DILATATION OF LIFE-TIMES OF MICRO-CLOCKS AND QUANTUM NONLOCALITY

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Dilatation effect of life-times of "micro-clocks" shows a controversy between the relativistic and quantum symmetries connected with quantum nonlocality that has recently been discussed in connection with Bell's inequalities violation. This controversy can be overcome within the hypothesis of internal spacetime R_4 of "relations" by making "events" of Minkowskian spacetime L_4 analyzable.

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1. Time dilatation and its measurement

Let us start with a general remark that proper-time and proper-space intervals, T_0 and L_0, of classical macro-clocks and macro-rods represent *two* internal absolute characteristics of those objects that also create the metrical spacetimes of both Galileo (G_4) and Minkowski (L_4). The relativization of time and space intervals by the L_4 geometry is thus secondary to the absoluteness of T_0 and L_0 and connected with measurement which, as such, introduces the outer world interacting with entities measured. However, empty L_4 deals with only *one* L-invariant four-interval $x^2 = (X_2 - X_1)^2$ ($X_{1,2}$ denote two events) and so the L_4 geometry has to deal with a negative balance "$-2+1 = -1$". The same balance of the G_4 geometry is equalized ("$-2+2 = 0$"), because G_4 deals with *two* G-invariant time and space intervals which directly parametrize the absolute characteristics T_0 and L_0. It turns out that the negative balance of the L_4 geometry leads to a controversy between the quantum symmetry Q and the relativistic-classical symmetry L while describing the dilatation effect of micro-clock. Our intension is to present that controversy and then, to solve it within the hypothesis of internal spacetime R_4 put forward in paper [1].

Let $\Delta t^* = T_0$ be the time interval of the "moving" clock in its rest-frame S^* and $\Delta t = T$ the corresponding time interval registered by the clocks of the reference frame S where our clock moves. As

$$\Delta t = \Gamma(\Delta t^* - V\Delta Z^*/c^2); \quad \Gamma = (1-V^2/c^2)^{-1/2}, \tag{1.1}$$

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the classical world-line attached to the clock implies that $\Delta Z^* = 0$ and hence, from (1.1), we obtain the dilatation effect

$$T = \Gamma T_0. \tag{1.2}$$

The classical world-line attached to the clock implies that together with sharp localization ($\Delta Z^* = 0$), its velocities V^* and V are sharply defined ($\Delta V^* = \Delta V = 0$) which implies that the dilatation factor Γ is sharply defined too ($\Delta\Gamma = 0$). However, the two constraints, $\Delta Z^* = \Delta V^* = 0$, are in conflict with finite inertia M of micro-clock A_M, since

$$\Delta Z^* \gtrsim \hbar/\Delta P^* \doteq (\hbar/M)(1/\Delta(\Gamma^* V^*)). \tag{1.3}$$

Indeed, if $\Delta Z^* \to 0$ then $\Delta(\Gamma^* V^*) \to \infty$ hence $\Delta\Gamma \to \infty$ and the dilatation factor, together with the dilatation effect, cease to be defined. On the other hand, if $\Delta V^* \to \Delta V \to 0$ which makes the dilatation factor sharply defined ($\Delta\Gamma \to 0$), then $\Delta Z^* \to \infty$. This quantum nonlocality makes it impossible to draw the dilatation effect (1.2) from (1.1). A paradox! As seen from (1.3), the two constraints $\Delta V^* = \Delta Z^* = 0$ remain consistent with the uncertainty relation provided $M \to \infty$. Then, however, one regains the classical macro--clock with classical world-line, like in the mathematical limit $\hbar \to 0$ which characterizes the classical framework, where both uncertainties ΔZ^* and ΔV^* are a priori equal to zero.

In order to explain why this paradox does not interfere with experiments which determine the life-times of "relativistic" micro-clocks and prove the dilatation effect, let us analyze two kinds of those experiments. They will correspond to relatively long and short life-times of micro-clocks A_M. In the first kind of experiments (*i*), T is measured *directly*, i.e. in the "x" spacetime of a given lab-system, while in the second (*ii*), it is measured *indirectly*, in the "p" language, by measuring the suitable dispersion (uncertainty) of energy of A_M.

