HIGHER-DIMENSIONAL COSMOLOGY AND THE GAUSS-BONNET TERM*

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We study cosmological consequences of the effective higher-dimensional action of gravity as resulting from superstring theories. Non-trivial fix points are found. Collapse of the compact space is seen to be possible for certain initial conditions, but for this to occur the Gauss-Bonnet term does not appear to be of direct importance.

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1. Introduction

Superstring theories [1] are promising candidates for a finite unified quantum theory of gravity and all matter interactions. Most of the interest in superstring theories results from their internal consistency properties, like the absence of anomalies. Conclusive experimental tests are not available and might not be easy to find, however, masses and coupling constants would be calculable by quasiclassical methods if strings would be weakly coupled (which they might not). Large classes of static compactified solutions to the effective low energy field equations are known to exist, Calabi-Yau spaces [2] as well as solutions with torsion [3], all preserving $N = 1$ supersymmetry. It is, however, not clear how to select a particular compactified solution. In any case no completely realistic example has been found so far.

In view of the absence of conclusive low energy tests, cosmological considerations may provide additional information. Various groups [4] have studied the evolution of cosmological scale factors in the framework of the 10-dimensional effective field theory, including higher derivative curvature terms. For the terms quadratic in the curvature the Gauss-Bonnet combination $R^2_{ABCD} - 4R^2_{AB} + R^2$ is usually taken, because it is this combination which leads to a ghostfree gravitation propagator near flat space [5]. String calculations of the low energy effective action are consistent with the appearance of the Gauss-

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-Bonnet combination, but these calculations are in fact not conclusive since neither the R_{AB}^2 nor the R^2 term does actually contribute on linear Einstein shell [6], but can be removed by local field redifinitions. For compactified dynamical solutions these terms are of course relevant. We shall adopt the Gauss-Bonnet prescription in the following. Justification or disproval from string theories would require a better understanding of their offshell formulation.

An expanding 4-dimensional radiation-dominated universe combined with an almost static Calabi-Yau space has been shown to be a stable solution of this model [4]. However, this observation does not provide any clue to the question of why the corresponding scale factors behave so differently, since the 10-dimensional Friedman universe is also a stable solution. There is a well-known solution [7] to this problem in usual Kaluza-Klein theories. Positive curvature of the compact manifold typically drives a universe starting from a Friedman singularity towards a Kasner singularity, i.e. the higher dimensional compact space will collapse after a finite time. This solution is not available in the present case, however, since Calabi-Yau spaces are Ricci-flat. In the following we shall essentially address the question of whether Kasner-type singularities nevertheless appear. We shall restrict our attention to the simplest case with a static dilation, no torsion, and the background gauge potential always identified with the spin connection of the Calabi-Yau manifold. More complex situations will be discussed briefly in Chapter 4.

It has become clear recently that Calabi-Yau spaces are not exact solutions to the tree level string equations of motion [8, 9]. There remains the possibility of systematically constructing solutions by computing higher and higher corrections to Calabi-Yau spaces. If these corrections should turn out to be small Calabi-Yau spaces would still provide a useful starting point for cosmological considerations. At present this is not clear, however. These corrections could very well diverge in such a way that the corresponding series could not be used to define exact solutions. In the following we assume that flat Calabi-Yau manifolds do provide a good approximation to solutions of the string equations of motion.

2. *The model*

The bosonic part of the effective action derived from the heterotic superstring theory [14] is given by

$$S = \int d^{10}x \sqrt{-g} \frac{1}{2\kappa^2} \Big\{ -R - \tfrac{1}{2}\, \partial_A D \partial^A D - \tfrac{1}{12}\, e^{-D} H_{ABC} H^{ABC} + \frac{\varkappa'}{2} e^{-D/2} (-\operatorname{tr} F_{AB} F^{AB} + R_{ABCD} R^{ABCD} + a R_{AB} R^{AB} + b R^2) \Big\} . \tag{1}$$

The indices $A, B, \ldots = 0, 1, \ldots, 9$ label the 10-dimensional space time with the metric g_{AB}, R_{ABCD}, R_{AB} and R are the corresponding curvature tensor, Ricci tensor and curvature scalar. D is the dilaton field, whereas F_{AB} and H_{ABC} represent the field strength of the Yang-Mills and antisymmetric tensor field, respectively. The 10-dimensional gravitational constant is denoted by $\kappa^2 = 8\pi G_{10}$, the inverse string tension by $\varkappa'$. We only consider the torsion-free case $H_{ABC} = 0$, and set $a = -4$, $b = 1$ (Gauss-Bonnet) as discussed in the introduction. Furthermore, we search for solutions with a constant field $D = D_0$.

