CLASSICAL YANG-MILLS THEORY IN PRESENCE OF EXTENDED ELECTRIC AND MAGNETIC SOURCES*

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The Yang-Mills field equations in presence of extended electric and magnetic sources have been considered and the solutions have been obtained by formulating the equations as an initial value problem in temporal gauge. For spherically symmetric electric and magnetic source distributions, the totally screened solutions with arbitrary low energy have been obtained. The static solutions for the non-spherically symmetric sources have also been obtained.

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1. *Introduction*

The Yang-Mills field equations in presence of static sources have been discussed by Sikivie and Weiss [1] in the temporal gauge as an initial value problem. Considering the static sources as both electric and magnetic, we have studied in an earlier paper [2] the classical Yang-Mills field equations and have formulated them as an initial value problem in the temporal gauge. The initial value problem was applied to discuss the Yang-Mills fields produced by a system of point dyons and it was observed that there exist different solutions differing in their total energy and isospin.

Extending this approach in the present paper, we incorporate the extended electric and magnetic sources and obtain the solutions to the Yang-Mills field equations. The extended electric and magnetic sources have been taken as both spherically symmetric and non-spherically symmetric ones. The former have been considered as having finite or infinite extensions. As regards the spherically symmetric source distributions it has been found that the totally screened Coulombian solutions with arbitrarily small energies are possible, while in the case of non-spherically symmetric sources the multipole moments exist which must vanish in order to allow static solutions of the Yang-Mills fields.

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2. The electric and magnetic sources and initial value problem in temporal gauge

We consider the nontopological electric and magnetic sources and introduce [2] the following non-Abelian field tensor to describe them:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ef^{abc}A^b_\mu A^c_\nu - \tfrac{1}{2}\,\delta_{\mu\nu\varrho\sigma}(\partial^\varrho B^{\sigma a} - \partial^\sigma B^{\varrho a} + gf^{abc}B^{\varrho b}B^{\sigma c}), \tag{1}$$

where A^a_μ and B^a_μ are the two non-Abelian potentials, e and g are the corresponding coupling parameters, f^{abc} are the structure constants of the gauge group SU(2) and $\delta_{\mu\nu\varrho\sigma}$ is the anti-symmetric tensor. The potentials A^a_μ and B^a_μ obey the gauge transformation

$$A'^a_\mu \to UA_\mu U^{-1} - \frac{1}{e}(\partial_\mu U)U^{-1} \tag{2a}$$

and

$$B^a_\mu \to U'B_\mu U'^{-1} - \frac{1}{g}(\partial_\mu U')U'^{-1}, \tag{2b}$$

where

$$U = e^{-ie\lambda^a(x)}T^a \tag{3a}$$

and

$$U' = e^{-ig\lambda'^a(x)}T^a \tag{3b}$$

in which $\lambda^a(x)$ and $\lambda'^a(x)$ are the constant matrices and T^a the group generators of SU(2). The field tensor (1) is form-invariant under the transformations (2):

$$F^a_{\mu\nu} \to UF_{\mu\nu}U^{-1} + U'F_{\mu\nu}U'^{-1} \tag{4}$$

provided the subsidiary conditions

$$U[\delta_{\mu\nu\varrho\sigma}(\partial^\varrho B^{\sigma a} - \partial^\sigma B^{\varrho a} + gf^{abc}B^{\varrho b}B^{\sigma c}]U^{-1} = 0 \tag{5a}$$

and

$$U'[\partial_\mu A_\nu - \partial_\nu A_\mu + ef^{abc}A^b_\mu A^c_\nu]U'^{-1} = 0 \tag{5b}$$

are observed.