(*i*) For relatively long-living A_M's direct measurement of the life-time T and its dilatation (in a given laboratory) is possible if the lengths l of tracks of A_M's fulfill strong inequality

$$l = VT \gg a, \tag{1.4}$$

where V is the velocity of A_M and a is the diameter of the spot (e.g. in the photographic plate) created by A_M colliding inelastically with "atoms" of the medium. For high-energy A_M's their energy losses for creating observable tracks and hence the decrease of the Γ factor are very small, $\Delta\Gamma/\Gamma \ll 1$ (though finite!), and so the dilatation factor remains defined quite well (though not sharply). This picture might give an impression that the classical trajectory (world-line) of a micro-clock is its adequate characteristic. However, that would question the completeness of quantum description of micro-object A_M by "ψ" function and does not solve the paradox itself. The point is that inelastic collisions *do not create* the dilatation effect, but merely make it directly observable, provided that the strong inequality (1.4) is fulfilled.

The clue to the success of quantum (complete) description is in what Heisenberg calls [2] the "quantum-potentiality" expressed by nonlocal "ψ" describing an individual micro--object and opposed to the local "classical-actuality" of any registration of a spot by macro-

-devices. Therefore, the dilatation effect must follow from the uncertainty relation

$$T = \hbar/\Delta E, \tag{1.5}$$

where ΔE is the energy uncertainty of an individual (unstable) micro-object A_M of the following meaning. If W denotes the internal energy of A_M then, in the rest-frame S^* of A_M, $E^* = W$ and $\Delta E^* = \Delta W$ hence,

$$T_0 = \hbar/\Delta W \tag{1.6}$$

should denote the proper life-time of A_M. Consequently, ΔE which enters (1.5) must be due to the uncertainty of the internal energy W only and so, the quantum description of A_M requires T to be subject to the dilatation effect; $T = \Gamma T_0$.

Let us emphasize that direct registration of the spots which create the track of A_M itself excludes the determination of ΔE. In other words, the measurement of the track of A_M excludes indirect measurement of its dilatation effect, which can be seen as follows. The energy uncertainty δE due to inelastic collisions resulting in a registable spot is of the order of

$$\delta E = (\partial E/\partial P)\Delta P \gtrsim V(\hbar/\Delta X) \cong V(\hbar/a),$$

where $\Delta X \cong a$ and hence, $a \gtrsim V(\hbar/\delta E)$. As $l = VT = V(\hbar/\Delta E) \gg a$,

$$\delta E \gg \Delta E \tag{1.7}$$

which just means that ΔE disappears in the energy fluctuations of the order of δE. (*ii*) For short-living A_M's $l = VT$ can be even smaller than the atomic radius and hence the condition (1.4) cannot be fulfilled. Then we are forced to determine T indirectly but, according to (1.7), this measurement must be performed on the decay products of A_M and not on A_M itself. Then let us analyze a typical measurement assuming that A_M decays into N stable particles A_J ($J = 1, \ldots, N$) with known masses m_J ($\Delta m_J = 0$). Let the asymptotic momenta $\boldsymbol{p}_J^{(k)}$ and hence the energies $e_J^{(k)} = c(m_J^2c^2 + \boldsymbol{p}_J^{(k)2})^{1/2}$ of A_J in the k-th decay-event of $A_M^{(k)}$, be measured with any accuracy. Thus $p_J^{(k)} = (\boldsymbol{p}_J^{(k)}; (i/c)e_J^{(k)})$ denotes the four-momentum of $A_J^{(k)}$ ($p_J^{(k)2} = -m_J^2c^2$) and, according to the energy-momentum conservation

