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CHIRAL SYMMETRY BREAKING IN BARYONIC ENVIRONMENT

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The Bogoliubov-Valatin variational technique is used to describe chiral symmetry breaking in QCD with a finite baryonic number density at zero temperature. Using the BCS-like trial vacuum state with the condensate wave function approximated by $\sim \exp(-\frac{1}{2}R^2(k-aK_F)^2)$ with a different from zero, it is shown that the chirally invariant vacuum is unstable for small baryonic density. Our results also indicate that in the high baryonic density regime chiral symmetry is restored, presumably in the first order phase transition.

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1. Introduction

An important question of quantum chromodynamics is whether the chiral symmetry is restored at finite baryonic density and temperature. There is an evidence from lattice simulation [1, 2], and from calculation in continuum [3, 4], that at finite temperature without baryonic background such a transition occurs. However, the effect remains elusive at finite baryonic number density [5, 6]. Suitable lattice methods have been developed only recently [14]. In continuum understanding of these nonperturbative effects relies on the QCD inspired models.

One such a model, which is based on analogy with superconductivity, was proposed by Finger et al. [8]. In this scheme nonzero baryonic number density may be simply achieved [10, 11]. The whole construction [11] is done in quasiparticle basis. The nuclear matter is built from colour singlet baryons containing three valence/constituent quarks, which gradually fill Fermi sea up to a given K_F. It was shown that the chirally symmetric vacuum becomes unstable for some finite value of baryon density, beyond which the perturbative vacuum turns out to be a stable one. The results are based on Bogoliubov-Valatin instability method. In this paper we carry out the instability analysis with a trial function $\phi \sim \exp(-\frac{1}{2}R^2(k-aK_F)^2))(a \neq 0)$. As far as the quark selfenergy is concerned, which

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is negative and plays the crucial role in triggering mechanism for the chiral symmetry breaking, this function has better justification thn thae case with $a = 0$ previously considered because in the situation ($a \neq 0$) there is fewer pairs destroyed by introduction of the additional quarks.

In Chapter 2 we outline the generalization of the Bogoliubov-Valatin method to the case of finite baryonic number density introduced in Ref. [11]. The results are presented in Chapter 3. In appendices we give mathematical proofs of some of the formulas introduced in Chapter 2.

2. *Review of the model*

Let us recall the method presented in Ref. [11], which is based on the variational approach of Amer et al. [7]. The model QCD hamiltonian for massless quarks interacting through an instantaneous, forth component Lorentz vector colour potential reads

$$H = \sum_{\vec{x}} \psi^+(\vec{x})(-i\vec{\alpha}\cdot\vec{\nabla})\psi(\vec{x}) + \tfrac{1}{2}\sum_{\vec{x},\vec{y},\beta} V(\vec{x}-\vec{y})\left(\psi^+(\vec{x})\frac{\lambda^\beta}{2}\psi(\vec{x})\right)\left(\psi^+(\vec{x})\frac{\lambda^\beta}{2}\psi(\vec{x})\right), \tag{2.1}$$

where $V(\vec{x}) = -V_0^2|\vec{x}|$ and the temporary lattice regulator is introduced. $\vec{\alpha}$, λ^β, $\beta = 1, ..., 8$ are Dirac and Gell-Mann matrices respectively. $\psi(\vec{x})$ is the coloured massless quark field, which can be expanded in the chirally invariant basis of free massless spinors

$$\psi_\alpha(\vec{x}) = \frac{1}{n^{3/2}}\sum_{\vec{k},s}[u^0_{s\alpha}(\vec{k})b^0_{s\alpha}(\vec{k}) + v^0_{s\alpha}(\vec{k})d^{0+}_{s\alpha}(-\vec{k})]e^{i\vec{k}\cdot\vec{x}}, \tag{2.2}$$

where $b^0_{s\alpha}(\vec{k})$ ($d^{0+}_{s\alpha}(\vec{k})$) is the annihilation (creation) operator of a quark (antiquark) with the helicity s and colour α. For the brevity we consider only one flavour $f = 1$.

The trial state $|\Omega\rangle$ is defined as coherent superposition of $q\bar{q}$ pairs [9]

$$|\Omega\rangle = \frac{1}{N}\prod_{\vec{k},s,\alpha}(1 - s\tau\beta(k)b^{0+}_{s\alpha}(\vec{k})d^{0+}_{s\alpha}(-\vec{k}))\,|0\rangle, \tag{2.3}$$

where $N = \prod_{\vec{k},s,\alpha}\sqrt{1+\tau\beta^2(k)}$ is the normalization factor, $\beta(k)$ is a trial wave function of a Cooper pair and τ denotes the volume element in the momentum space. $|0\rangle$ stands for the perturbative vacuum. Equivalently one can use the Bogoliubov-Valatin transformation to define new creation and annihilation operators

$$b_s(\vec{k}) = \cos\frac{\phi(k)}{2}\,b^0_s(\vec{k}) + s\sin\frac{\phi(k)}{2}\,d^{0+}_s(-\vec{k}),$$

$$sd_s(\vec{k}) = -\sin\frac{\phi(k)}{2}\,b^{0+}_s(-\vec{k}) + s\cos\frac{\phi(k)}{2}\,d^0_s(\vec{k}). \tag{2.4}$$