The Lagrangian density for the system may now be written as

$$\mathscr{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + j^a_\mu A^{\mu a} - k^a_\mu B^{\mu a} \tag{6}$$

in which j^a_μ and k^a_μ are respectively the electric and magnetic source densities obeying

$$j'_\mu \to Uj_\mu U^{-1} \tag{7a}$$

and

$$k'_\mu \to U'k_\mu U'^{-1}. \tag{7b}$$

The Lagrangian density (6) is now invariant under the transformations (2), (4) and (7) and its Euler-Lagrange variation gives us the following symmetric field equations

$$D_\mu F^{\mu\nu a} = j^{\nu a} \tag{8a}$$

and

$$D'_\mu \tilde{F}^{\mu\nu a} = k^{\nu a}, \tag{8b}$$

where $\tilde{F}^{\mu\nu a}$ is the dual of the field tensor (1)

$$\tilde{F}^{\mu\nu a} = \tfrac{1}{2}\,\delta_{\mu\nu\varrho\sigma}F^{\varrho\sigma a} \tag{9}$$

and

$$D_\mu = \partial_\mu + ef^{abc}A^b_\mu \tag{10a}$$

$$D'_\mu = \partial_\mu + gf^{abc}B^b_\mu \tag{10b}$$

are the covariant derivatives. It may be observed from the gauge transformations (2) that the gauge functions U and U' allow us to consider

$$A^0 = A^{0a}T^a = 0 \tag{11a}$$

and

$$B^0 = B^{0a}T^a = 0 \tag{11b}$$

simultaneously as the temporal gauge conditions for the system. Now, considering the field equation (8) under the temporal gauge conditions (11) we may obtain

$$\partial_0 F^{0ia} = \partial_j F^{jia} + ef^{abc}A^b_j F^{jic} \tag{12a}$$

$$\partial_i F^{0ia} + ef^{abc}A^b_i F^{0ic} = j^{0a} \tag{12b}$$

$$\partial_0 A^{ia}(\vec{x}, t) = F^{0ia}(\vec{x}) + \varepsilon_{ijk}\partial_j B^a_k(\vec{x}, t) + \frac{g}{2}f^{abc}\varepsilon_{ijk}B^b_j(\vec{x}, t)B^c_k(\vec{x}, t) \tag{12c}$$

and

$$\partial_0 \tilde{F}^{0ia} = \partial_j \tilde{F}^{jia} + gf^{abc}B^b_j \tilde{F}^{jic} \tag{13a}$$

$$\partial_i \tilde{F}^{0ia} + gf^{abc}B^b_i \tilde{F}^{0ic} = k^{0a} \tag{13b}$$

$$\partial_0 B^{ia}(\vec{x}, t) = \tilde{F}^{0ia}(\vec{x}) - \varepsilon_{ijk}\partial_j A^a_k(\vec{x}, t) - \frac{e}{2}f^{abc}\varepsilon_{ijk}A^b_j(\vec{x}, t)A^c_k(\vec{x}, t), \tag{13c}$$

where the sources have been taken as static. If we now assume that the small fluctuations in potentials are allowed and potentials obey [3], then:

$$A^a_i(\vec{x}, t) = A^a_i(\vec{x}) + a^a_i(\vec{x})e^{+S\omega_1 t} \tag{14a}$$

and

$$B_i^a(\vec{x}, t) = B_i^a(x) + b_i^a(\vec{x})e^{+S\omega_2 t}, \tag{14b}$$

where $a_i^a(\vec{x})$ and $b_i^a(\vec{x})$ are the small perturbations on the potentials; $S = \sqrt{-1}$, ω_1 and ω_2 are the frequencies of eigen-modes which determine [4] the Abelian and non-Abelian character of potentials. Now, substituting (14b) into (12c) we may obtain

$$\partial_0 A^{ia}(\vec{x}, t) = P(\vec{x}) + Q(\vec{x})e^{S\omega_2 t} + R(\vec{x})e^{2S\omega_2 t}, \tag{15}$$

where

$$P(\vec{x}) = F^{0ia}(\vec{x}) + \varepsilon_{ijk}\left[\partial_j B^{ka}(\vec{x}) + \frac{g}{2} f^{abc} B^{jb}(\vec{x}) B^{kc}(\vec{x})\right] \tag{16a}$$

$$Q(\vec{x}) = \varepsilon_{ijk}\left[\partial_j b^{ka}(\vec{x}) + \frac{g}{2} f^{abc} B^{jb}(\vec{x}) b^{kc}(\vec{x}) + b^{jb}(\vec{x}) B^{kc}(\vec{x})\right] \tag{16b}$$

and

$$R(\vec{x}) = \frac{g}{2} f^{abc} \varepsilon_{ijk} b^{jb}(\vec{x}) b^{kc}(\vec{x}). \tag{16c}$$