$$P^{(k)} = \sum_{J/1}^{N} p_J^{(k)} \tag{1.8}$$

is the four-momentum of $A_M^{(k)}$ after its decay which thus realizes a sharply defined mass $M^{(k)}$ of $A_M^{(k)}$ determined by $P^{(k)}$. However, the statistical nature of quantum predictions forces one to deal with a rich *ensemble* of $A_M^{(k)}$'s in order to determine the energy uncertainty of an *individual* micro-object A_M. Meanwhile, in a fixed laboratory we deal with short-living A_M's of different and a priori uncontrollable (though a posteriori measurable) momenta $\boldsymbol{P}^{(k)}$. Therefore, the only quantity determined by $P^{(k)}$'s and free of uncontrollable spread of $\boldsymbol{P}^{(k)}$'s is the L-invariant internal energy $W = Mc^2$ of A_M where,

$$W^{(k)} = c(-P^{(k)2})^{1/2} = M^{(k)}c^2. \tag{1.9}$$

The sufficiently rich sample of $W^{(k)}$'s determines the invariant internal energy $W_0 = M_0c^2$ as the mean value of $W^{(k)}$'s, and the dispersion $\Delta W = \Delta Mc^2$ of $W^{(k)}$'s which, by virtue of (1.6), determines the proper life-time T_0 of an individual particle A_M.

Thus indirect, much like direct measurements of T are free of the paradox and consistent with the relativistic symmetry, however, and here is the point, indirect measurements cannot prove whether T from (1.5) is or is not subject to the dilatation effect, because we can only determine T_0. In the next Section we show that the statistical aspect of the measurements of T conceals a controversy between the relativistic symmetry L and the quantum symmetry Q which concerns an individual object A_M described by "ψ".

2. *Quantum-relativistic controversy*

The wave function is characteristic of an individual isolated micro-object and hence the uncertainty ΔM of the mass of A_M must be attached to each micro-object A_M. According to the L_4-*framework* which regards L_4 as the first background not only of the classical-actuality of events ("Alles ist Coincidenz") but also of the quantum-potential relations and by virtue of (1.8), A_M should be in a state superposed of different P's, as $P^2 = -M^2c^2$ or,

$$E = (W^2+c^2\boldsymbol{P}^2)^{1/2}, \tag{2.1}$$

where E is the total energy of A_M. The Legendre transformation

$$\boldsymbol{V} = \partial E/\partial \boldsymbol{P} = c^2(\boldsymbol{P}/E), \tag{2.2}$$

which introduces the classical language of velocities and constitutes the classical Hamiltonian equations, implies that

$$E = \Gamma W, \quad \boldsymbol{P} = (W/c^2)\Gamma \boldsymbol{V}; \quad \Gamma = (1-V^2/c^2)^{-1/2} \tag{2.3}$$

and

$$\partial \boldsymbol{P}/\partial W = \Gamma(\boldsymbol{V}/c^2) \neq 0 \text{ (except } S^* \text{ where } \boldsymbol{V} = 0). \tag{2.4}$$

Assuming that A_M is at rest in S^* hence $P^* = (0, 0, 0; (i/c)W)$, the same relations (2.3) follow from the Lorentz transformation of P. Thus $\boldsymbol{V}$ denotes a sharply defined velocity between S^* and S and hence, the uncertainty of energy ΔE from (1.5) and the corresponding uncertainty of the momentum which follow from (2.3) are equal to

$$\Delta E^{\mathrm{L}} = \Gamma \Delta W, \quad \Delta \boldsymbol{P}^{\mathrm{L}} = (\Delta W/c^2)\Gamma \boldsymbol{V}. \tag{2.5}$$

The index "L" has been added to the uncertainties of the energy and the momentum in order to emphasize their relativistic origin. Note that according to the classical language of velocities of the L symmetry, the momentum $\boldsymbol{P}$ depends a priori on the internal energy W of A_M as stated in (2.4). Finally, from (1.5) and (2.5)

$$T^{\mathrm{L}} = \hbar/\Delta E^{\mathrm{L}} = T_0/\Gamma \neq \Gamma T_0 \text{ (except } S^* \text{ where } \Gamma = 1) \tag{2.6}$$

which just expresses the quantum-relativistic controversy. This controversy can also be seen as a consequence of the negative balance of the L_4 geometry which has to deal with only *one* L-invariant P^2. The life-time problem, however, calls for *two* L-absolute quantities W_0 and ΔW. As seen from (2.4), in the nonrelativistic (NR) limit $c \to \infty$ when $L_4 \to G_4$, the Galilean momentum $\boldsymbol{P}^G$ becomes independent of the internal energy W^G of A_M, i.e.,

$$\partial \boldsymbol{P}^G / \partial W^G = 0. \tag{2.7}$$

Thus, there is no quantum-NR controversy, as it was to be expected, because the equalized balance of the G_4 geometry is strictly connected with the absoluteness of Newtonian time.