The state $|\Omega\rangle$ can be considered as a quasiparticle vacuum provided that

$$\sin\frac{\phi(k)}{2} = \frac{\beta(k)}{\sqrt{1+\beta^2(k)}} \text{ and } \cos\frac{\phi(k)}{2} = \frac{1}{\sqrt{1+\beta^2(k)}}. \tag{2.5}$$

The Bogoliubov-Valatin transformation consists in writing the quark field $\psi(\vec{x})$ in terms of an arbitrary spinors u, v

$$\psi(\vec{x}) = \frac{1}{n^{3/2}}\sum_{\vec{k},s}[u_s(\vec{k})b_s(\vec{k})+v_s(\vec{k})d_s^+(-\vec{k})]e^{i\vec{x}\cdot\vec{k}}. \tag{2.6}$$

Now the spinors $u_s(\vec{k})$, $v_s(\vec{k})$ need not satisfy masless Dirac equation.

We now introduce the state with finite baryonic number

$$|B\rangle = \prod_{\substack{s_1s_2s_3\\ |\vec{p}|<K_F}} \frac{1}{3!}(b_{s_1a}^+(\vec{p})b_{s_2b}^+(\vec{p})b_{s_3c}^+(\vec{p})\varepsilon_{abc})\,|\Omega\rangle, \tag{2.7}$$

with $|\Omega\rangle$ and $b_{as}^+(\vec{p})$ defined by (2.3) and (2.4) respectively. The colour singlet character of $|B\rangle$ is evident from (2.7). We did not have to introduce the chemical potential to the description because baryonic number is a constant of motion and we work in zero temperature. The state $|B\rangle$ automatically represents the sector in Hilbert space with a finite baryonic background because [11]

$$\varrho_B = \sigma_f\frac{K_F^3}{6\pi^2}, \tag{2.8}$$

where σ_f is the number of baryonic states which can be built from three quarks and equals four (twenty) for one (two) flavour(s). This property of state (2.7) is very important because this allows for easy monitoring of the influence of ϱ_B on the stability of various vacua.

In the variational method one looks for the stationary point of the average energy

$$E[\phi] = \langle B|H|B\rangle. \tag{2.9}$$

The resulting gap equation $\frac{\delta E}{\delta\phi} = 0$ is difficult to solve analytically for a linear potential. Instead, we may try to see if the system with chiral invariant vacuum ($\phi \equiv 0$) is stable. To this end, we must first calculate the average energy (2.9). The quantity $\langle B|H|B\rangle$ can be computed with the aid of Wick's theorem[1]. The latter implies that H may be rewritten in the form

$$H = \mathscr{E} + :H_2: + :H_4:,$$

[1] Introduction of the normal ordering to the model was briefly considered in papers [7, 9, 12].

where

$$\mathscr{E} = 3\sum_{\vec{k}} \mathrm{Tr}\,[(\vec{\alpha}\cdot\vec{k})\Lambda_-(\vec{k})] + \frac{4}{(an)^3}\sum_{\vec{k},\vec{p}} \tfrac{1}{2}\,\tilde{V}(\vec{k}-\vec{p})\,\mathrm{Tr}\,[\Lambda_+(\vec{k})\Lambda_-(\vec{p})],$$

$$H_2 = \tfrac{2}{3}\frac{1}{n^3}\sum_{\vec{x},\vec{y},\vec{k}} V(\vec{x}-\vec{y})\,[\psi^+(\vec{x})\,(\Lambda_+(\vec{k})-\Lambda_-(\vec{k}))\psi(\vec{y})]e^{i\vec{k}\cdot(\vec{x}-\vec{y})}$$

$$+\sum_{\vec{x}} \psi^+(\vec{x})\,(-i\vec{\alpha}\cdot\vec{\nabla})\psi(\vec{x}),$$

$$H_4 = \sum_{\vec{x},\vec{y},\beta} \tfrac{1}{2}\,V(\vec{x}-\vec{y})\left(\psi^+(\vec{x})\frac{\lambda^\beta}{2}\psi(\vec{x})\right)\left(\psi^+(\vec{y})\frac{\lambda^\beta}{2}\psi(\vec{y})\right), \tag{2.10}$$

$\mathscr{E}$ is the energy of the trial $B-V$ state (2.3). $\Lambda_\pm$ denote the projection operators ($\hat{k} = \vec{k}/|\vec{k}|$):

$$\Lambda_\pm(\vec{k}) = \tfrac{1}{2}\,(1\pm\beta\sin\phi(k)\pm\vec{\alpha}\cdot\hat{k}\cos\phi(k)),$$

and $\tilde{V}(\vec{k})$ is the Fourier transform of the potential

$$\tilde{V}(\vec{k}) = a^3\sum_{\vec{x}} V(\vec{x})e^{i\vec{k}\cdot\vec{x}}.$$

In the case with finite baryonic background ($K_F \neq 0$), all operators: $\mathscr{E}$, H_2, H_4 contribute to $\langle B|H|B\rangle$. Secondly, to prove the instability, it is enough to find such a test function ϕ that the following expression is *negative*

$$\Delta = \langle B|H|B\rangle|_{\phi\neq 0}-\langle B|H|B\rangle|_{\phi=0}.$$

In this paper we take into account the following normalized test function

$$\phi(k) = \frac{1}{A}\,e^{-\frac{1}{2}R^2(k-aK_F)^2}, \tag{2.11}$$

where

$$A^2 = \frac{1}{(4\pi R^2)^{3/2}}\left[\frac{2}{\sqrt{\pi}}\,a\varrho e^{-a^2\varrho^2}+(1+\mathrm{erf}\,(\sqrt{2}a\varrho))\,(1+2a^2\varrho^2)\right]$$

and

$$\mathrm{erf}\,(x) = \sqrt{\frac{2}{\pi}}\int_0^x dt e^{-\frac{t^2}{2}}. \tag{2.12}$$