Now, integrating equation (15) from the initial time t_0 to time t, we may obtain

$$A^{ia}(\vec{x}, t) = P(\vec{x})(t - t_0) - \frac{SQ(\vec{x})}{\omega_2}(e^{S\omega_2 t} - e^{S\omega_2 t_0}) - \frac{SR(\vec{x})}{2\omega_2}(e^{2S\omega_2 t} - e^{2S\omega_2 t_0}), \tag{17}$$

from which we may obtain the initial time value of

$$A^{ia}(\vec{x}, t) \quad \text{at} \quad t = t_0, \quad \text{as}$$

$$A^{ia}(\vec{x}, t)|_{t=t_0} = 0. \tag{18}$$

Similarly, substitution of (14a) into (13c) would yield the vanishing initial value for $B^{ia}(\vec{x}, t)$:

$$B^{ia}(\vec{x}, t)|_{t=t_0} = 0. \tag{19}$$

These vanishing initial values would help eliminate the nonlinearity from the Yang Mills field equations.

3. The extended static sources

When the point particle is assumed to carry both the electric and magnetic charges we have the case of point dyon [2]. Now we consider that electric and magnetic charges of dyon have no δ-function singularities but instead have spherically symmetric distributions

$$q_e(r) = q_e(\vec{x}) = C \exp(-cr) \tag{20a}$$

$$q_g(r) = q_g(\vec{x}) = D \exp(-dr) \tag{20b}$$

for $r = |\vec{x}| \to \infty$,
where C and D are constants, and c and d are the positive quantities. Since, in view of equations (20), the electric and magnetic charge distributions have infinite extensions, the total electric and magnetic charges may be given by

$$Q_e = \int_0^\infty 4\pi r^2 q_e(r) dr \tag{21a}$$

and

$$Q_g = \int_0^\infty 4\pi r^2 q_g(r) dr. \tag{21b}$$

The total charges (21) may, however, be looked upon as the sum of the charges between origin and a certain radius r and charges between r and ∞. If $h_e(r)$ and $h_g(r)$ be the respective fractions of total electric and total magnetic charges outside the radius r, we may write

$$h_e(r) = \frac{1}{Q_e} \int_r^\infty 4\pi r^2 q_e(r) dr \tag{22a}$$

and

$$h_g(r) = \frac{1}{Q_g} \int_r^\infty 4\pi r^2 q_g(r) dr. \tag{22b}$$

It may be observed from equations (20) that as $r \to \infty$, $q(r) \to 0$. Therefore, equations (22) imply

$$q_e = -\frac{Q_e}{4\pi r^2} \frac{dh_e(r)}{dr} \tag{23a}$$

and

$$q_g = -\frac{Q_g}{4\pi r^2} \frac{dh_g(r)}{dr}. \tag{23b}$$

Since we are considering the source distributions as static, and static source distribution only 'rotates' in the internal isospin space [1], we may assume that electric charge rotates about the δ^{a1} axis while the magnetic charge about δ^{a3} and thus the general electric and magnetic charge distributions may be given by

$$q_e^a(r) = q_e(r) [\delta^{a3} \cos(2\pi n h_e(r)) + \delta^{a2} \sin(2\pi n h_e(r))], \tag{24a}$$

and

$$q_g^a(r) = q_g(r) [\delta^{a1} \cos(2\pi n h_g(r)) + \delta^{a2} \sin(2\pi n h_g(r))], \tag{24b}$$

where n is an integer and δ^{ai}, $i = 1, 2, 3$ denote isotopic spin directions. Let us assume that there exists a gauge in which electric charges is aligned along δ^{a3} and magnetic charge along δ^{a1}. That is in this gauge