Let us neglect the relativistic symmetry L for a moment. Then the quantum symmetry Q enables one to regard $\boldsymbol{P}$ as an independent variable and, at the same time, to maintain the expression (2.1) of the energy E. In fact, the momentum operator $\hat{\boldsymbol{P}} = -i\hbar\partial/\partial \boldsymbol{X}$ is a priori independent of the internal state and hence, of the internal energy of A_M and so its eigenvalues (in a fixed S!) become independent variables, which, much like in the NR--framework, cf. (2.7), implies

$$\partial \boldsymbol{P} / \partial W = 0. \tag{2.8}$$

Taking into account (2.1) and (2.8), $\Delta E = \Delta E^Q$ amounts to

$$\Delta E^Q = (\partial E/\partial W)\Delta W = \Delta W/\Gamma; \quad \Gamma = (1 + c^2\boldsymbol{P}^2/W^2)^{1/2} \tag{2.9}$$

which, by virtue of (1.5) and (1.6), results in the correct dilatation effect

$$T^Q = \hbar/\Delta E^Q = \Gamma T_0. \tag{2.10}$$

The quantum-relativistic controversy (2.6) together with the proper answer (2.10) as to the dilatation question of T given by Q symmetry (2.8) strongly favour the hypothesis of internal spacetime R_4 of "relations", which would precede the external spacetime L_4 of "events". According to that hypothesis, quantum states "ψ" do not represent "event--shapes" in L_4 but "relation-shapes" in R_3 — cf. Sect. 3. As a matter of fact, the same suggestion follows from the spacetime nonlocality of ψ recognized for the first time by Einstein [3] and proved in a spectacular way by recent experiments [4] violating Bell's inequalities [5]. The peculiarity of microphysics consists in the fact that the registration of an individual micro-process (such as the decay of micro-object A_M) can be done only once by a single lab-system [6], say, on a given photographic plate. Therefore, in spite of the fact that the decay products of a single particle A_M create the four-momentum $P^{(k)}$ (in L_4) determining a sharp mass $M^{(k)}$, cf. (1.9), the state ψ of an individual unstable A_M which contains ("potentially") different masses M is not necessarily embedded in L_4. According to R_4, ψ represents a relation-shape in R_4, which, as will be shown, remains consistent with the Einsteinian principle of relativity. Let us remember that indirect measurements of T do not question the L_4 geometry of measurement but, as a rule, provide us with $T = T_0$ without proving or disproving the dilatation effect. Thus the statistical nature of quantum predictions would conceal the controversy (2.6) ensuring that the L_4 geometry is adequate for any measurement, but it does not cancel this controversy, because it concerns an individual micro-object A_M.

3. R_4 geometry of relations

In order to explain how a physical continuum different from L_4 can be introduced without violating the Einsteinian principle of relativity, let us remember the difference between the L_4 and G_4 geometries. Note namely that the metrical relations of both G_4 and L_4 space-times can coincide in some fixed reference frame S^*, which means that

$$X^* = X^{G*}, \quad t^* = t^{G*}, \tag{3.1}$$

where (X, t) and (X^G, t^G) denote the events in L_4 and G_4, respectively. However, in spite of the coincidence of *metrical* relations (3.1) in S^*, the L_4 and G_4 spacetimes are different because of the difference of the *symmetries* of their points = events. Otherwise, we could never discover the relativity theory, i.e. the true symmetry L of events suggested by the L symmetry of the Maxwell equations. The difference between R_4 and L_4 also concerns their symmetries only, however; the analogy ceases to work here, as the hypothesis of R_4 must be consistent with the L symmetry of *directly observable* events.

