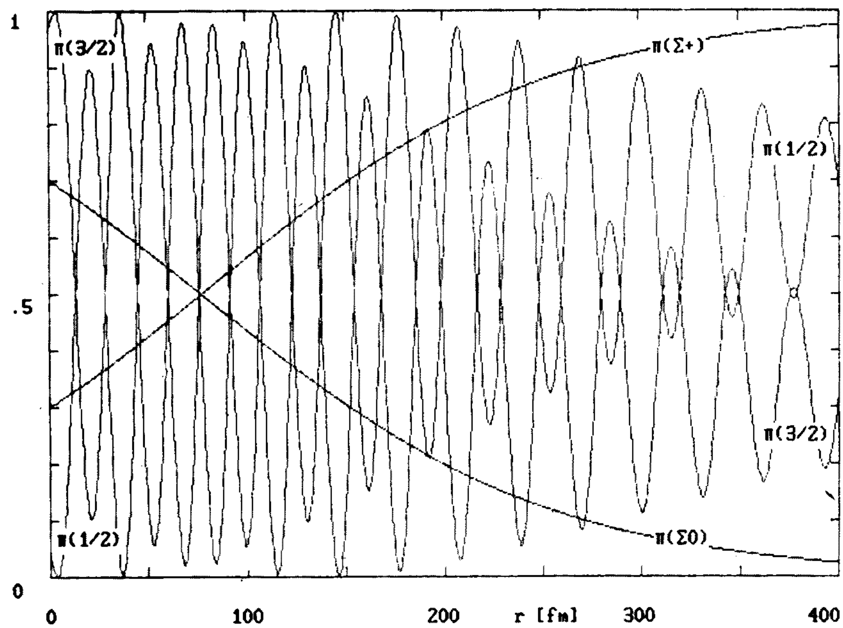

Fig. 9. The same as Fig. 8 but for $V_t = 12.7$ MeV

state), which makes the amplitude of the isospin oscillations big. The probability $\pi_{3/2}$ is close to 1 within the nuclear core. As r increases, $\pi_{3/2}$ must reach eventually the limit of $\pi_{3/2}(r \to \infty) = 1/3$. However, before reaching this limit, it oscillates with only a weak damping and with a big amplitude. The probability $\pi_{1/2} = 1 - \pi_{3/2}$ behaves correspondingly, and in certain regions $\pi_{1/2} \gg \pi_{3/2}$. The resulting total probabilities are: $P_{\Sigma^0} = 0.74$ and $P_{3/2} = 0.86$. One factor which certainly increases P_{Σ^+} (and decreases P_{Σ^0}) is the bigger depth of the attractive real potential $V_\Sigma - V_t/2$ in the Σ^+ channel.

The relative probabilities π_Σ and π_T in the case of a strong coupling $V_t = 12.7$ MeV is shown in Fig. 9. Their behaviour agrees with the discussion in Sect. 4. The resulting

TABLE I

Comparison of the "Σ^+" and "Σ^0" states for $V_t = 5\,(12.7)$ MeV, $V_{\Sigma^+} = V_{\Sigma^0} = -30$ MeV, $x = 1.5$. All energies are in MeV, momenta in fm^{-1}, and λ in fm

	"Σ^+"	"Σ^0"
Γ	6.0(8.3)	3.2(0.7)
E_{Σ^0}	0.2(−2.8)	3.6(4.7)
E_{Σ^+}	4.8(1.7)	8.2(9.3)
$k_{\Sigma^0 R}$	−0.30(−0.25)	−0.46(−0.51)
$k_{\Sigma^+ R}$	−0.53(−0.41)	−0.67(−0.71)
$k_{\Sigma^0 I}$	0.28(0.46)	0.10(0.02)
$k_{\Sigma^+ I}$	0.15(0.28)	0.06(0.01)
Δk_I	0.13(0.18)	0.04(0.01)
Δk_R	0.24(0.17)	0.22(0.20)
λ	26.4(37.5)	29.0(30.7)

total probabilities are: $P_{\Sigma^0} = 0.63$, $P_{3/2} = 0.61$. Consequently, the state is certainly not an approximate $T = 3/2$ state.

