FORMATION TIME IN THE LUND MODEL*

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The Lund string model is used in estimation of the average time needed for the formation of final hadrons in high energy collisions. The energy-dependence of the formation time and sensitivity of results to the choice of the scaling function of the model is studied.

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1. *Introduction*

It is now well established that the appearance of final particles in high energy collisions requires a certain time necessary for their formation [1–7]. It is generally assumed that this formation time increases linearly with increasing energy of the system and therefore may be large at sufficiently high energies. The concept of finite formation time was introduced by Landau, Pomeranchuk and Feinberg in their analysis of bremsstrahlung of fast electrons passing through dense amorphous media [1–3]. They argued that the photon of energy ω and momentum $\vec{k}$ radiated by a fast electron moving with velocity $\vec{v}$ appears in the final state at the time

$$\tau_f = \frac{1}{\omega - \vec{k} \cdot \vec{v}} \tag{1.1}$$

after interaction of the electron. This time can be large for soft, collinear photons emitted from relativistic electrons [4].

In hadronic interactions the effects of finite formation time are most clearly manifested in high energy interactions with nuclear targets. The absence of intra-nuclear cascading of fast particles observed in experimental data [5, 8] is attributed to the fact that due to the large formation times the fast final hadrons are formed mostly beyond the target, hence do not interact with nuclear matter. Recently proposed methods of determination of the

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formation time from experimental data [9–12] are based on analysis of attenuation of leading hadrons produced on nuclear targets.

Assuming the production mechanism similar to the bremsstrahlung ("bremsstrahlung analogy" [13]) we obtain the following expression for the formation time [6]:

$$\tau_f = \frac{1}{\omega - k_{\parallel}v} \cong \frac{2k_{\parallel}}{(Mx)^2 + k_{\perp}^2 + \mu^2}, \tag{1.2}$$

where: μ, ω — mass, energy of emitted particle, $k_{\parallel}$, $k_{\perp}$ — its longitudinal, transverse momentum, M, E, v — mass, energy, velocity of primary particle, $x = k_{\parallel}/E$ — momentum fraction of primary particle carried by the final particle.

The bremsstrahlung-like production mechanism seems to work well for soft particles but is not certain if it is correct for those with momenta close to that of the original particle. Unfortunately, our present understanding of theory does not allow one to calculate the formation time from Quantum Chromodynamics. In view of that it seems interesting to obtain this quantity using model of hadronization other than the "bremsstrahlung analogy".

The aim of this work is to estimate the formation time in the framework of the Lund hadronization model [14]. This model reproduces well experimental data [15] and provides a definite space-time description of hadronization [16]. The paper is organized as follows. The main assumptions of the Lund string model specifying the space-time description of hadronization are reviewed in Sect. 2. Our calculation of the formation time are outlined in Sect. 3. Sect. 4 is devoted to the discussion of the x-dependence of the formation time. We present our conclusions in the last Section.

2. *Space-time picture of hadronization*

We apply to the description of hadronization a simple semiclassical picture proposed by the Lund string model [16].

According to this model, in a high energy hadronic collisions coloured objects (quarks, diquarks, antiquarks) are separated. The separated sources of colour create a thin string-like colour field between them. This field is constant along the string, i.e. the amount of the field energy per unit length is constant:

$$E_s = \kappa l, \tag{2.1}$$

where E_s is the energy of the string, l — its length, κ — string tension, $\kappa = 1$ GeV/fm [14]. The field can be broken by the production of $q\bar{q}$ pair; this splits a meson from the string (cf. Fig. 1). It is assumed that in each process

$$\text{string}_0 \rightarrow \text{string}_1 + \text{meson} \tag{2.2}$$

there is a universal way of partition of the energy of the string_0 between the meson and the remainder-string_1. It is described in terms of scaling function $f(z)$ defined as [17]:

$f(z)dz$ = probability that the meson will take the fraction z of string_0 momentum. (2.3)