Under these assumptions the action simplifies to

$$S = \int d^{10}x \sqrt{-g} : \frac{1}{2K^2} \{-R + \varkappa(-\mathrm{tr}\, F_{AB}F^{AB} + R_{ABCD}R^{ABCD} - 4R_{AB}R^{AB} + R^2)\} \tag{2}$$

with $\varkappa = \varkappa' \cdot \exp(-D_0/2)/2$. We further identify the Yang-Mills vector potential with the spin connection of the 6-dimensional compact manifold with metric $\tilde{g}_{\alpha\beta}$ ($\alpha, \beta = 4, \ldots, 9$). In particular, this leads to the relation

$$\mathrm{tr}\,(F^{\lambda}{}_{\alpha} F_{\lambda\beta}) = \tilde{R}_{\alpha\sigma\lambda\varrho} \tilde{R}_{\beta}{}^{\sigma\pi\varrho} \tag{3}$$

with $\tilde{R}_{\alpha\beta\sigma\nu}$ the Riemann tensor constructed from the metric $\tilde{g}_{\alpha\beta}$. Under these conditions Calabi-Yau spaces yield static solutions to the vacuum equations of motion. $\tilde{g}_{\alpha\beta}$ is not explicitly known because of the complicated structure of Calabi-Yau spaces. Fortunately, we do not need any detailed information about $\tilde{g}_{\alpha\beta}$.

We now consider an incoherent matter contribution (e.g. thermal radiation) described by the energy-momentum tensor

$$T_{AB} = (p+g)u_A u_B + p g_{AB} \tag{4}$$

as a perfect fluid. Solutions to the modified Einstein equations will now be time-dependent, and may be found by the usual metric ansatz

$$g_{AB} = (-1, a_1^2(t)\tilde{g}_{ij}, a_2^2(t)\tilde{g}_{\alpha\beta}) \tag{5}$$

with $\tilde{g}_{ij} = \delta_{ij}$. These equations of motion derived from the effective action (2) are given in the appendix. Using the Ricci-flatness $\tilde{R}_{\alpha\beta} = 0$ of the Calabi-Yau spaces, and with space dimensions $d_1 = 3$ and $d_2 = 6$, resp., we find the following equations

$$3 + 15u + 18u^2 - w(72u + 540u^2 + 720u^3 + 180u^4) = \varrho/y_1^2 \tag{6a}$$

$$\begin{aligned} 3 + 12u + 21u^2 + \dot{y}_1/y_1^2[2 - w(48u + 120u^2)] + \dot{y}_2/y_1^2[6 - w(24 + 240u \\ + 240u^2)] - w(48u + 324u^2 + 720u^3 + 420u^4) = -p_1/y_1^2 \end{aligned} \tag{6b}$$

$$\begin{aligned} 6 + 15u + 15u^2 + \dot{y}_1/y_1^2[3 - w(12 + 120u + 120u^2)] + \dot{y}_2/y_1^2[5 - w(60 + 240u \\ + 120u^2)] - w(12 + 180u + 540u^2 + 600u^3 + 180u^4) = -p_2/y_1^2, \end{aligned} \tag{6c}$$

where we have defined $y_i = \dot{a}_i/a_i$, $u = y_2/y_1$ and $w = \kappa y_1^2$. This system of equations contains one constraint equation (6a) and two first order non-linear differential equations for y_1, y_2. They will be discussed further in the next chapter.