The relevant parameters which determine the different properties of the "Σ^+" and "Σ^0" states for $V_t = 5$ and 12.7 MeV are collected in Table I.

5.2. Comparison with CERN data

The experimental data obtained by Bertini et al. [1] in the (K^-, $\pi^\pm$) reactions on ^{12}C were interpreted by the authors in the following way:

(*i*) The peak in the π^+ distribution, observed in the (K^-, π^+) reaction on ^{12}C at $\Delta M = 279$ MeV (with $\Gamma = 4 \pm 1$ MeV) was assigned to the $(p3/2, p3/2^{-1})_{\Sigma^- p}$ configuration of $^{12}_{\Sigma^-}Be$. The corresponding energy $E_{\Sigma^-} = 4.0$ MeV.

(*ii*) The peak at $\Delta M = 270$ MeV, observed in the (K^-, π^-) reaction at $p_{K^-} = 400$ MeV/c and 450 MeV/c was assigned to the $(p3/2, p3/2^{-1})_{\Sigma^+ p}$ configuration of $^{12}_{\Sigma^+}C$. The corresponding energy $E_{\Sigma^+} = 3.0$ MeV.

(*iii*) The (less visible) peak at $\Delta M = 275$ MeV, observed in the (K^-, π^-) reaction at $p_{K^-} = 450$ MeV/c was assigned to the $(p3/2, p3/2^{-1})_{\Sigma^0 n}$ configuration of $^{12}_{\Sigma^0}C$. The corresponding energy $E_{\Sigma^0} = 3.4$ MeV.

It was pointed out in [3] that the interpretation of the peaks (*ii*) and (*iii*) as corresponding to pure charge states implies breaking of charge invariance of the ΣN interaction. If one splits each of the energies E_Σ into nuclear (E^N_Σ) and Coulomb energies, then assuming that the Coulomb energy of Σ^- in $^{12}_{\Sigma^-}Be$ is the same (except for sign) as the (positive) Coulomb energy E_C of Σ^+ in $^{12}_{\Sigma^+}C$, one gets

$$E_{\Sigma^-} = E^N_{\Sigma^-} - E_C, \quad E_{\Sigma^0} = E^N_{\Sigma^0}, \quad E_{\Sigma^+} = E^N_{\Sigma^+} + E_C. \tag{5.16}$$

Relations (5.16) imply that $(E^N_{\Sigma^-} + E^N_{\Sigma^+})/2 = (E_{\Sigma^-} + E_{\Sigma^+})/2 = 3.5$ MeV. Since $E_{\Sigma^0} = 3.4$ MeV, we see that $E_{\Sigma^0} = E^N_{\Sigma^0} \cong (E^N_{\Sigma^-} + E^N_{\Sigma^0})/2$, which suggests that

$$E^N_{\Sigma^-} = E_{\Sigma^0} + E_{CB}, \quad E^N_{\Sigma^+} = E_{\Sigma^0} - E_{CB}, \tag{5.17}$$

where E_{CB} is the charge invariance breaking (CB) part of E^N_Σ.

In the three hypernuclei considered, we have two nuclear cores: ^{11}B with $T_3^c = -1/2$, and ^{11}C with $T_3^c = 1/2$. It appears then that the simplest way of obtaining relations (5.17) is to assume for the depth parameter V_Σ, Eq. (5.7):

$$V_\Sigma = V_0 + V_{CB} T_3^c t_3^\Sigma, \tag{5.18}$$

i.e.,

$$V_{\Sigma^-} = V_0 + V_{CB}/2, \quad V_{\Sigma^0} = V_0, \; V_{\Sigma^+} = V_0 - V_{CB}/2. \tag{5.19}$$

If we wanted to derive the s.p. potential $v_\Sigma(r)$ from the underlying two-body ΣN interaction $v_{\Sigma N}(r_{\Sigma N})$, we could do it by assuming a charge independence breaking part of $v_{\Sigma N}$,

$$\{v_{\Sigma N}\}_{CB} = v_{CB}(r_{\Sigma N}) t_3^N t_3^\Sigma. \tag{5.20}$$

Notice that the breaking of the charge (or isospin) invariance introduced by expressions (5.19) and (5.20) preserves the charge symmetry.