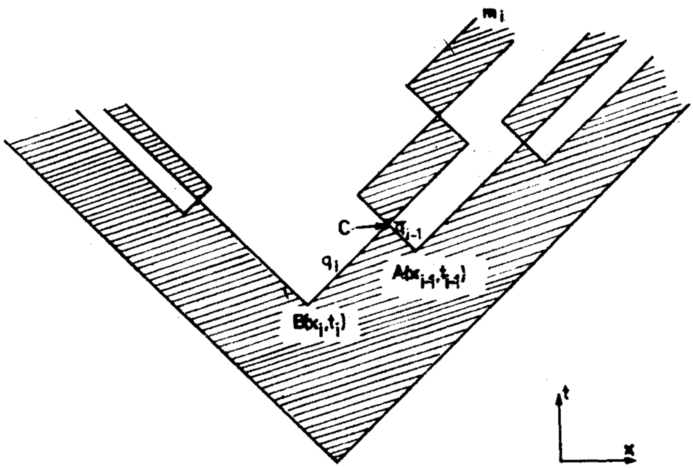

Fig. 1. Space-time picture of hadronization in the Lund model. A quark q_{i-1} produced at the point A and an antiquark $\bar{q}_i$ produced at the point B form at the point C a meson m_i

Space-time and energy-momentum space descriptions of hadronization are closely related; if we introduce the light-cone variables:

$$x^{\pm} = t \pm x,$$

$$p^{\pm} = E \pm p, \tag{2.4}$$

we obtain the following relations between momentum of a given meson and space-time coordinates of production points of partons — its constituents [16]:

$$p_i^+ = \kappa(x_{i-1}^+ - x_i^+),$$

$$p_i^- = \kappa(x_i^- - x_{i-1}^-). \tag{2.5}$$

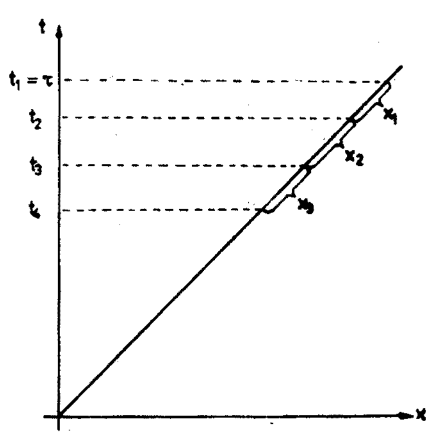

Fig. 2. Space-time description of hadronization in massless approximation

Following [16] as a hadron production point we define the point of the first crossing of partons — its constituents (e.g. point C in Fig. 1).

The formation time is a frame-dependent quantity. We discuss the situation corresponding to the case of leptoproduction in the laboratory system. In that case the string of colour field is stretched between a quark q_0 kicked out by the current with initial energy E and remaining diquark. We will use the simplified version of the Lund model with one type of quark and one type of meson, we will also neglect the masses both of the quark and of the meson. In this approximation all the $q\bar{q}$ pair and hadron production points will lie on the line $x^- = 0$ (Fig. 2).

3. *Estimation of the formation time*

Consider a hierarchy sequence of mesons [17, 14]:

$$\begin{matrix} q_0\bar{q}_1 & q_1\bar{q}_2 & q_2\bar{q}_3 \dots q_{i-1}\bar{q}_i \dots \\ m_1 & m_2 & m_3 \dots m_i \quad \dots \end{matrix}$$

arising from the subsequent breaking of the string through creations of $q\bar{q}$ pairs; the first in rank meson contains the original quark q_0. If the i-th in rank particle carries the fraction x_i of the initial energy E, the values of scaling variables at successive production steps are:

$$z_i = \frac{x_i}{1-\sum\limits_{k=1}^{i-1} x_k}. \tag{3.1}$$

Inverting (3.1) we obtain:

$$x_n = z_n \prod_{i=1}^{n-1} (1-z_i). \tag{3.2}$$

According to the definition of hadron production point, the first in rank meson is produced at the time $t_1 = \tau = E/\kappa$. The time t_i of the production of i-th in rank meson coincides in massless approximation with the time of the production of $(i-1)$-th $q\bar{q}$ pair, hence we obtain (cf. Fig. 2):

$$t_i = \tau(1-\sum_{k=1}^{i-1} x_k) = \tau \prod_{k=1}^{i-1} (1-z_k). \tag{3.3}$$