3. *Various solutions*

There are a number of interesting questions to be asked at this point, all concerning the influence of the Gauss-Bonnet corrections. The first one is whether it would destabilize previous solutions. Another one is the possible appearance of qualitatively new asymptotic solutions. We first have to decide which equation of state $p_i(\varrho)$ to use. Since our approach is essentially perturbative in $\varkappa y_1^2$ it is reliable only for temperature well below the Planck mass. String contributions to the equation of state are exponentially small in this case, such that only massless particles are relevant. The equation of state will therefore be either

$$p_1 = p_2 = \varrho/9 \tag{7a}$$

or

$$p_1 = \varrho/3, \qquad p_2 = 0 \tag{7b}$$

depending on whether the size of the compact manifold is large or small compared to the inverse temperature. We start with a discussion of the symmetric case (7a). String theory fixes the sign of $\varkappa$ to be positive. In order to get rid of any explicit dependence on the string tension $\varkappa$ we redefine $\tilde{y}_i \equiv \sqrt{\varkappa}\, y_i$ and $\tilde{t} \equiv t/\sqrt{\varkappa}$. The flow diagram for trajectories in the $\tilde{y}_1$, $\tilde{y}_2$ plane is shown in Fig. 1. Since we are interested in an expanding 3-universe

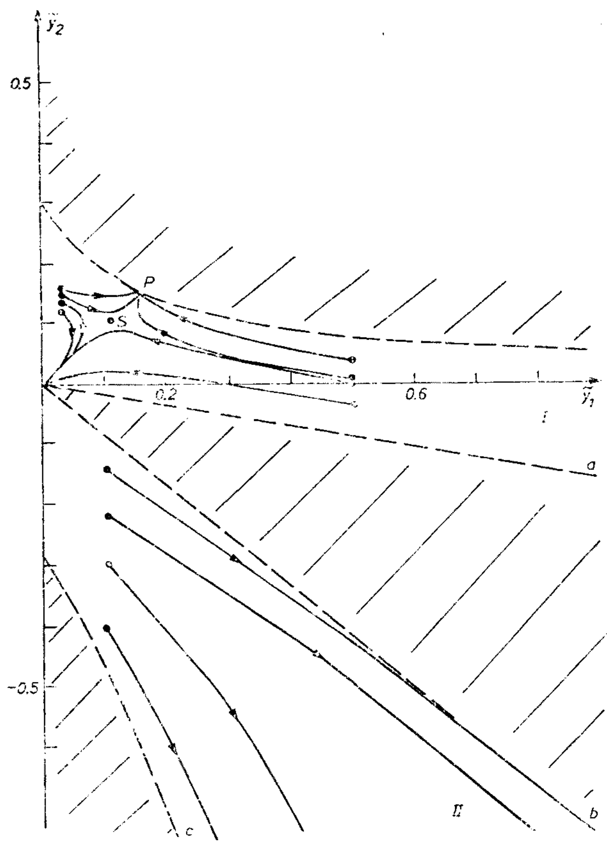

Fig. 1. $\tilde{y}_1 \geqslant 0$ — half plane ($\varkappa > 0$) for the equation of state (7a). We show groups of trajectories with equa initial data $\tilde{y}_1^0$ but different $\tilde{y}_2^0$. P is the attractive fix point, S the isolated singularity. The asymptotics of a, b and c are given by: a: $\tilde{y}_2 = -0.17\,\tilde{y}_1$; b: $\tilde{y}_2 = -0.76\,\tilde{y}_1$; c: $\tilde{y}_2 = -3.06\,\tilde{y}_1$. The shadowed areas are the forbidden regions as determined by the constraint (6a) with zero r.h.s.

we consider only the half-plane $\tilde{y}_1 > 0$. There are two allowed regions (I, II), i.e. regions with positive energy density. The boundaries are found from the constraint equation (6a) with zero r.h.s.. In region (I) all trajectories are attracted by either of the fix points $(\tilde{y}_1, \tilde{y}_2) = (0, 0)$ or $(0.15, 0.15)$. Between them, there is an isolated singularity at $(\tilde{y}_1, \tilde{y}_2) = (0.109, 0.109)$. The first fix point leads to a power-like symmetric expansion of all space dimensions. Higher curvature terms soon become irrelevant. The second one could be quite interesting since it leads to a de Sitter-like expansion at zero matter density ϱ. The existence of this second fix point is related to the observation of Boulware and Deser [10] that a pure gravity theory with Gauss-Bonnet term (but no gauge field condensate) has flat as well as anti de Sitter vacuum solutions. The gauge field condensate changes this into flat and de Sitter solutions. The de Sitter fix point could provide a useful source of inflation if an efficient exit mechanism could be found. Quantum instability of the de Sitter space [11] has been conjectured in this context but this question is not settled. The fact that terms quadratic in the curvature may simulate an effective cosmological constant has also been emphasized by Starobinsky [12]. We note in passing that for the other sign of $\varkappa (\varkappa < 0)$ one finds a fix point at $\tilde{y}_1 = 0.87$, $\tilde{y}_2 = -0.14$. This would correspond to