Let us expose individual contributions to Δ:

$$\Delta = \Delta\mathscr{E}+\Delta E, \tag{2.13}$$

where

$$\Delta\mathscr{E} = \Delta\mathscr{E}_{\rm kin} + \Delta\mathscr{E}_{\rm self} + \Delta\mathscr{E}_{\rm int} \tag{2.14}$$

resulting from interaction in the vacuum state and

$$\Delta E = \Delta E_{\rm kin} + \Delta E_{\rm self} + \Delta E_{\rm int} \tag{2.15}$$

coming from the mutual interactions among baryonic constituents and the sea. Accordingly

$$\Delta E_{\rm self} = \Delta E^{\rm sp}_{\rm self} + \Delta E^{\rm pp}_{\rm self} \quad \text{and} \quad \Delta E_{\rm int} = \Delta E^{\rm sp}_{\rm int} + \Delta E^{\rm pp}_{\rm int}. \tag{2.16}$$

Sandwiching $:H_2:$ and $:H_4:$ with $|B\rangle$ and collecting nonvanishing contractions allow us to identify all contributions listed in Eqs (2.14) and (2.16). Linearizing for small $\phi(k)$ one obtains, after simple Dirac and colour algebra,

$$\Delta\mathscr{E}_{\rm kin} = 3\int \frac{d^3p}{(2\pi)^3}\, p\phi^2(p), \tag{2.17}$$

$$\Delta\mathscr{E}_{\rm self} = \int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\, \tilde{V}(\vec{p}-\vec{k})\,(\phi^2(p)+\phi^2(k))\hat{p}\cdot\hat{k}, \tag{2.18}$$

$$\Delta\mathscr{E}_{\rm int} = -2\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\, \tilde{V}(\vec{p}-\vec{k})\phi(p)\phi(k), \tag{2.19}$$

$$\Delta E_{\rm kin} = -3\int \frac{d^3p}{(2\pi)^3}\, p\phi^2(p)\theta(K_{\rm F}-p), \tag{2.20}$$

$$\Delta E^{\rm sp}_{\rm self} = -2\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\, \tilde{V}(\vec{p}-\vec{k})\,[\phi^2(p)+\phi^2(k)]\hat{k}\cdot\hat{p}\theta(K_{\rm F}-p), \tag{2.21}$$

$$\Delta E^{\rm pp}_{\rm self} = \int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\, \tilde{V}(\vec{p}-\vec{k})\,[\phi^2(p)+\phi^2(k)]\hat{k}\cdot\hat{p}\theta(K_{\rm F}-p)\theta(K_{\rm F}-k), \tag{2.22}$$

$$\Delta E^{\rm sp}_{\rm int} = 4\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\, \tilde{V}(\vec{p}-\vec{k})\phi(p)\phi(k)\theta(K_{\rm F}-p), \tag{2.23}$$

$$\Delta E^{\rm pp}_{\rm int} = -2\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\, \tilde{V}(\vec{p}-\vec{k})\phi(p)\phi(k)\theta(K_{\rm F}-p)\theta(K_{\rm F}-k). \tag{2.24}$$

With Gaussian ansatz (2.11) for $\phi(p)$ most of the integrals can be calculated analytically. Calculation of $\Delta\mathscr{E}_{\rm kin}$, $\Delta\mathscr{E}_{\rm self}$ and $\Delta E_{\rm kin}$ are easily obtained. In calculation of $\Delta\mathscr{E}_{\rm self}$ we used the result [7]

$$\int \frac{d^3p}{(2\pi)^3}\, \tilde{V}(\vec{p}-\vec{k})\hat{p}\cdot\hat{k} = -\frac{4V_0^2}{\pi k}, \tag{2.25}$$

and the integral (2.19) is done in Appendix A. The results are

$$\Delta\mathscr{E}_{\text{kin}} = \frac{3}{(2\pi)^2 R^4 A^2}\left((1+a^2\varrho^2)e^{-a^2\varrho^2}+\sqrt{\pi}\,a\varrho(\tfrac{3}{2}+a^2\varrho^2)\left(\text{erf}\,(\sqrt{2}\,a\varrho)+1\right)\right), \tag{2.26}$$

$$\Delta\mathscr{E}_{\text{self}} = \frac{2V_0^2}{\pi^3 R^2 A^2}\left[e^{-a^2\varrho^2}+\sqrt{\pi}\,a\varrho(1+\text{erf}\,(\sqrt{2}a\varrho))\right], \tag{2.27}$$

$$\Delta\mathscr{E}_{\text{int}} = \frac{-2V_0^2}{\pi^3 R^2 A^2}\int_0^\infty dp \int_0^\infty \frac{dq}{q^3}\, G(p, q, a\varrho)pe^{-\frac{1}{2}(p-a\varrho)^2}, \tag{2.28}$$