As the values of the scaling variables z_i at each production step are independent, the probability of creating a hierarchy sequence of mesons in which k-th in rank has the fraction x_k in dx_k of initial momentum is [17]:

$$\text{Prob}\,(x_1, x_2, \dots, x_k, \dots)dx_1 dx_2 \dots dx_k \dots = \prod_{i=1}^{\infty} f(z_i)dz_i, \tag{3.4}$$

where $f(z)$ is the scaling function defined in (2.3), the relations between x_i and z_k are given by (3.1) and (3.2).

Consequently, the inclusive probability that the n-th in rank meson will carry the fraction x in dx of the initial momentum is:

$$P_n(x)dx = dx \int_0^1 dz_1 \ldots \int_0^1 dz_n f(z_1) \ldots f(z_n) \delta(x - z_n \prod_{k=1}^{n-1} (1 - z_k)). \qquad (3.5)$$

The average number of mesons carrying the fraction x of the initial momentum regardless of their position in rank is the sum:

$$D(x)dx = \sum_{n=1}^{\infty} P_n(x)dx. \qquad (3.6)$$

The function $D(x)$ is nothing but the quark fragmentation function [18]. In a similar way we obtain the expressions for the probability that n-th in rank meson carries the fraction x in dx of the initial momentum and is produced at the time t in dt:

$$P'_n(x, t)dxdt = dxdt \int_0^1 dz_1 \ldots \int_0^1 dz_n f(z_1) \ldots f(z_n)$$

$$* \delta(x - z_n \prod_{k=1}^{n-1} (1 - z_k)) \delta(t - \tau \prod_{k=1}^{n-1} (1 - z_k)), \qquad (3.7)$$

and for the average number of mesons carrying the fraction x of initial momentum and produced at the time t regardless their position in rank:

$$D'(x, t)dxdt = \sum_{n=1}^{\infty} P'_n(x, t)dxdt. \qquad (3.8)$$

The ratio:

$$f_x(t) = \frac{D'(x, t)}{D(x)} \qquad (3.9)$$

is the probability distribution for production at the time t a meson carrying the fraction x of the initial momentum.

We will estimate the formation time τ_f by the average time of the production of a meson with the fraction x of the initial momentum:

$$\tau_f(x) = \int_0^{\infty} dt t f_x(t) = \frac{\int_0^{\infty} dt t D'(x, t)}{D(x)}. \qquad (3.10)$$

Because of the δ-function of time in (3.8) and (3.9) the integral in the numerator of (3.10) can be easily performed and we end up with the formula:

$$\tau_f = \tau \frac{T(x)}{D(x)} = \frac{E}{\kappa} \frac{T(x)}{D(x)}, \qquad (3.11)$$

where

$$T(x) = \sum_{n=1}^{\infty} T_n(x),$$

$$T_n(x) = \int_0^1 dz_1 \dots \int_0^1 dz_n \delta(x-z_n \prod_{k=1}^{n-1} (1-z_k)) * f(z_n) \prod_{i=1}^{n-1} (1-z_i) f(z_i). \tag{3.12}$$

It is clear from (3.11) that the formation time depends on:

1. the initial energy of the system. This dependence is very simple, $\tau_f \sim E$,
2. the momentum fraction x carried by the produced meson. The x-dependence is given by the function

$$g(x) = \frac{T(x)}{D(x)}. \tag{3.13}$$

4. *The x-dependence of the formation time*

In order to obtain the detailed form of the x-dependence of the formation time we have to determine both $D(x)$ and $T(x)$ (cf. (3.5), (3.6), (3.12)). To this end we apply the standard Mellin transformation technique and calculate their transforms $\tilde{D}(p)$ and $\tilde{T}(p)$. The infinite series defining $D(x)$ and $T(x)$ are transformed into simple geometrical series and we acquire compact expressions for the Mellin transforms:

$$\tilde{D}(p) = \frac{I_0(p)}{1-I_1(p)}, \tag{4.1}$$

$$\tilde{T}(p) = \frac{I_0(p)}{1-I_1(p+1)}. \tag{4.2}$$

The functions $I_0(p)$, $I_1(p)$ are expressed in terms of the scaling function $f(z)$ as follows:

$$I_0(p) = \int_0^1 dz z^{p-1} f(z), \tag{4.3}$$

$$I_1(p) = \int_0^1 dz (1-z)^{p-1} f(z). \tag{4.4}$$

Inverting $\tilde{D}(p)$ and $\tilde{T}(p)$ and applying (3.13) one obtains the shape of the x-dependence of the formation time.