In what follows we shall consider V_{CB} as a phenomenological parameter which we shall try to adjust to the experimental data.

In [3] only the pure charge states were used. However, the full s.p. potential, Eq. (2.3), contains the Lane term which couples the charge states. This coupling was not considered in [3] although it is expected to be important. This point was discussed by Dover, Gal and Millener [18], who applied the mass matrix method [19]. The method involves simplifying approximations (constant matrix elements, real $\mathscr{V}$) and predicts only mass splitting. Here we shall analyze the CERN data [1] by applying the theory outlined in Sect. 2 with the full s.p. potential, Eq. (2.3), which includes the Lane coupling and the CB term, Eqs (5.18–19).

To reproduce the best established state of ${}^{12}_{\Sigma^-}$Be, the peak observed by Bertini et al. [1] at $\Delta M(\Sigma^-)_B = 279$ MeV with $\Gamma(\Sigma^-)_B = 4 \pm 1$ MeV, we find it necessary that

$$V_0 + V_t/2 - V_{CB}/2 = -21.4 \text{ MeV}. \tag{5.21}$$

Furthermore, we fix the value of x at $x = 2$ which leads to $\Gamma(\Sigma^-) = 3.1$ MeV.

The energy of the "Σ^+" and "Σ^0" states is calculated as a function of V_t in two cases: (*I*) with $V_{CB} = 0$; (*II*) with V_{CB} fixed at each value of V_t by the requirement that the calculated energy of the "Σ^+" state agrees with the position of the peak at $\Delta M(\Sigma^+)_B = 270$ MeV.

Results obtained for ΔM are shown as solid curves (case *I*) and broken curves (case *II*) in Fig. 10. In both cases results for $\Delta M(\Sigma^-)$ coincide with the CERN value $\Delta M(\Sigma^-)_B$. In case *II* results for ΔM("Σ^+") coincide with $\Delta M(\Sigma^+)_B$. Numbers below the V_t axis indicate values of V_{CB} obtained in case *II* at the corresponding values of V_t.

The best agreement (ΔM("Σ^0") = 275.4 MeV) with the CERN data is obtained at $V_t = 0$, with a strong $V_{CB} = 17.9$ MeV ($V_{\Sigma^+} = -39.3$ MeV, $V_{\Sigma^0} = -30.4$ MeV, $V_{\Sigma^-} = -21.4$ MeV), in accordance with [3]. Here, the calculated widths are: Γ("Σ^0") = 7.1 MeV, Γ("Σ^+") = 10.2 MeV. At $V_t = 5$ MeV (the value calculated in [9]) the agreement is worse (ΔM("Σ^0") = 275.9 MeV) but still acceptable. Here the required $V_{CB} = 11.7$ MeV ($V_{\Sigma^+} = -35.6$ MeV, $V_{\Sigma^0} = -29.8$ MeV, $V_{\Sigma^-} = -23.9$), and the

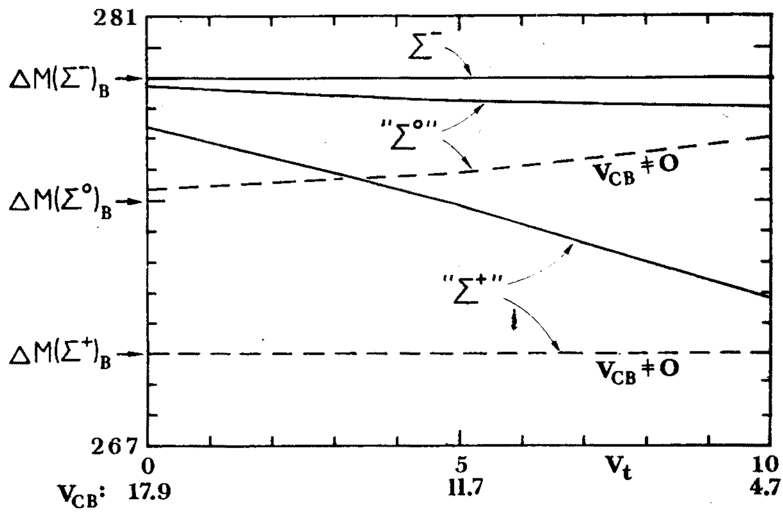

Fig. 10. Results for ΔM ($x = 2$) compared with the CERN data. All numbers are in MeV.