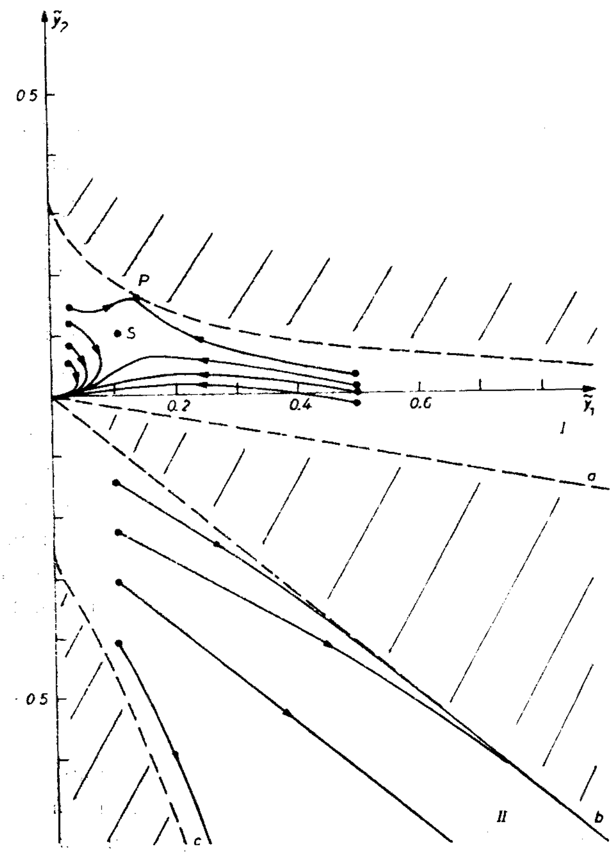

Fig. 2. The corresponding trajectories (cf. Fig. 1) for the equation of state (7b)

a universe with exponentially expanding 3-space, and exponentially collapsing compact 6-space. However, this model is unphysical in leading to naked singularities, and from the point of view of string theories $\varkappa < 0$ is the wrong sign.

We turn the discussion of region (II) of Fig. 1. With initial conditions in this region, all trajectories flow towards infinity. The situation is qualitatively the same as for the Kasner singularity found with the Einstein term alone, a will shrink to zero within a finite time. One could hope that string corrections would stabilize a_2 against collapse, but this cannot be decided at present.

In Fig. 2 we present the corresponding flow diagram for the effective 3-dimensional equation of state (7b). The allowed regions are the same as in the case (7a). The axis $\tilde{y}_2 = 0$ is a stable solution of the equations of motion for the case $\varkappa = 0$. The Gauss-Bonnet correction, however, drives the trajectory with initial data $y_2 = 0$ into the $\tilde{y}_2 > 0$ region towards the fix point at $\tilde{y}_1 = 0$, $\tilde{y}_2 = 0$. Fig. 2 shows that the axis $\tilde{y}_2 = 0$, nevertheless, attracts the trajectories with the appropriate initial conditions (region I). The region of initial data for trajectories tending to (0, 0) is larger than for the case of the equation of state (7a) (cf. Fig. 1). The qualitative behaviour of the scale factors in region II is unchanged.

4. *Summary and outlook*

Our discussion has been limited to the simplest possible situation, i.e. no torsion, a constant dilaton field and a Yang-Mills background field strictly identified with the spin connection of the Calabi-Yau manifold. Under these restrictions we have found that 'compactified' solutions ($y_2 = 0$) are stable against small perturbations (in agreement with previous results [4]) but not globally. If the perturbation is large enough the solution will approach symmetric de Sitter expansion. This is the new qualitative feature introduced by the Gauss-Bonnet term. Other solutions will in general appear if further higher derivative terms will be included in the effective action. Kasner-type solutions of the Einstein equations are only slightly modified by the Gauss-Bonnet term.