$$q_e^a(r) = q_e(r)\delta^{a3} \tag{25a}$$

and

$$q_g^a(r) = q_g(r)\delta^{a1}. \tag{25b}$$

This description is reminiscent of Zwangiger's [5] generalized charge vector, whose two vector components represent the electric and magnetic charges on the particle. The configuration (25) may be obtained from the general charge distributions (24) by their rotations about respective axes. It may be observed from equation (24) that when $h_e(r)$ and $h_g(r)$ have values 1 and 0, the equations (24) transform into (25). Also from the equations (22) and (21) we may notice that when $r \to 0$, $h_e(r) \to 1$ and when $r \to \infty$, $h_e(r) \to 0$ and so does the $h_g(r)$. Therefore, the electric charges are aligned along δ^{a3} and magnetic charges along δ^{a1} at both $r = 0$ and $r = \infty$. However, $h_e(r)$ and $h_g(r)$ may also have the values between 1 (at $r = 0$) and 0 (at $r = \infty$), such as

$$h_e(r) = \frac{1}{4n} = h_g(r) \tag{26}$$

resulting that the configuration (25) will not be obtained from (24). This indicates that between $r = 0$ and $r = \infty$, the alignment of electric and magnetic charges would change from the one along δ^{a3} and δ^{a1} axes, respectively. Therefore, to have the charges maintain their alignments along the desired axes all through, we must ascribe large values to the integer n, such that $\frac{1}{4n}$ of Eq. (26) tends to zero.

4. *The solutions*

(a) Spherically symmetric source distributions having finite extension

Let us consider that the electric and magnetic charges are spherically symmetric and extend up to a distance r_0 only, from the origin. To obtain the solutions of the Yang-Mills field equations (8), the nonlinear terms must be taken care of. For the Coulomb solutions to the field equations (8), the nonlinear terms may be eliminated by setting

$$A_\mu^a(x) = \delta^{a3} V_\mu(x) \tag{27a}$$

and

$$B_\mu^a(x) = \delta^{a1} W_\mu(x), \tag{27b}$$

where the source distributions are given by the configurations (25). When the electric and magnetic source distributions are spherically symmetric but confined to a radius r_0, the electric and magnetic fields produced by these sources may be expected both inside and outside r_0. For $r > r_0$, the outside region, we may observe that the ansatz (27) reduces field tensors (1) to its Abelian counterpart

$$f^a_{\mu\nu} = (\partial_\mu V_\nu - \partial_\nu V_\mu) - \tfrac{1}{2}\,\delta_{\mu\nu\varrho\sigma}(\partial^\varrho W^\sigma - \partial^\sigma W^\varrho)\delta^{a1} \tag{28a}$$

and its dual to

$$\tilde{f}^a_{\mu\nu} = (\partial_\mu W_\nu - \partial_\nu W_\mu) + \tfrac{1}{2}\,\delta_{\mu\nu\varrho\sigma}(\partial^\varrho V^\sigma - \partial^\sigma V^\varrho)\delta^{a3} \tag{28b}$$

such that the Yang-Mills field equations in temporal gauge conditions (11) assume the linear forms

$$\partial_i f^{0ia} = j^{0a} \tag{29a}$$

and

$$\partial_i \tilde{f}^{0ia} = k^{0a}, \tag{29b}$$

where $j^{0a} = q^a_e(\vec{x})$ and $k^{0a} = q^a_g(\vec{x})$ denote the static sources. From these equations the electric and magnetic fields produced by static electric and magnetic sources may be calculated by using equations (23) and integrating them

$$f^{0ia} = \frac{Q_e}{4\pi r^2}\,\hat{r}^i\delta^{a3} \tag{30a}$$

and

$$\tilde{f}^{0ia} = \frac{Q_g}{4\pi r^2}\,\hat{r}^i\delta^{a1}, \tag{30b}$$

where we have used $h(0) = 1$ and $h(r > r_0) = 0$.