where

$$G(p, q, \varrho) = e^{-\frac{1}{2}(|p-q|-\varrho)^2} - e^{-\frac{1}{2}(p+q-\varrho)^2} + \sqrt{\frac{\pi}{2}}\,\varrho(\text{erf}\,(p+q-\varrho)$$
$$-\text{erf}\,(|p-q|-\varrho)) - 2pqe^{-\frac{1}{2}(p-\varrho)^2},$$

and

$$\Delta E_{\text{kin}} = \frac{-3}{4\pi^2 R^4 A^2}\left[(1+a^2\varrho^2)e^{-a^2\varrho^2}-(1+(1+a+a^2)\varrho^2)e^{-(1-a)^2\varrho^2}\right.$$
$$\left.+\sqrt{\pi}\,a\varrho(\tfrac{3}{2}+a^2\varrho^2)\left(\text{erf}\,(\sqrt{2}\,a\varrho)+\text{erf}\,(\sqrt{2}\,(1-a)\varrho)\right)\right]. \tag{2.29}$$

It is convenient to calculate $\Delta E^{\text{sp}}_{\text{self}}$ by rewriting the integral in the configuration space (cf. Appendix B). One obtains

$$\Delta E^{\text{sp}}_{\text{self}} = \frac{2V_0^2}{\pi^3 R^2 A^2}\left\{\varrho^2 \int_0^\infty dr r \left[\left(\frac{\sin\frac{r\varrho}{2}}{\frac{r\varrho}{2}}\right)^2 - \frac{\sin(r\varrho)}{r\varrho}\right] F(r, a\varrho)\right.$$
$$\left. - e^{-(1-a)^2\varrho^2}+e^{-a^2\varrho^2}+\sqrt{\pi}\,a\varrho(\text{erf}\,(\sqrt{2}\,a\varrho)+\text{erf}\,(\sqrt{2}\,(1-a)\varrho))\right\}, \tag{2.30}$$

where

$$F(r, \varrho) = \left(\varrho\sin(r\varrho)+\left(\frac{1}{r}+\frac{r}{2}\right)\cos(r\varrho)\right)e^{-\varrho^2}e^{-\frac{1}{4}r^2}\int_0^{\frac{r}{2}} dt e^{t^2}\cos(2\varrho t)$$
$$+\left[\left(\tfrac{1}{2}e^{-\varrho^2}e^{-\frac{1}{4}r^2}\int_0^{\frac{r}{2}} dt\sin(2\varrho t)e^{t^2}+\tfrac{1}{4}\sqrt{\pi}\,e^{-\frac{1}{4}r^2}\,\text{erf}\,(\sqrt{2}\,\varrho)\right)+\tfrac{1}{4}\sqrt{\pi}e^{-\frac{1}{4}r^2}\right]$$
$$\times\left[\left(r+\frac{2}{r}\right)\sin(r\varrho)-2\varrho\cos(r\varrho)\right]-\tfrac{1}{2}e^{-\varrho^2}. \tag{2.31}$$

$\Delta E^{\rm pp}_{\rm self}$ is more difficult to derive. Careful treatment of the product of three distributions $\tilde{V}(\vec{p}-\vec{k})$, $\theta(K_F-p)$ and $\theta(K_F-k)$ which appear under the integral in (2.22) is required. The model has the good feature of using only colourless trial states (2.7), thus making the results independent of any constant added to the potential (2.1)[2]. This insures a good infrared behaviour [7, 9]. After lengthy calculations [11] one obtains the following result

$$\Delta E^{\rm pp}_{\rm self} = \frac{-4V_0^2}{\pi^3R^2A^2}\int\limits_0^{\varrho} dx x e^{-(x-a\varrho)^2} f\left(\frac{\varrho}{x}\right), \tag{2.32}$$

where

$$f(y) = \tfrac{y}{4}\ln\frac{y+1}{y-1}+\tfrac{3}{2}w(y)-\tfrac{1}{2}\frac{1}{y+1}-v(y)\left(w^2(y)-\frac{1}{(1+y)^2}\right)+g(y),$$

with

$$g(y) = \begin{cases} \tfrac{1}{2}\left(\mathscr{P}\dfrac{1}{y-1}-1\right)-\tfrac{1}{24}(4+y+y^2)\left(\mathscr{P}\dfrac{1}{y-1}-y+1\right) & \text{for} \quad 1<y<2, \\ 1-\dfrac{1}{y-1}-\tfrac{1}{3}\left(1-\dfrac{1}{(1-y)^2}\right) & \text{for} \quad 2<y<\infty, \end{cases}$$

and

$$w(y) = \min\left(1, \frac{1}{y-1}\right), \quad v(y) = \tfrac{1}{24}(4+3y+y^3).$$

Addition of the principial value prescription to the naive (divergent) result is the final outcome of the regularization discussed in [11].