The major unanswered question is how the initial conditions could be realized such that an effective 3-dimensional universe would result from the following evolution. This can only be studied by going closer to the critical temperature, including string effects in the equation of state, and considering corrections to Ricci-flat compact spaces. This last point should also solve a problem raised by Weiss [13] that solutions with static Ricci--flat 6-space become unstable when the universe becomes dust-dominated. Interesting effects could also be caused by fermion condensates and gauge field condensate temporarily not at their static values. Work along this line is in progress.

APPENDIX

We give some useful formulae necessary to obtain the equations of motion of the scale factors $a_1(t)$ and $a_2(t)$. The general $D = d_1+d_2+1$-dimensional case is considered with the metric ansatz (3). The two subspaces ($\tilde{g}_{ij}$, $\tilde{g}_{\alpha\beta}$) are not specified further. Latin indices run from 1, 2, ... to d_1, greek indices from d_1+1, ... to $D-1$. The equations of motion derived from (2) are

$$(R_{AB}-\tfrac{1}{2}\,g_{AB}R)+\varkappa(\tfrac{1}{2}\,g_{AB}(R_{CDEF}R^{CDEF}-4R_{CD}R^{CD}+R^2)$$
$$-2R_{ACDE}R_B{}^{CDE}+4R_{AC}R^C{}_B+4R^{CD}R_{CADB}-2RR_{AB}) = -8\pi G_{10}T_{AB}. \tag{A.1}$$

The convention $R^A{}_{BCD} = -\Gamma^A{}_{BD,C}+\ \dots,\ R_{AB} = R^C{}_{ACB},\ R = R^A{}_A$ is used. Inserting the metric (3) into the curvature tensor terms we get non-zero components:

$$R_{ABCD}\colon\quad R_{titj} = \ddot a_1 a_1 \tilde g_{ij}$$
$$R_{t\alpha t\beta} = \ddot a_2 a_2 \tilde g_{\alpha\beta}$$
$$R_{ijkl} = a_1^2\tilde R_{ijkl}-a_1^2\dot a_1^2(\tilde g_{ki}\tilde g_{jl}-\tilde g_{il}\tilde g_{jk})$$
$$R_{\alpha\beta\gamma\delta} = a_2^2\tilde R_{\alpha\beta\gamma\delta}-a_2^2\dot a_2^2(\tilde g_{\alpha\gamma}\tilde g_{\beta\delta}-\tilde g_{\alpha\delta}\tilde g_{\beta\gamma})$$
$$R_{\alpha i\beta j} = -\dot a_1\dot a_2 a_1 a_2\tilde g_{\alpha\beta}\tilde g_{ij} \tag{A.2}$$

$$R_{AB}\colon\quad R_{tt} = d_1\ddot a_1/a_1+d_2\ddot a_2/a_2$$
$$R_{ij} = \tilde R_{ij}-((d_1-1)\dot a_1^2+\ddot a_1 a_1+d_2\dot a_1\dot a_2 a_1/a_2)\tilde g_{ij}$$
$$R_{\alpha\beta} = \tilde R_{\alpha\beta}-((d_2-1)\dot a_2^2+\ddot a_2 a_2+d_1\dot a_1\dot a_2 a_2/a_1)\tilde g_{\alpha\beta} \tag{A.3}$$

$$R\colon\quad R = -(d_1\ddot a_1/a_1+d_2\ddot a_2/a_2)$$
$$+(\tilde R^{(1)}-d_1((d_1-1)\dot a_1^2+\ddot a_1 a_1+d_2\dot a_1\dot a_2 a_1/a_2))/a_1^2$$
$$+(\tilde R^{(2)}-d_2((d_2-1)\dot a_2^2+\ddot a_2 a_2+d_1\dot a_1\dot a_2 a_2/a_1))/a_2^2. \tag{A.4}$$

All tensors with tilde "$\sim$" are built from the metric $\tilde g_{ij}$, $\tilde g_{\alpha\beta}$ respectively, $\tilde R^{(i)}$ is the curvature scalar of the d_i-dimensional subspace and $\dot a_i = \frac{d}{dt}\,a_i(t)$. Combined with the energy-momentum tensor we find the equations of motion:

$A = B = t$:

$$\tfrac{1}{2}\,\dot a_1^2/a_1^2 d_1(d_1-1)+\tfrac{1}{2}\,\dot a_2^2/a_2^2 d_2(d_2-1)+\dot a_1\dot a_2(/a_1a_2)d_1d_2-\tfrac{1}{2}\,\tilde R^{(1)}/a_1^2-\tfrac{1}{2}\,\tilde R^{(2)}/a_2^2$$
$$-\varkappa\{\dot a_1^4/a_1^4 d_1(-3+\tfrac{11}{2}\,d_1-3d_1^2+\tfrac{1}{2}\,d_1^3)$$
$$+\dot a_1^3\dot a_2/(a_1^3a_2)d_1(4d_2-6d_1d_2+2d_2d_1^2)$$
$$+\dot a_1^2\dot a_2^2/(a_1^2a_2^2)3d_1d_2(1-d_1-d_2+d_1d_2)$$
$$+\dot a_1\dot a_2^3/(a_1a_2^3)d_2(4d_1-6d_1d_2+2d_2^2d_1)$$
$$+\dot a_2^4/a_2^4 d_2(-3+\tfrac{11}{2}\,d_2-3d_2^2+\tfrac{1}{2}\,d_2^3)$$
$$+\tfrac{1}{2}\,\tilde R_{\alpha\beta\gamma\delta}\tilde R^{\alpha\beta\gamma\delta}/a_2^4+\tfrac{1}{2}\,\tilde R_{ijkl}\tilde R^{ijkl}/a_1^4$$

$$-2\tilde{R}_{\gamma\beta}\tilde{R}^{\alpha\beta}/a_2^4-2\tilde{R}_{ij}\tilde{R}^{ij}/a_1^4+\dot{a}_1^2\tilde{R}^{(1)}/a_1^4(-6+5d_1-d_1^2)$$

$$+\dot{a}_2^2\tilde{R}^{(2)}/a_2^4(-6+5d_2-d_2^2)$$

$$+\dot{a}_1\dot{a}_2/(a_1a_2)\,(\tilde{R}^{(1)}/a_1^2(4d_2-2d_1d_2)+\tilde{R}^{(2)}/a_2^2(4d_1-2d_1d_2))$$

$$+\dot{a}_2^2\tilde{R}^{(1)}/(a_1^2a_2^2)\,(d_2-d_2^2)+\dot{a}_1^2\tilde{R}^{(2)}/(a_1^2a_2^2)\,(d_1-d_1^2)$$

$$+\tfrac{1}{2}\,\tilde{R}^{(1)2}/a_1^4+\tfrac{1}{2}\,\tilde{R}^{(2)2}/a_2^4+\tilde{R}^{(1)}\tilde{R}^{(1)}/(a_1^2a_2^2)\}\;=\;8\pi G_{10}(\varrho-T_{tt}^F) \qquad \text{(A.5)}$$

$A\,=\,i,\;B\,=\,j$:

$$\tilde{g}_{ij}(\ddot{a}_1/a_1\cdot(d_1-1)+\dot{a}_1^2/a_1^2(1-\tfrac{3}{2}\,d_1+\tfrac{1}{2}\,d_1^2)+\dot{a}_1\dot{a}_2/(a_1a_2)\,(-d_2+d_1d_2)$$

$$+\ddot{a}_2/a_2d_2+\dot{a}_2^2/a_2^2(-\tfrac{1}{2}\,d_2+\tfrac{1}{2}\,d_2^2)-\tfrac{1}{2}\,\tilde{R}^{(1)}/a_1^2-\tfrac{1}{2}\,\tilde{R}^{(2)}/a_2^2)+\tilde{R}_{ij}$$

$$-\varkappa\{\ddot{a}_1/a_1[\tilde{g}_{ij}(\dot{a}_1^2/a_1^2(12-22d_1+12d_1^2-2d_1^3)$$

$$+\dot{a}_2^2/a_2^2(-2d_2+2d_1d_2+2d_2^2-2d_1d_2^2)+\dot{a}_1\dot{a}_2/(a_1a_2)\,(-8d_2+12d_1d_2-4d_1^2d_2)$$

$$+\tilde{R}^{(1)}/a_1^2(-6+d_1)+\tilde{R}^{(2)}/a_2^2(-2+2d_1))+\tilde{R}_{ij}(4-4d_1)]$$

$$+\ddot{a}_2/a_2[\tilde{g}_{ij}(\dot{a}_1^2/a_1^2(-4d_2+6d_1d_2-2d_1^2d_2)$$

$$+\dot{a}_2^2/a_2^2(-4d_2+6d_2^2-2d_2^3)+\dot{a}_1\dot{a}_2/(a_1a_2)\,(-4d_2+4d_1d_2+4d_2^2-4d_1d_2^2)$$