We have determined the function $g(x)$ for different scaling functions $f(z)$ used in the Lund model[1].

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[1] The different scaling functions we have used are plotted in Fig. 3.

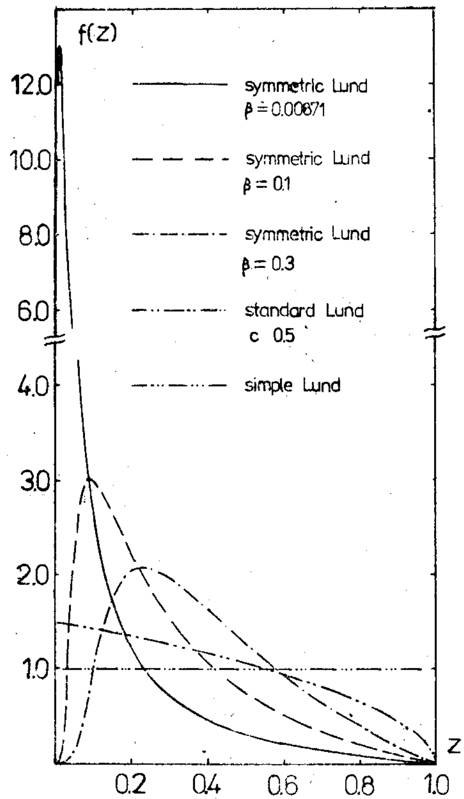

Fig. 3. Scaling functions of different versions of the Lund model used in calculations of the formation time

A. Simple Lund model

This version of the model uses flat scaling function:

$$f(z) = 1. \tag{4.5}$$

The function $g(x)$ corresponding to (4.5) is:

$$g(x) = x(1-\ln(x)). \tag{4.6}$$

B. Standard Lund model

The appropriate scaling function has the form:

$$f(z) = (c+1)(1-z)^c, \quad c = 0.3-0.5. \tag{4.7}$$

The corresponding x-dependence of the formation time for $c = 0.5$ reads:

$$g(x) = x\left\{\frac{3}{2}(1-x)^{-1/2}\ln\left|\frac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}}\right| -2\right\}. \tag{4.8}$$

The curves corresponding to the function $g(x)$ for simple and standard version of the model are plotted in Fig. 4.