Calculation of $\Delta E^{\rm sp}_{\rm int}$ is analogous to that for $\Delta\mathscr{E}_{\rm int}$. The answer is

$$\Delta E^{\rm sp}_{\rm int} = \frac{4V_0^2}{\pi^3R^2A^2}\int\limits_0^{\varrho} dp\int\limits_0^{\infty} dq\frac{p}{q}e^{-\frac{1}{2}(p-a\varrho)^2}G(p,q,a\varrho). \tag{2.33}$$

Calculation of $\Delta E^{\rm pp}_{\rm int}$ is done in Appendix C. The result reads

$$\Delta E^{\rm pp}_{\rm int} = \frac{-2V_0^2}{\pi^3R^2A^2}\int\limits_0^{\varrho} dp p^2 j(p,\varrho,a)e^{-\frac{1}{2}(p-a\varrho)^2},$$

$$j = j^{\rm I}+j^{\rm II}+j^{\rm III}_{I},$$

$$j_{\rm I}(p,\varrho,a) = \int\limits_0^{\varrho-p} dq\left\{\frac{G(p,q,a\varrho)}{qp}-2e^{-\frac{1}{2}(p-a\varrho)^2}\right\},$$

[2] The contribution from a constant potential is proportional to the Casimir operator of the colour symmetry group and vanishes between colour singlets.

$$j^{\mathrm{II}}(p, \varrho, a) = \int\limits_{\varrho-p}^{p+\varrho} \frac{dq}{q^2} \left\{ \frac{1}{pq} \left[e^{-\frac{1}{2}(|p-q|-a\varrho)^2} - e^{-\frac{1}{2}(p-a\varrho)^2} \right.\right.$$

$$\left.\left. + a\varrho \sqrt{\frac{\pi}{2}} \left(\mathrm{erf}\,(p-a\varrho) - \mathrm{erf}\,(|p-q|-a\varrho) \right) \right] - e^{-\frac{1}{2}(p-a\varrho)^2} \right\},$$

$$j^{\mathrm{III}}(p, \varrho, a) = -\frac{1}{\varrho-p} \left(e^{-\frac{1}{2}(p-a\varrho)^2} - e^{-\frac{1}{2}\varrho^2(1-a)^2} \right) - \frac{1}{\varrho+p} e^{-\frac{1}{2}(p-a\varrho)^2}. \tag{2.34}$$

Collecting together all formulas one obtains the following structure for the total energy difference

$$\Delta = \frac{A(\varrho)}{R} + RV_0^2 B(\varrho), \tag{2.35}$$

where the dimensionless functions $A(\varrho)$, $B(\varrho)$ can be easily read off from Eqs (2.26)–(2.34) and are discussed in the next Chapter.

3. *Results*

The first conclusion which follows from Eqs (2.26–2.34) is that for zero density chiral symmetry is spontaneously broken because (the result was obtained in Ref. [7])

$$\Delta = \frac{6}{\sqrt{\pi} R} \left[1 - \tfrac{2}{3} R^2 V_0^2 \left(\frac{4}{\pi} - 1 \right) \right] \tag{3.1}$$

can be made negative by a suitable choice of the trial function (i.e. of R). For large densities, however, $\Delta \to 0$ because of the Pauli blocking. There is no difference between chirally symmetric and asymmetric state $|B\rangle$ in the limit ($K_F \to \infty$), since introduction of a quark with momentum $\vec{k}$ destroys the pair with this momentum in vacuum state. It is apparent from the definition $|\Omega\rangle$ (2.3). Hence, the answer to the question whether chiral symmetry is restored in this model depends on the behaviour of Δ for intermediate densities and this question is studied in this Chapter.

At first we calculate the separate contributions to the total energy Δ which are plotted in Fig. 1 versus $K_F R$. We perform numerical calculation for $a = 0.5$. For small K_F the large dependence Δ of R is stabilized due to the increase of $B(K_F R)$. Hence, there exists a finite range in R in which Δ is negative. When density raises the kinetic term (function $A(K_F R)$) gives larger contribution to Δ, and Δ becomes positive for all R. Therefore, chiral symmetry is restored in this model, what confirms earlier intuitive arguments [5] and the mean field results [6]. The critical value K_F^C at which the instability of the perturbative vacuum towards the Bogoliubov-Valatin one vanishes is $K_F^C = 0.12\, V_0$. Substituting $V_0 = 400$ MeV [13] we have $K_F^C = 48$ MeV. This in turn gives $\varrho_B = (33\ \mathrm{MeV})^3$. Similar value was found in Ref. [11], where the case $a = 0$ was considered, although, the various contributions to the total energy behave in quite different way. The increase of kinetic energy $A(K_F R)$ near $K_F R = 0$ (see Fig. 2), in the case $a \neq 0$, partly cancels the decrease of interaction energy $B(K_F R)$ producing in the whole small effect. It seems that the model is not very sensitive to the shape of the test function. However, the value (ϱ_B) is much smaller than the generally expected $\varrho_B = (250 \text{ MeV})^3$. Such discrepancies were also obtained by

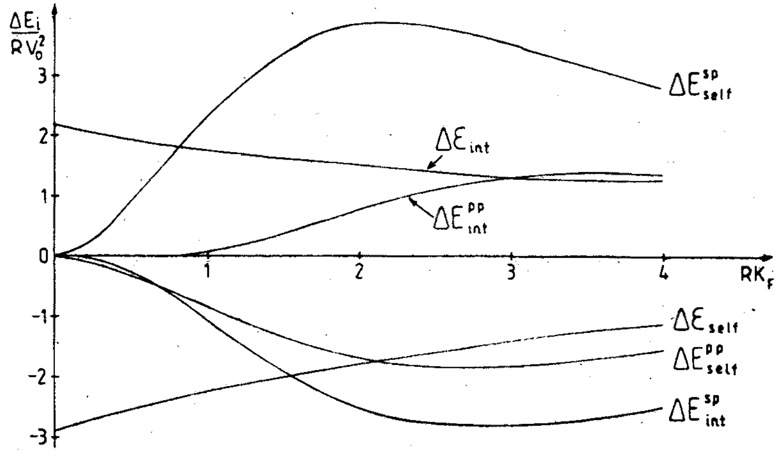

Fig. 1. The energy differences $\Delta\mathcal{E}_{\text{self}}$, $\Delta\mathcal{E}_{\text{int}}$, $\Delta E^{\text{sp}}_{\text{self}}$, $\Delta E^{\text{sp}}_{\text{int}}$, $\Delta E^{\text{pp}}_{\text{self}}$, $\Delta E^{\text{pp}}_{\text{int}}$ as obtained from Eqs (2.27)–(2.34)