Now we make use of the observation [1] that in SU(2), a source aligned along $+\delta^{ai}$ may be locally changed to $-\delta^{ai}$. Therefore, if we assume that the region $0 \leqslant r \leqslant r_0$ is divided into even number of shells and if a source is aligned along a particular isotopic axis in a shell, the alignment will be in the reverse direction in the adjacent shells. Thus, when each of the shell carries an equal total isotopic charge the net charge in the region $0 \leqslant r \leqslant r_0$ would be zero. In the present case, the sources are both electric and magnetic and the total isotopic charge is

$$I = Q_e\delta^{a3} + Q_g\delta^{a1}. \tag{31}$$

Since source alignments along either of the isospin axes may be reversed locally, the net electric and net magnetic charges in the R.H.S. of the field equations (29) for the region $0 \leqslant r \leqslant r_0$ would be zero and obviously the solutions of field equation (29) would have vanishing electric and magnetic fields. If we now write the constraints (12b) and (13b) as

$$\partial_i F^{0ia} = j^{0a} - e f^{abc} A^b_i F^{0ic} \tag{32a}$$

and

$$\partial_i \tilde{F}^{0ia} = k^{0a} - g f^{abc} B^b_i \tilde{F}^{0ic}, \tag{32b}$$

where nonlinear terms which also act as charge distributions have opposite sign to the source distributions j^{0a} and k^{0a}, we observe that these charge distributions provide screening to the source distributions and when they become equal, the R.H.S. of equations (32) become zero and the screening is total. In the present case the nonlinear terms vanished due to the ansatz (27) and source terms due to their locally changeable alignments. Thus the solutions discussed above are obtained for the totally screened electric and magnetic fields and their energy may be made arbitrarily small by making the number of shells large.

(b) The spherically symmetric distributions with infinite extensions

We considered in (a) that the extension of the sources was finite and that beyond $r > r_0$ the source distribution vanished. Now, if the source distributions have infinite extension, a certain fraction of the total charge would be outside r_0. These fractions have been defined by $h_e(r)$ and $h_g(r)$ given by equation (22) and the corresponding electric and magnetic source distributions are expressed by equations (24) for which the field equation (29) may be written as

$$\partial_i f^{0ia} = -\frac{Q_e}{4\pi r^2}(\delta^{a3}\cos(2\pi n h_e(r))+\delta^{a2}\sin(2\pi n h_e(r)))\frac{dh_e(r)}{dr} \tag{33a}$$

$$\partial_i \tilde{f}^{0ia} = -\frac{Q_g}{4\pi r^2}(\delta^{a1}\cos(2\pi n h_g(r))+\delta^{a2}\sin(2\pi n h_g(r)))\frac{dh_g(r)}{dr}, \tag{33b}$$

where we have used equation (23) in source equation (24). Integrating equation (33) and using $h_e(0) = 1 = h_g(0)$ the solution to the field equations may be obtained as

$$f^{0ia} = \frac{Q_e}{4\pi r^2}\frac{r^i}{2\pi n}\{\delta^{a2}(\cos(2\pi n h_e(r))-1)-\delta^{a3}\sin(2\pi n h_e(r))\} \tag{34a}$$

and

$$\tilde{f}^{0ia} = \frac{Q_g}{4\pi r^2}\frac{r^i}{2\pi n}\{\delta^{a2}(\cos(2\pi n h_g(r))-1)-\delta^{a1}\sin(2\pi n h_g(r))\}. \tag{34b}$$

Under ansatz (27) the equations for the potentials viz. (12c) and (13c) become

$$A_i^a = [F^{0ia}(\vec{x})+\delta^{a1}\varepsilon_{ijk}\partial_j W_k(\vec{x})]t \tag{35a}$$

and

$$B_i^a = [\tilde{F}^{0ia}(\vec{x})-\delta^{a3}\varepsilon_{ijk}\partial_j V_k(\vec{x})]t. \tag{35b}$$

Therefore, for the source distributions (24) the complete solutions are given by the temporal gauge conditions (11) and the equations (34) and (35). Now, the configurations (25) may be obtained from (24) by rotating them in the opposite sense through $2\pi n h(r)$. The corresponding solutions may be obtained from equations (34) and (35) as

$$A_0^a = 0$$

$$A_i^a(\vec{x}, t) = [f^{0ia}(\vec{x})+\delta^{a1}\varepsilon_{ijk}\delta_j W_k(\vec{x})]t-\delta^{a1}e^{-1}\vec{\nabla}(2\pi n h_e(r))$$

$$f^{0ia}(\vec{x}) = \frac{Q_e}{4\pi r^2}\frac{r^i}{2\pi n}\{\delta^{a2}(1-\cos(2\pi n h_e(r)))-\delta^{a3}\sin(2\pi n h_e(r))\} \tag{36a}$$