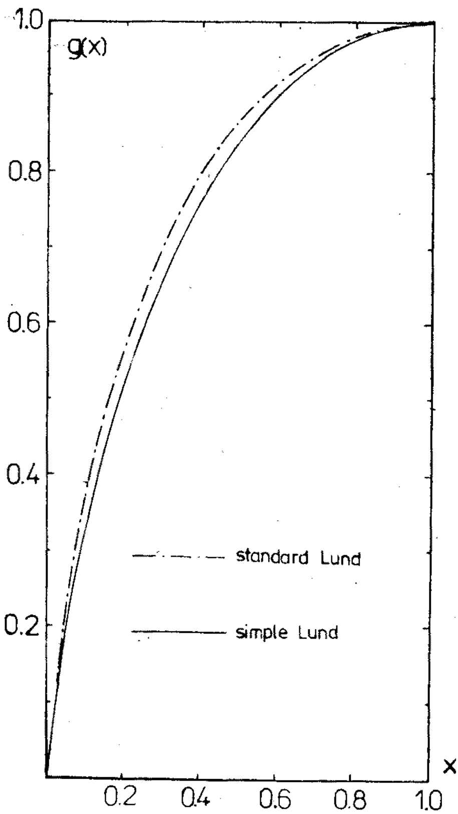

Fig. 4

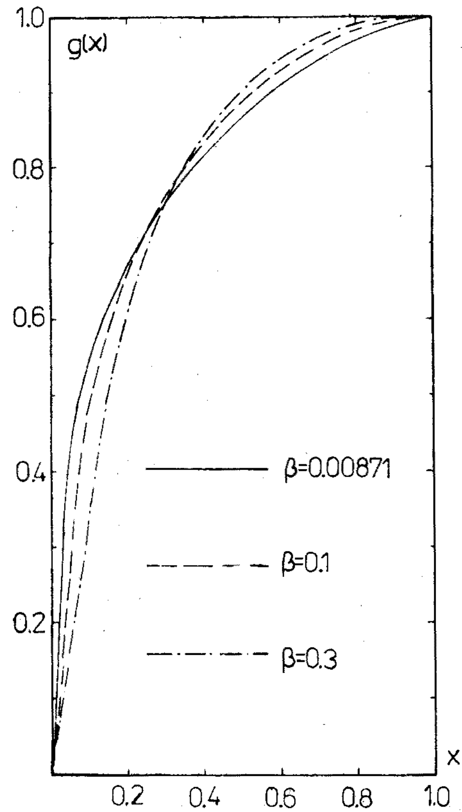

Fig. 5

Fig. 4. The x-dependence of the formation time in simple and standard version of the model
Fig. 5. The x-dependence of formation time in the symmetric Lund model

C. Symmetric Lund model

Recently introduced symmetric version of the Lund model [19] uses the following scaling function:

$$f(z) = N\frac{1}{z}(1-z)^a \exp(-\beta/z). \tag{4.9}$$

In that case the Mellin transforms $\tilde{T}(p)$, $\tilde{D}(p)$ are too complicated to be inverted analytically and we inverted them numerically [20]. We considered the scaling function (4.9) for $a = 1$[2] and several values of β. The resulting curves for $g(x)$ are plotted in Fig. 5.

Although obtained from a very different scaling functions (cf. Fig. 3) all the curves of $g(x)$ are rather similar to each other. They grow monotonically in the whole range of x. This gives a specific time-ordering of production processes : slow particles are pro-

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[2] This value of a is normally used in standard applications of the model [14].

duced earlier than fast ones [5, 6, 7, 16]. The x-dependence of the formation time in the Lund model is not linear. The curves of $g(x)$ grow much faster at small (for $x = 0.2$ $g(x) \geqslant 0.5$) than at large values of x. Consequently, the formation time is not proportional to the final energy of the produced hadron.

The detailed shape of $g(x)$ is rather weakly dependent on the choice of the scaling function $f(z)$; the space-time relations of particle productions are determined mostly by kinematics of the model.

5. *Conclusions*

The formation time is an important and interesting parameter of hadronic interactions. We have estimated it in the framework of the Lund string model. Our results can be summarized as follows:

1. The formation time of a final particle depends on the initial energy E of the system and on the fraction x of this energy carried by the particle. This dependence factorizes in the form:

$$\tau_f(E, x) = \frac{E}{\kappa} g(x).$$

2. At fixed initial energy E the formation time rises with increasing x, it means that soft particles are produced earlier in time than the fast ones.

3. The formation time is a nonlinear function of x. Consequently, it is not proportional to the energy of a produced particle.

4. The shape of x-dependence of the formation time is not very sensitive to the choice of the scaling function $f(z)$.

We close the paper with a few comments:

a) We neglect in our calculations the mass of the produced mesons. Inclusion of the mass effects will make the formation time the function of mass and Q^2 also. We expect however, that this dependence (especially for large x) should not modify our predictions significantly.

b) The process of hadron formation seems to be determined by the dynamics of colour confinement. Consequently, the time scale of the formation time should be given by the quantity characterizing confinement. This makes the role of κ in our formulae quite natural.

c) The measurements of nuclear attenuation of fast particles leptoproduced from nuclear targets [10] should allow one to determine the formation time from the data. These measurements can help to answer the question if the space-time picture of hadronization provided by the Lund string model is correct.

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