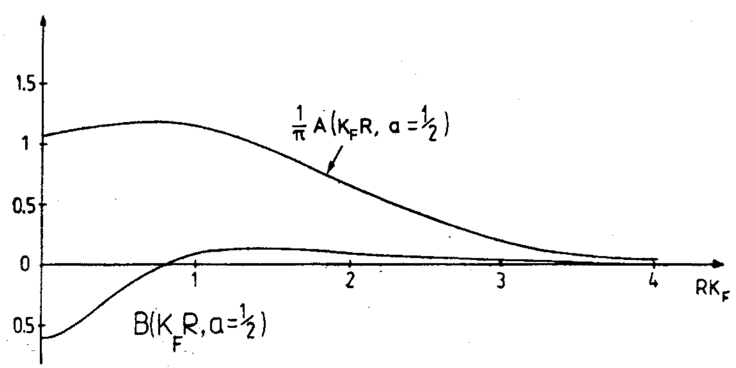

Fig. 2. The kinetic (A) and potential (B) energy differences as functions of RK_F (cf. Eq. (2.35))

several authors [7, 9, 10], who indicate that a potential with additional short-range attraction is needed to describe the quark interaction in the high density regime. We currently investigate the problem. We believe that the addition of the Coulomb potential significantly improves the results.

We may extend our discussion and look at the optimal size of the $q\bar{q}$ pair — the minimum in R dependence of Δ. The optimal radius depends weekly on K_F showing instability around the critical point — similar behaviour was found in Ref. [10, 11]. From Fig. 4 we

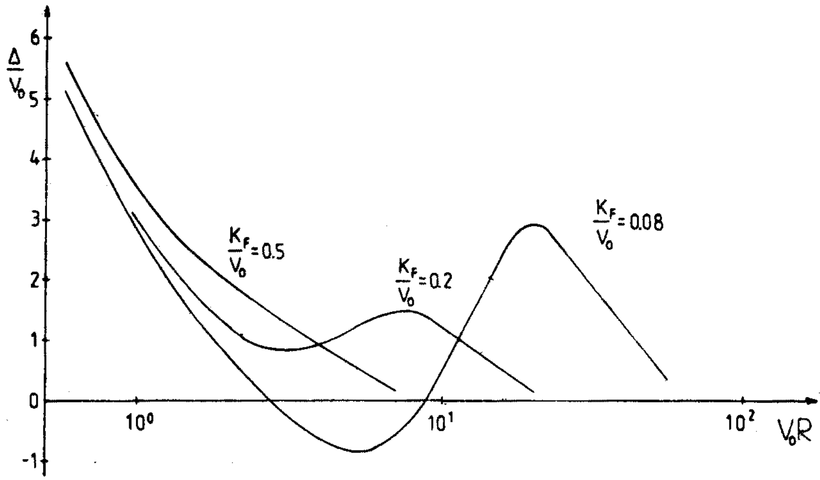

Fig. 3. Total energy difference Δ plotted vs V_0R for three values of K_F/V_0

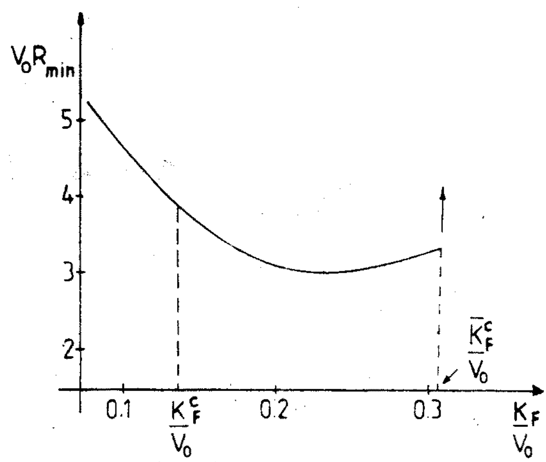

Fig. 4. V_0R_{min} vs K_F/V_0

conclude that the restoration of chiral symmetry occurs through the first order phase transition in the model. This is in agreement with earlier predictions [5, 6]. There also exists a metastable state for densities $0.12\,V_0 < K_F < \overline{K}_F^C = 0.31\,V_0$. However, the conclusions about the order of phase transition and the metastable state may be artifacts of the linear approximation.