The R_4 geometry recognizes the *directly unobservable* "relations" between micro--objects as preceding "events" of L_4. According to an old thesis of Landau and Peierls [7], free four-momenta P_A of particles "A" scattered to the asymptotic zone of infinitely heavy "bases" (in Bohr's terminology [8]) are the only directly observable quantities of quantum--relativistic objects. The same is assumed in the S-matrix theory as parametrized by the Mandelstam variables. This, together with the non-commutative algebra of the "$x-p$" canonical variables, enables one to introduce the internal spacetime R_4 of absolute relations. According to [7] it must be required that if relations are referred to infinitely heavy "bases" (measuring tools), as implied by any measurement, then the symmetry $R \neq L$ of relations must convert into the symmetry L of events of L_4. In this way, events would become analyzable in terms of the more elementary relations of the R_4 geometry.

The internal space R_3 extends in a way the absoluteness of classical space relations referred to a given "basis" which remains static (in its own rest-frame) even that interacting with another entity. Consequently, the hypothesis of R_4 must take for granted the fact that isolation of a micro-system precedes its observation (measurement) by the outer world equipped with the "bases". Following the Landau-Peierls thesis, let us consider the class of auxiliary L-invariant observables $\tilde{G}(p^2)$ where p is the four-momentum built from P_A's. We start with defining the three-dimensional space $\tilde{R}_3$ of L-absolute momenta $\boldsymbol{q}$ as a space where the functions $\tilde{F}(\boldsymbol{q}^2)$ are embedded, such that

$$\tilde{F}(\boldsymbol{q}^2) = \tilde{G}(p^2 = \boldsymbol{q}^2 \geqslant 0). \tag{3.2}$$

The unitary Fourier transform

$$F(\boldsymbol{y}^2) = (2\pi\hbar)^{-3} \int d^3q \tilde{F}(\boldsymbol{q}^2) \exp\left[i/\hbar(\boldsymbol{q}\boldsymbol{y})\right] \tag{3.3}$$

which expresses the non-commutative, quantum-canonical algebra thus determines the L-absolute *relation-shape* $F(\boldsymbol{y}^2)$ embedded, by definition, in the three-dimensional internal space R_3 of "relations" $\boldsymbol{y}$. At the same time, the L_4 four-geometry determines the corresponding L-invariant function (distribution)

$$G(x^2) = (2\pi\hbar)^{-4} \int d^4p\tilde{G}(p^2) \exp [i/\hbar(px)], \tag{3.4}$$

where, following the translation invariance of $\tilde{G}(p^2)$ in L_4, $x = X_2 - X_1$ denotes the relative four-coordinate in L_4. As spanned on the L-invariant four-interval x^2, $G(x^2)$ does not represent any event-shape and, according to (3.2), we shall say that $F(y^2)$ and $G(x^2)$ represent "the same" relation-shape in R_3 and L_4, respectively. This will be denoted henceforth by "$\doteq$",

$$F(y^2) \doteq G(x^2). \tag{3.5}$$

Definitions (3.2)–(3.4) imply the numerical identity

$$\int_{-\infty}^{+\infty} dx_0 G(x^2 - x_0^2) = F(x^2 = y^2). \tag{3.6}$$

From now on, R_3 is recognized as an independent space that precedes L_4. The L-absoluteness of R_3 means that any object embedded in R_3 is automatically L-absolute. Thus R_3 is different from any space $E_3^{(S)}$ of the reference frame S in L_4. Note that the so-called "semi--relativistic" models [9] which assume the L_4-framework distinguish some reference frames S^*. As pointed out by Dirac et al. [10] and other authors e.g. [11], these models conflict with the Einsteinian principle of relativity.

In spite of "the sameness" (3.5) of c-numbers F and G, laws of motion in R_3 can go beyond those in L_4, because q-numbers in R_3,

$$\hat{Q} = \hat{Q}(y, \hat{q}); \quad \hat{q} = -i\hbar\partial/\partial y, \tag{3.7}$$

cannot be translated into the L_4 geometry language. In spite of that, the L-absolute laws in R_3 expressed by $\hat{Q}$'s will provide us with L-absolute c-number characteristics, which can be translated into the language of the L_4 geometry of measurement — see Appendix I.