$$+2\tilde{R}^{(1)}/a_1^2+\tilde{R}^{(2)}/a_2^2(-4+2d_2))-4d_2\tilde{R}_{ij}]$$

$$+\tilde{g}_{ij}[\dot{a}_1^4/a_1^4(-12+25d_1-\tfrac{35}{2}\,d_1^2+5d_1^3-\tfrac{1}{2}\,d_1^4)$$

$$+\dot{a}_1^3a_2/(a_1^3a_2)\,(12d_2-22d_1d_2+12d_1^2d_2-2d_1^3d_2)$$

$$+\dot{a}_1^2\dot{a}_2^2/(a_1^2a_2^2)\,(6d_2-9d_1d_2+3d_1^2d_2-6d_2^2+9d_1d_2^2-3d_1^2d_2^2)$$

$$+\dot{a}_1\dot{a}_2^3/(a_1a_2^3)\,(4d_2-4d_1d_2-6d_2^2+6d_1d_2^2+2d_2^3-2d_1d_2^3)$$

$$+\dot{a}_2^4/a_2^4(3d_2-\tfrac{11}{2}\,d_2^2+3d_2^3-\tfrac{1}{2}\,d_2^4)$$

$$+\dot{a}_1^2\tilde{R}^{(1)}/a_1^4(8-7d_1+d_1^2)+\dot{a}_2^2\tilde{R}^{(2)}/a_2^4(6-5d_2+d_2^2)$$

$$+\dot{a}_1^2\tilde{R}^{(2)}/(a_1^2a_2^2)\,(2-3d_1+d_1^2)+\dot{a}_2^2\tilde{R}^{(1)}/(a_1^2a_2^2)\,(-d_2+d_2^2)$$

$$-\tfrac{1}{2}\,\tilde{R}_{\alpha\beta\sigma\lambda}\tilde{R}^{\alpha\beta\sigma\lambda}/a_2^4-\tfrac{1}{2}\,\tilde{R}_{ijkl}\tilde{R}^{ijkl}/a_1^4+2\tilde{R}_{\alpha\beta}\tilde{R}^{\alpha\beta}/a_2^4$$

$$+2\tilde{R}_{ij}\tilde{R}^{ij}/a_1^4+\dot{a}_1\dot{a}_2\tilde{R}^{(1)}/(a_1^3a_2)\,(-6d_2+2d_1d_2)$$

$$+\dot{a}_1\dot{a}_2\tilde{R}^{(2)}/(a_1a_2^3)\,(4-4d_1-2d_2+2d_1d_2)$$

$$-\tfrac{1}{2}\,\tilde{R}^{(1)^2}/a_1^4-\tfrac{1}{2}\,\tilde{R}^{(2)^2}/a_2^4-\tilde{R}^{(1)}\tilde{R}^{(2)}/(a_1^2a_2^2)]$$

$$+\tilde{R}_{ij}/a_1^2[\dot{a}_1^2/a_1^2(-24+14d_1-2d_1^2)+\dot{a}_1\dot{a}_2/(a_1a_2)\cdot(4d_2-4d_1d_2)$$

$$+\dot{a}_2^2/a_2^2(2d_2-2d_2^2)+2\tilde{R}^{(1)}/a_1^2+2\tilde{R}^{(2)}/a_2^2]$$

$$-4\tilde{R}_{il}\tilde{R}^l_j/a_1^4+2\tilde{R}_{iklm}\tilde{R}_j{}^{klm}/a_1^4+4\tilde{R}^{mn}\tilde{R}_{minj}/a_1^4\}\;=\;-8\pi G_{10}(p_1\tilde{g}_{ij}+T_{ij}^F/a_1^2) \qquad \text{(A.6)}$$

with

$$T^F_{AB} = \tfrac{1}{4}\, g_{AB}F^{CD}F_{CD} - F_A{}^C F_{CB}.$$

The $A = \alpha$, $B = \beta$ — equations of motion are obtained from (A.6) by interchanging the indices $1 \leftrightarrow 2$ and $i, j, \ldots \leftrightarrow \alpha, \beta, \ldots$.

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