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APPENDIX A

In this appendix we calculate $\Delta\mathscr{E}_{\text{int}}$ as defined by Eq. (2.19). We use the same definition of the Fourier transform of the linear potential as Ammer et al. [7]

$$\tilde{V}(\vec{q}) = \lim_{m\to 0} 8\pi V_0^2 \left[\frac{1}{q^2(q^2+m^2)} - \frac{2\pi^2}{m}\delta^{(3)}(\vec{q})\right]. \tag{A1}$$

$\Delta\mathscr{E}_{\text{int}}$ may be written as ($\vec{q} = \vec{k}-\vec{p}$)

$$\Delta\mathscr{E}_{\text{int}} = \frac{8V_0^2}{(2\pi)^5 A^2 R^2}\int d^3q \left[\frac{1}{q^2(q^2+m^2)} - \frac{2\pi^2}{m}\delta^{(3)}(\vec{q})\right] F(q), \tag{A2}$$

where ($\varrho = RK_{\text{F}}$)

$$F(q) = 2\pi \int_0^\infty dp\, p^2 e^{-\frac{1}{2}(p-a\varrho)^2} G(p), \tag{A3}$$

and

$$G(p, q, a\varrho) = \int_{-1}^{1} dt\, e^{-\frac{1}{2}[(q^2+p^2+2qpt)^{1/2}-a\varrho]^2}. \tag{A4}$$

After performing elementary integration

$$G(p, q, \varrho) = \frac{1}{pq}\left\{e^{-\frac{1}{2}(|p-q|-\varrho)^2} - e^{-\frac{1}{2}(p+q-\varrho)^2} + \sqrt{\frac{\pi}{2}}\,\varrho(\operatorname{erf}(p+q+\varrho) - \operatorname{erf}(|p-q|-\varrho))\right\}. \tag{A5}$$

$\Delta\mathscr{E}_{\text{int}}$ may be written in the form

$$\Delta\mathscr{E}_{\text{int}} = \frac{16V_0^2}{(2\pi)^4 A^2 R^2}\left\{\left(\int_0^\infty dq\, \frac{F(q)}{q^2+m^2}\right) - \frac{\pi}{2m}F(0)\right\}, \tag{A6}$$

Inserting for $\dfrac{\pi}{m} = 2\displaystyle\int_0^\infty dq\, \frac{1}{q^2+m^2}$, we obtain the infrared finite expression for $\Delta\mathscr{E}_{\text{int}}$

$$\Delta\mathscr{E}_{\text{int}} = \frac{16V_0^2}{(2\pi)^4 A^2 R^2}\int_0^\infty \frac{dq}{q^2+m^2}(F(q)-F(0)), \tag{A7}$$

so that, the limit $m \to 0$ can be taken. Substituting (A5) and (A3) into (A7) we arrive at result (2.28).

APPENDIX B

The self-energy contribution ΔE^{sp}_{self} is defined by Eq. (2.21). Using Eq. (2.25) the first part of the integral may be easily obtained ($\varrho = RK_F$)

$$-2\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\tilde{V}(\vec{p}-\vec{k})\phi^2(p)\hat{k}\cdot\hat{p}\theta(K_F-p) = \frac{2V_0^2}{\pi^3R^2A^2}$$

$$\times[e^{-a^2\varrho^2}-e^{-(1-a)^2\varrho^2}+\sqrt{\pi}\,a\varrho(\mathrm{erf}(\sqrt{2}\,a\varrho)+\mathrm{erf}(\sqrt{2}\,(1-a)\varrho))]. \tag{B1}$$

It is convenient to calculate the second part of ΔE^{sp}_{self} by rewriting the integral in the configuration space. One obtains

$$-2\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\tilde{V}(\vec{p}-\vec{k})\phi^2(k)\hat{k}\cdot\hat{p}\theta(K_F-p)$$

$$= -2\int d^3rV_0^2r\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}e^{i\vec{r}(\vec{p}-\vec{k})}\phi^2(k)\hat{k}\cdot\hat{p}\theta(K_F-p). \tag{B2}$$

At first we perform the integration over $\vec{k}$. It is useful to define

$$I = \int d^3k\phi^2(k)\hat{k}\cdot\hat{p}e^{-i\vec{r}\cdot\vec{k}}. \tag{B3}$$

I can be rewritten in the form

$$I = f(r, a\varrho)\hat{r}\cdot\hat{p}. \tag{B4}$$

The function $f(r)$ may be obtained by setting $\vec{r} = \vec{p}$ in (B3) and after elementary calculation one arrives at the result (2.31) ($f(r,\varrho) = \frac{-i4\pi}{A^2}F(r,\varrho)$). The subsequent integration, over $\vec{p}$, is easy and we obtain (2.30).

APPENDIX C

In this appendix we calculate the term ΔE^{pp}_{int}, which is given by (2.24). From this definition one easily has

$$\Delta E^{pp}_{int} = -\frac{2V_0^2}{\pi^3R^2A^2}\int_0^{\varrho} dpp^2j(p,\varrho,a)e^{-\frac{1}{2}(p-a\varrho)^2}, \tag{C1}$$

where $j(p,\varrho,a)$ is equal to

$$j(p,\varrho,a) = \int_0^{\infty}\frac{dq}{q^2}(G(q)-G(0)), \tag{C2}$$

with $G(q)$ defined by

$$G(p, q, \varrho) = \int_{-1}^{1} dt e^{-\frac{1}{2}[\sqrt{q^2+p^2-2qpt}-a\varrho]^2}\theta(\varrho - \sqrt{q^2+p^2-2pqt}). \tag{C3}$$