The internal energy operator $\hat{h}$ of a two-body system is an example of an L-absolute q-number in R_3, which, according to the Einsteinian energy-mass relation, can be taken in the form

$$\hat{h} = c[(M_0^2c^2 + \hat{q}^2)^{1/2} + (M_1^2c^2 + \hat{q}^2)^{1/2}] + V(y^2). \tag{3.8}$$

The relation-shape $V(y^2)$ denotes the potential of the internal force that acts at a distance in R_3 and fulfills the third Newtonian law. According to (3.5), $V(y^2)$ describes "the same" relation-shape as the L-invariant function (distribution) $U(x^2) \doteq V(y^2)$ in L_4. The L-absoluteness of R_3 implies that the Schroedinger equation,

$$i\hbar\partial/\partial\tau\psi(y, \tau) = \hat{h}\psi(y, \tau) \tag{3.9}$$

defines the L-absolute internal time continuum $\mathscr{T}(\tau)$ which completes R_3 to the four--dimensional internal spacetime R_4. As the internal symmetry R of Eq. (3.9) and hence of R_4 consists of rotations in R_3 and translations in τ, $R_4 = R_3(y) \times \mathscr{T}(\tau)$.

The internal angular-momentum $\hat{j} = y \wedge \hat{q}$ represents another L-absolute q-number resulting in L-absolute internal angular-momenta ($\hbar j, j = 0, 1, \ldots$) which, very much like spins, represent off-spacetime characteristics in agreement with $R_4 \neq L_4$. Weak four-parameter symmetry R of R_4 makes room for a much wider class of dynamical models than does the strong ten-parameter symmetry L which is exceedingly restrictive for any dynamics [12] and in the classical and canonical relativistic mechanics it even results in the "no interaction theorem" [13]. With the factorization of the R_4 relations from the outer world of L-symmetric "bases" a problem arises that of the relationship between the characteristics obtained in R_4 and their measurement. This is analyzed in more details in [1] and here let us illustrate it by the following example.

The internal energy eigenvalues $W_n = M_n c^2$ of $\hat{h}$ of an isolated system "$A_M + A_1$" are a priori L-absolute but, since R_4 precedes L_4, no four-momentum is a priori attached to "$A_M + A_1$". It is only a posteriori, when the eigenproblem $\hat{h}\psi_n = W_n \psi_n$ is solved in R_3 that the relativization of W_n can be performed. It consists in attaching to W_n the four-momentum P_n such that $P_n^2 = -M_n^2 c^2$. Only now can we speak of the rest-frame S^* of "$A_M + A_1$" in L_4, where $E_n^* = W_n$ and $P_n^* = (0, 0, 0; iM_n c)$.

In the NR limit ($c \to \infty$) $y \to x = X_2^G - X_1^G$, where the analytic form of x^2 is independent of the reference frame S in G_4, like the analytic forms of $G(x^2) \doteq F(y^2)$ are independent of S in L_4. At the same time $\tau \to \tau^G$, where τ^G coincides (up to an arbitrary additive constant) with the G-absolute Newtonian time. Thus $R_4^G = \lim_{c \to \infty} R_4$ becomes embedded in G_4 and hence the hypothesis of internal spacetime becomes superfluous. The equalized balance of the G_4 geometry makes the question of priority of relations over events in G_4 physically meaningless. Nevertheless, since NR quantum mechanics gives, as a rule, the relation-characteristics (as parametrized by the relative space coordinates in G_4), it strongly favours the R_4-relationism.