widths are: Γ("Σ^0") = 6.2 MeV, Γ("Σ^+") = 10.4 MeV. Clearly the strength of the Lane coupling and the magnitude of the CB interaction necessary to reproduce the data are interrelated. As is seen in Fig. 10 without the CB interaction one cannot reproduce the peak at 275 MeV seen by Bertini et al. [1] — even for a very strong $V_t \cong 10$ MeV one gets ΔM("Σ^0") $\cong$ 278 MeV.

Let us notice that for $V_t = V_{CB} = 0$, we have $E_{\Sigma^0} - E_{\Sigma^-} = 3.14$ MeV and $E_{\Sigma^+} - E_{\Sigma^0} = 3.15$ MeV. This would suggest that the Coulomb energy $E_C \cong 3.1$ MeV.

5.3. Comparison with KEK data

These data were obtained at KEK with negative kaons stopped in ^{12}C. At the 1985 Brookhaven Symposium, Yamazaki et al. [2] reported the observation of three peaks in both π^+ and π^- distributions, denoted by A, B and C. The positions and widths of the A peaks, interpreted as predominantly the (p3/2, p3/2^{-1})$_{\Sigma N}$ states, are: $\Delta M(\Sigma^-)_A = 277.4 \pm 0.4$ MeV, $\Delta M(^{12}_{\Sigma}\mathrm{C})_A \equiv M(\Sigma^{+/0})_A = 271.2 \pm 0.4$ MeV, with $\Gamma_A = 6 \pm 0.5$ (the same for both peaks). We also quote the position and width of the peak B in the π^- distribution: $\Delta M(\Sigma^{+/0})^0 = 277.6 \pm 0.4$ MeV, $\Gamma_B = 4 \pm 0.5$ MeV. This peak appears to be a candidate for our "Σ^0" state, although in [2] it was tentatively interpreted as the (p1/2, p3/2^{-1})$_{\Sigma N}$ state.

In the interpretation of these data, we proceed as in the case of the CERN data. The lower value of $\Delta M(\Sigma^-)_A$ compared to the CERN result, permits the use of a smaller factor x. Actually, we use two values of x: 2 and 1.5. On the right hand side of condition (5.21) we have now — 26.1 MeV for $x = 2$ and -23.8 MeV for $x = 1.5$, which leads to $\Gamma(\Sigma^-) = 5.6$ MeV for $x = 2$ and $\Gamma(\Sigma^-) = 2.3$ MeV for $x = 1.5$. Our results are shown in Fig. 11 for $x = 2$, and in Fig. 12 for $x = 1.5$.

The $\Delta M(\Sigma^{0/+})_A$ peak (together with the $\Delta M(\Sigma^-)_A$ peak) may be reproduced without breaking the isospin invariance with $V_t = 7.7$ (8.6) MeV for $x = 2(1.5)$. Here, the calculated width Γ("Σ^+") = 9.9(6.7) MeV for $x = 2(1.5)$. To reproduce the $\Delta M(\Sigma^{0/+})_A$ peak