The integration over t is simple. The result reads

$$G(p, \varrho, a) = \frac{\theta(\varrho+p-q)}{pq}\Bigg[e^{-\frac{1}{2}(|p-q|-a\varrho)^2} - e^{-\frac{1}{2}(\min(\varrho(1-a),p+q-a\varrho))^2}$$

$$+a\varrho\sqrt{\frac{\pi}{2}}\{\mathrm{erf}\,(\min\,(\varrho(1-a),\,p+q-a\varrho))-\mathrm{erf}\,(|p-q|-a\varrho))\Bigg]. \tag{C4}$$

The integral (C1) is logarithmically divergent at $p = K_F$. We single out the divergent part explicitly. To do this, it is convenient to split q-integration into three parts

$$AI: \qquad 0 < q \leqslant K_F - p,$$

$$AII: \quad K_F - p < q \leqslant K_F + p,$$

$$AIII: \quad K_F + p < q \leqslant \infty.$$

Accordingly

$$j = J^{I} + J^{II} + J^{III} \quad \text{and} \quad J^{II} = J^{II}_{SING} + J^{II}_{REG}.$$

J^{I} and J^{III} are regular at $p = K_F$ (j^{I} defined in Chapter 2 is equal to J^{I}). Expanding $G(q)$ in the ranges AI and $AIII$ one has

$$j^{I}(p, \varrho, a) = \int_{0}^{\varrho-p} dq \left\{\frac{G(p, q, a\varrho)}{qp} - 2e^{-\frac{1}{2}(p-a\varrho)^2}\right\}, \tag{C5}$$

$$J^{III}(p, \varrho, a) = -\frac{2}{\varrho+p}e^{-\frac{1}{2}(p-a\varrho)^2} \tag{C6}$$

and

$$J^{II} = \int_{\varrho-p}^{\varrho+p} \frac{dq}{q^2}\left\{\frac{1}{pq}\Bigg[e^{-\frac{1}{2}(|p-q|-a\varrho)^2} - e^{-\frac{1}{2}(\varrho(1-a))^2}\right.$$

$$\left.+a\varrho\sqrt{\frac{\pi}{2}}(\mathrm{erf}\,(\varrho(1-a))-\mathrm{erf}\,(|p-q|-a\varrho))\Bigg] - 2e^{-\frac{1}{2}(p-a\varrho)^2}\right\}. \tag{C7}$$

Whole singularity comes from the J^{II} term. In order to separate the most divergent contribution we add and subtract $\frac{1}{pq}e^{-\frac{1}{2}(p-a\varrho)^2} + a\varrho\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\,(p-\varrho a)$ under the integral in (C7).

J^{II} splits into parts

$$J^{\mathrm{II}}_{\mathrm{REG}} = \int\limits_{\varrho-p}^{\varrho+p} \frac{dq}{q^2}\left\{\frac{1}{pq}\left[e^{-\frac{1}{2}(|p-q|-a\varrho)^2} - e^{-\frac{1}{2}(p-a\varrho)^2}\right.\right.$$

$$\left.\left. + a\varrho\sqrt{\frac{\pi}{2}}\,(\mathrm{erf}\,(p-\varrho a)) - \mathrm{erf}\,(|p-q|-a\varrho))\right] - e^{-\frac{1}{2}(p-a\varrho)^2}\right\}, \tag{C8}$$

which has logarithmic singularity at $p = K_{\mathrm{F}}(j^{\mathrm{II}} = J^{\mathrm{II}}_{\mathrm{REG}})$, and

$$J^{\mathrm{II}}_{\mathrm{SING}} = \int\limits_{\varrho-p}^{\varrho+p} \frac{dq}{q^2}\left\{\frac{1}{pq}\left[e^{-\frac{1}{2}(p-a\varrho)^2} - e^{-\frac{1}{2}(1-a)^2\varrho^2}\right.\right.$$

$$\left.\left. + a\varrho\sqrt{\frac{\pi}{2}}\,(\mathrm{erf}\,(\varrho(1-a)) - \mathrm{erf}\,(p-a\varrho))\right] - e^{-\frac{1}{2}(p-a\varrho)^2}\right\}. \tag{C9}$$

Linear divergence of $J^{\mathrm{II}}_{\mathrm{SING}}$ in $p = K_{\mathrm{F}}$ is explicit after performing the integral (C9)

$$J^{\mathrm{II}}_{\mathrm{SING}} = \frac{1}{p}\left[e^{-\frac{1}{2}(p-a\varrho)^2} - e^{-\frac{1}{2}(1-a)^2\varrho^2} + a\varrho\sqrt{\frac{\pi}{2}}\,(\mathrm{erf}\,(\varrho(1-a))\right.$$

$$\left. - \mathrm{erf}\,(p-a\varrho))\right]\left(\frac{1}{2(\varrho-p)^2} - \frac{1}{2(\varrho+p)^2}\right) - e^{-\frac{1}{2}(p-a\varrho)^2}\left(\frac{1}{\varrho-p} - \frac{1}{\varrho+p}\right).$$

According to Ref. [11] the naive result should be supplemented with the principal value ($\mathscr{P}$) prescription for the integration of p in (C1) ($j^{\mathrm{III}} = J^{\mathrm{III}} + \mathscr{P}(J^{\mathrm{II}}_{\mathrm{SING}})$). Collecting all formulas together gives (2.34).

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