The second limit $M_1 \to \infty$, when one end (A_1) of the relation y between stable objects A_M and A_1 becomes a "basis", is crucial for the hypothesis of R_4. After subtracting from $\hat{h}$ the infinite term $M_1 c^2$ representing the internal energy of A_1, Eq. (3.9) converts into a one-body (A_M) equation which can be rewritten in the L-covariant form. The "basis" A_1 drops from equations of motion and the following metrical coincidences take place:

$$y = X^*, \quad \tau = t^* + a_0; \quad q = P^*, \tag{3.10}$$

where S^* is the reference frame where the "basis" A_1 is at rest and localized at the origin of $E_3^{(S^*)}$. Thus X^* and P^* denote the space coordinate and the momentum, respectively, of A_M in S^* but, as known from (3.1), this does not imply that R_4 converts into L_4. The reason that besides the metrical coincidences (3.10), S^* can be identified with one of the reference frames in L_4 is the L-covariant structure of the limiting equation (3.9). Thus the symmetry L of R_4 makes R_4 coinciding with L_4. Consequently, the *real* "basis" A_1 can be replaced by the *mathematical* reference frames S in L_4 and this *Lorentz limit* of R_4 creates the outer world of A_M, while A_1 gets a classical world-line which in (3.10) became identified with the t^*-axis of the rest-frame S^* of A_1. Together with it, X^*, t^* can be identified with the event represented in S^* and P^*, with the space part of the four-momentum $P = (P; iE/c)$ also represented in S^*, where $P^2 = -M^2c^2$. Finally, the internal potential

$V(y)$ in R_4 can be relativized by the four-potential U where,

$$U^* = (0, 0, 0; (i/c)V(y = X^*)). \tag{3.11}$$

Thus, from the inside of the isolated system "$A_M + A_1$" the L_4 geometry has been reproduced of the asymptotic zone of the measuring "bases".

A most striking consequence of the Lorentz limit of R_4 which creates the outer world, and which would explain a well-known discontinuity between the two-body Bethe-Salpeter and the one-body Dirac equations [14] consists in a transition from the relation-shape $V(y^2) \doteq U(x^2)$ off spacetime to the event-shape $U(X)$ which, as seen from (3.11), remains static in S^*.

For noninteracting and stable particles, i.e. in the case of pure kinematics, the Lorentz limit of R_4 does not impose any physical constraints, because, without changing the physics, A_1 can be arbitrarily heavy and hence, the resulting equations of motion in R_4 become L-invariant. The equivalence of the R_4 and L_4 kinematics was to be expected because of the assumed structure of $\hat{h}$ and by the very fact that kinematics makes no use of the quantum non-commutative algebra. However, if the system contains an unstable particle, the limit $M_1 \to \infty$ will preserve the metrical coincidences (3.10), but it will not result in the Lorentz limit of R_4 — see Sect. 4.

Let us emphasize that the R_4 geometry deals with *two* L-absolute intervals,

$$r = |y| \quad \text{and} \quad \Delta\tau, \tag{3.12}$$

which proves that, except for the Lorentz limit of R_4, the points (y, τ) of internal spacetime R_4 are directly unobservable. Indeed, directly observable events determine only *one* L-absolute four-interval x^2 (negative balance of the L_4 geometry).

A singular situation occurs in the presence of an external field which is an event-shape such as U from (3.11). Then U defines the four-velocity $u = cn$ of the "basis" A_1, where $n^* = (0, 0, 0; i)$ and we gain the second L-invariant (nx) which together with x^2 equalizes the negative balance of the L_4 geometry. Indeed,

$$|\mathbf{x}^*| = (x^2 + (nx)^2)^{1/2} \quad \text{and} \quad \Delta t^* = -(nx)/c \tag{3.13}$$

represent *two*, space and time, proper intervals of S^*. This would explain the well-known success of relativistic dynamical models dealing with external fields. Of course, the very presence of any external field (event-shape) in equations of motion spoils the full isolation of the physical system in question. The corresponding laws of motion are then L-covariant, thus consistent with the theory of relativity, but the symmetry L of empty L_4 ceases to be the internal symmetry of those laws.

4. *Dilatation effect in R_4 framework*

The equalized balance of the R_4 geometry, cf. (3.12), enables one to overcome the quantum-relativistic controversy (2.6). As the "R_4-relationism" rules out the one-body problem (as an elementary one) let us introduce the second, besides the unstable A_M, auxiliary but stable object A_1 with mass $M_1(\Delta M_1 = 0)$, thus creating a two-body system "A_M+A_1". We also assume that A_1 does not interact with A_M, which guarantees that the structure of A_M and hence $T_0 = \hbar/\Delta Mc^2$ remain unchanged. An example of weakly interacting unstable system is discussed in Appendix II.