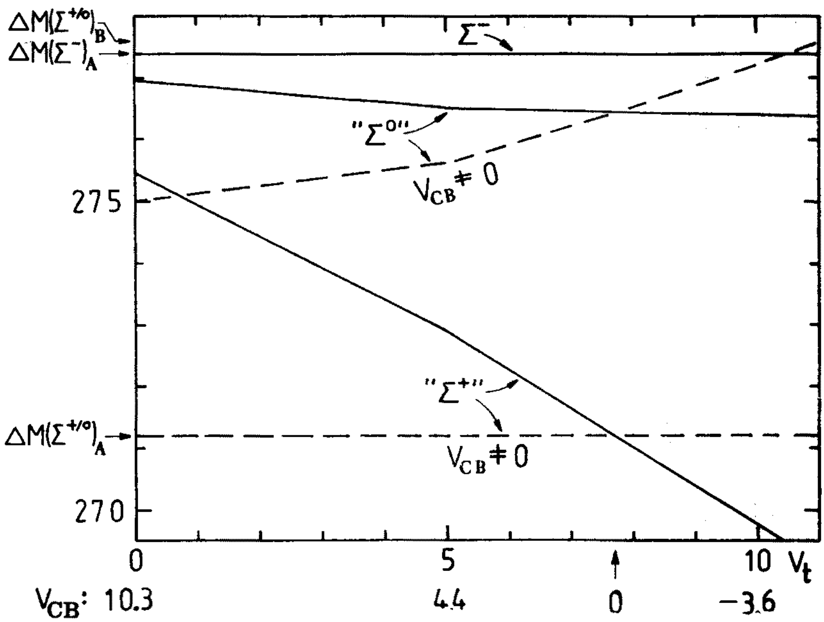

Fig. 11. Results for ΔM ($x = 2$) compared with the KEK data. All numbers are in MeV

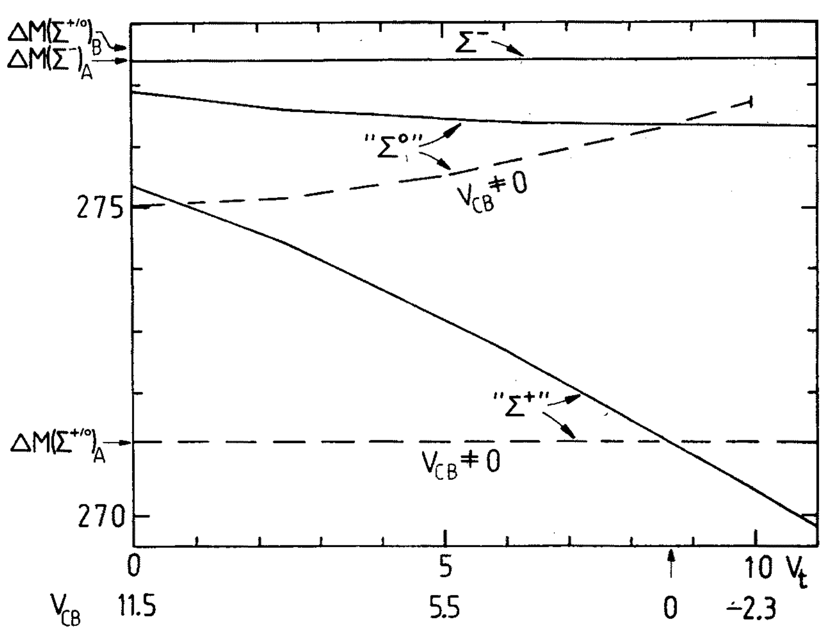

Fig. 12. The same as Fig. 11 but for $x = 1.5$

with $V_t = 5$ MeV, a relatively small V_{CB} is required: $V_{CB} = 4.4(5.5)$ MeV for $x = 2(1.5)$. Here, the calculated width Γ("Σ^+") $= 9.7(6.4)$ MeV for $x = 2(1.5)$.

As far as the "Σ^0" state is concerned, we see in Fig. 11 that for $x = 2$, $V_t = 11$ MeV, and $V_{CB} = -5.6$ MeV, our calculated ΔM("Σ^0") coincides with $\Delta M(\Sigma^{+/0})_B$. Here, the calculated widths are: Γ("Σ^0") $= 2.5$ MeV and Γ("Σ^+") $= 10.4$ MeV. Thus for $x = 2$, we are able to reproduce all the three peaks (except for the width Γ("Σ^+") which appears too big), provided the $\Delta M(\Sigma^{+/0})_B$ peak is interpreted as our "Σ^0" state, and assuming a very strong Lane coupling. On the other hand, Fig. 12 hints at the possibility that the "Σ^0" state might not exist. Namely, with $x = 1.5$ no "Σ^0" bound state exists for $V_t > 10$ MeV, as indicated by the short vertical line at the end of the broken "Σ^0" line. This situation would be consistent with a different interpretation of the $\Delta M(\Sigma^{0/+})_B$ peak, e.g., the configuration $(p1/2, p3/2^{-1})_{\Sigma N}$ suggested in [2].