and

$$B_0^a = 0$$

$$B_i^a(\vec{x}, t) = [\tilde{f}^{0ia}(x)-\delta^{a3}\varepsilon_{ijk}\partial_j V_k(\vec{x})]t-\partial^{a3}g^{-1}\vec{\nabla}(2\pi n h_g(r))$$

$$\tilde{f}^{0ia}(\vec{x}) = \frac{Q_g}{4\pi r^2}\frac{r^i}{2\pi n}\{\delta^{a2}(1-\cos(2\pi n h_g(r))-\delta^{a1}\sin(2\pi n h_g(r))\}, \tag{36b}$$

where $r = |\vec{x}|$.

In the limit of $h_e(r)$ and $h_g(r) \to 0$ these fields reduce to the totally screened electric and magnetic fields obtained under sources with finite extension. A similar result is obtained for $n \to \infty$, i.e. when the number of shells is large. Since we require large n, so that the alignment of the sources is maintained throughout the space, the fields (36) will essentially be totally screened. The energy of the solutions (36) may be computed from

$$H = \int [f_{0i}^a(\vec{x})f_{0i}^a(\vec{x})+\tilde{f}_{0i}^a(\vec{x})\tilde{f}_{0i}^a(\vec{x})]d^3\vec{x} \tag{37}$$

to give

$$H = \frac{1}{\pi(2\pi n)^2}\left[Q_e^2\int_0^\infty \frac{\sin^2(\pi n h_e(r))}{r^2}dr+Q_g^2\int_0^\infty \frac{\sin^2(\pi n h_g(r))}{r^2}dr\right]. \tag{38}$$

The integration of (38) shows that for $r = 0$, the energy becomes infinite. For the energy to be finite we must have

$$h_e(0)-h_e(r) \approx r^{>\frac{1}{2}} \approx h_g(0)-h_g(r). \tag{39}$$

We may also observe that since n is large, the energy (38) of the solutions (36) tends to smaller values when $n \to \infty$, $H \to 0$ which suggests that arbitrary small values of energy are possible for the solutions (36).

(c) The nonspherically symmetric sources

When the source distributions deviate from spherical symmetry, the ansatz (27) cannot be used to eliminate the non-linearities in the field equations (8). For such non-spherical source distributions, the Yang-Mills field equations (8) may be solved through the temporal gauge conditions (11). Under these conditions the field equations (8) assume the forms (12b) and (13b), respectively, which can be solved through the initial value problem. For this purpose we observe from the field equations (8) that

$$D_\nu D_\mu F^{\mu\nu} = D'_\nu D'_\mu \tilde{F}^{\mu\nu} = 0 \tag{40}$$

and from (12a) and (13a) in temporal gauge

$$D_\mu F^{j\mu} = D'_\mu \tilde{F}^{j\mu} = 0. \tag{41}$$

The equations (40) and (41) then tell us that under the temporal gauge conditions (11),

$$\partial_0 (D_i F^{0i})^a = \partial_0 j^{0a}(\vec{x}) = 0 \tag{42a}$$

and also

$$\partial_0 (D'_i \tilde{F}^{0i})^a = \partial_0 k^{0a}(\vec{x}) = 0. \tag{42b}$$

These equations imply that the constraints (12b) and (13b) are time independent and therefore it will be sufficient for A^{ia}, B^{ia}, F^{0ia} and $\tilde{F}^{0ia}$ to satisfy these constraints at $t = t_0$ to satisfy the constraints for all time [1]. Considering equations (12b) and (13b) at the initial time $t = t_0$, their nonlinear terms may be eliminated by using vanishing initial values of potentials given by equations (18) and (19). The equations (12b) and (13b) then become

$$\partial_i \chi^{ia} = q_e^a(\vec{x}), \tag{43a}$$

$$\partial_i \tilde{\chi}^{ia} = q_g^a(\vec{x}), \tag{43b}$$

where χ^{ia} and $\tilde{\chi}^{ia}$ are the initial time values of the fields F^{0ia} and $\tilde{F}^{0ia}$, respectively, $q_e^a(\vec{x})$ and $q_g^a(\vec{x})$ are not spherically symmetric now. The solutions for these equations may therefore be obtained at $t = t_0$, as [6]:

$$A_0^a = 0, \quad A_i^a = 0,$$
$$F^{0ia} = \frac{1}{4\pi} \int d^3\vec{x}' q_e^a(\vec{x}) \frac{\vec{x}_i - \vec{x}'_i}{|\vec{x} - \vec{x}'|^3} \tag{44a}$$

and

$$B_0^a = 0, \quad B_i^a = 0,$$
$$\tilde{F}^{0ia} = \frac{1}{4\pi} \int d^3\vec{x}' q_g^a(\vec{x}) \frac{\vec{x}_i - \vec{x}'_i}{|\vec{x} - \vec{x}'|^3}. \tag{44b}$$

The energy corresponding to these solutions will be given by equation (27), F_{0i}^a and $\tilde{F}_{0i}^a$ are taken from the solutions (44). It may then be observed that due to nonspherical symmetry, $q^a(\vec{x}) \not\equiv q^a(r)$ implying that for same $|\vec{x}| = r$, there will be different $q^a(\vec{x})$, so that the energy of the solutions which correspond to the same $|\vec{x}| = r$ will be different. Obviously, the corresponding total isotopic charge will also be different. The solutions corresponding to the time development may be obtained from (12a), (13c) and (13a), (13c) and they will not be static in general. However, static solutions may be obtained if an arbitrary number of multipole moments of both electric and magnetic sources vanish. To obtain this, we consider that electric and magnetic sources may be represented as

$$q_e^a(r) = \delta^{a3} q_e^3(r) \tag{45a}$$

and

$$q_g^a(r) = \delta^{a1} q_g^1(r), \tag{45b}$$

where $q_e^3(r) = +q_e(r)$ in some regions S_+ and equal to $-q_e(r)$ in regions S_-. We assume that similar situation exists for magnetic charges as well, in regions S_+ and S_-. The multipole moments of these source distributions may be written as

$$Q^e_{1m} = \delta^{a3} \int r^1 Y^*_{1m}(\theta, \phi) q_e^3(r) d^3r \tag{46a}$$

and

$$Q^g_{1m} = \delta^{a1} \int r^1 Y^*_{1m}(\theta, \phi) g_g^1(r) d^3r, \tag{46b}$$

where $Y_{1m}(\theta, \phi)$ denote the normalized spherical harmonics, and asterisks denote complex conjugates. If the regions S_+ and S_- are chosen in such a way that the arbitrary multipole moments vanish, i.e.

$$Q_{nn'} = \delta^{a3} \int q^3(r) Y^*_{nn'}(\theta, \phi) \left[r^n - r'^n\right] d^3r = 0, \tag{47}$$

where r is for the region S_+ and r' for S_-, then the configuration (44) will yield static solutions.

5. *Discussion*

We have obtained the solutions to the Yang-Mills fields in presence of extended electric and magnetic sources. The sources considered were both spherically symmetric and non-spherically symmetric. In the former case, the general electric and magnetic source distributions were described by equations (24) which were then aligned along respective axes as in equations (25). It has been observed that the alignment of sources is maintained according to (25) at both $r = 0$ and $r = \infty$, while a change in alignment is observed when equation (26) is valid. However, the same has been restored by ascribing large value to the integer.

The nonlinearities in the Yang-Mills field equations (8) have been overcome through ansatz (27) in the case of spherically symmetric source distributions and through the initial values (18) and (19) in the case of non-spherically symmetric sources. The solutions (35) differ from the purely electric case [1] in that we have an additional term in terms of potentials of equation (27). A similar observation may be made with respect to solutions (26). The energy of the solutions given by equation (38) has been found to be finite if $1-h(r) \simeq r^{\varepsilon}$, where $\varepsilon > \frac{1}{2}$, which implies that $q_e(r)$ and $q_g(r)$ must be more singular than $r^{-5/2}$ in the case of non-spherically symmetric source distributions. The deviation from spherical symmetry gives rise to multipole moments whose arbitrary number must vanish in order to obtain the static solutions.

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