6. Concluding remarks

The last Section may be summarized as follows:

(*i*) In interpreting experimental data, the Lane potential is important, as was first pointed out by Dover, Gal and Millener [18].

(*ii*) The KEK data appear compatible with a reasonable magnitude of the Lane potential and with a relatively weak breaking of the isospin invariance of the ΣN interaction. The CERN data suggest a serious breaking of isospin invariance and a rather weak Lane potential.

(*iii*) More precise and consistent data are needed for a conclusive analysis.

A novel aspect of the present work are the radial oscillations of the isospin distribution in the substitutional states in $^{12}_{\Sigma}C$. The occurrence of these isospin oscillations affects the probability P_T of finding $^{12}_{\Sigma}C$ in the isospin T state. In particular, the "Σ^0" state is not an approximate $T = 3/2$ state. E.g., in the case considered in Section 5.1 ($V_\Sigma = -30$ MeV, $x = 1.5$) for $V_t = 5$ MeV/12.7 MeV $P_{1/2}$("Σ^+") $= 0.89/0.98$ and $P_{3/2}$("Σ^0") $= 0.86/0.61$, contrary to the values $P_{1/2}$("Σ^+") $= P_{3/2}$("Σ^0") $= 0.99$ suggested in [18]. The problem of a direct experimental detection of the isospin oscillations in $^{12}_{\Sigma}C$ is difficult because they occur outside the nuclear core and a process sensitive to the isospin probably would involve interaction of Σ with the nuclear core. It would be probably easier to detect the effects of the coupling of the two charge states in Σ atoms (e.g., in the ^{3}He-Σ^- atomic system coupled with the ^{3}H-Σ^0 system, or in the $^{11}C+\Sigma^-$ atomic system coupled with the $^{11}B+\Sigma^0$ system) which are presently investigated.

In the present paper, the formation of the Σ^-, "Σ^0" and "Σ^+" states has not been discussed, nevertheless a problem with the formation should be mentioned here.

In all cases considered in this paper, we have $\Delta M(^{12}_{\Sigma}Be) > 275.0$ MeV, and consequently $E_{\Sigma^-} > 0$ (see Eq. (2.14)). (An exception is a small range of negative values of V_t in Fig. 2). This means that our Σ^- states are the "unstable bound states embedded in the continuum" (UBS), first noticed by Stępień-Rudzka and Wycech [20] and discussed in detail by Gal et al. [13]. For the "Σ^0" and "Σ^+" states, we have $E_{\Sigma^+} > 0$ for $\Delta M(^{12}_{\Sigma}C) > 267.0$ MeV, and $E_{\Sigma^0} > 0$ for $\Delta M(^{12}_{\Sigma}C) < 271.6$ MeV (see Eqs (2.20)). Consequently for all our "Σ^0" states considered $E_{\Sigma^+} > 0$ and $E_{\Sigma^0} > 0$, and thus all our "Σ^0" states are of the UBS type. With the "Σ^+" states the situation is such that although we always have $E_{\Sigma^+} > 0$, but some of our "Σ^+" states have $E_{\Sigma^0} < 0$ and are not completely of the UBS type.

Recently, the identification of the UBS with the peaks in the pion distribution observed in the (K^-, π^+) reaction has been criticized by Morimatsu and Yazaki [21]. Their work, however, involves an approximate expression for the formation cross-section in terms of an approximate Green function. Furthermore, their criticism does not apply to bound states, and the properties of the Σ hypernuclear states discussed in the present paper do not change drastically when E_Σ become negative — the situation expected in hypernuclei heavier than $^{12}_{\Sigma}C$.

All the computations of the present work were performed on the Olivetti M21 Personal Computer with an essential help of Dr J. Rożynek.

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