Making use of complex "internal energy", the eigenstate of the L-absolute momentum $\hat{\boldsymbol{q}}$ of our two-body system in R_4 takes the form:

$$\Psi = A\exp\left[i/\hbar[\boldsymbol{q}y - c\{[(M_0 - i\Delta M/2)^2c^2 + \boldsymbol{q}^2]^{1/2} + (M_1^2c^2 + \boldsymbol{q}^2)^{1/2}\}\tau]\right]. \tag{4.1}$$

In order to solve the problem of the relationship between the prediction of the R_4-framework as concern the dilatation effect and the measurement (always performed by the classical "bases") let us assume that $M_1 \to \infty$. The resulting metrical equalities (3.10) then solve this problem although, $M_1 \to \infty$ does not result in the Lorentz limit of R_4. The point is that complex M implies complex "momenta" $\boldsymbol{P}$ and hence, divergent solutions for $|X| \to \infty$, which means that the Klein-Gordon or Dirac equations with complex mass--parameter are not L-covariant. In other words, in spite of the limit $M_1 \to \infty$ we are restricted to the symmetry R of R_4, which means that the real "basis" A_1 cannot be replaced by mathematical reference frames parametrizing L_4. Consequently, assuming that $\Delta M/M \ll 1$ and introducing instead of Ψ, $\psi = \Psi \exp[i/\hbar(M_1c^2\tau)]$, after $M_1 \to \infty$ we get

$$\psi = A\exp\{i/\hbar[\boldsymbol{q}y - c(M_0^2c^2 + \boldsymbol{q}^2)^{1/2}\tau]\}\exp(-\tau/2T^*), \tag{4.1'}$$

with

$$T^* = (1 + \boldsymbol{q}^2/M_0^2c^2)^{1/2}T_0 = (1 + \boldsymbol{P}^{*2}/M_0^2c^2)^{1/2}T_0 = \Gamma^* T_0,$$

but ψ remains an L-absolute relation-shape of A_M and real "basis" A_1 in R_4. This is consistent with the fact that indirect measurements of T do not prove the dilatation effect. By taking into account that $\tau = t^* + a_0$, from (4.1') we get,

$$|\psi|^2 = |B|^2 \exp(-t^*/T^*), \tag{4.2}$$

where $T^* = \Gamma^* T_0$ is the interval of the L-absolute internal time of the two-body system "A_M+A_1". Thus T^* denotes the relaxation-(life-)time of the state ψ of that system. Since A_1 is stable, the instability of the whole system "A_M+A_1" is due to the instability of A_M only and hence, T^* describes the dilatation effect of the life-time of A_M with regard to the "basis" A_1. Of course, one may just as well take another "basis" A_1' instead of A_1, which would result in another dilatation factor $\Gamma^{*\prime}$. However, the new life-time $T^{*\prime} = \Gamma^{*\prime}T_0$ again means an L-absolute relation-characteristic of the two-body problem of the "A_M+A_1'" system and not a relative one-body characteristic of the L_4 geometry. The latter would bring us back to our controversy (2.6).

Taking into account that complex "mass" of "A_M+A_1" is equal to

$$\mathscr{M} = [(M_0 - i\Delta M/2)^2 + \boldsymbol{q}^2/c^2]^{1/2} + (M_1^2 + \boldsymbol{q}^2/c^2)^{1/2} = \mathscr{M}_0 - i\Delta\mathscr{M}/2, \tag{4.3}$$

where

$$\Delta\mathscr{M} = \Delta M/\Gamma^*,$$

we see that the two life-times T_0 and T^* have in fact the same geometrical nature of the proper life-times of A_M and "A_M+A_1", respectively, as

$$T_0 = \hbar/\Delta Mc^2, \quad \text{while} \quad T^* = \hbar/\Delta\mathcal{M}c^2. \tag{4.4}$$

Both life-times are determined by L-absolute uncertainties of the corresponding L-absolute masses M and $\mathcal{M}$. In Appendix II we make use of this fact in evaluating the proper life-times $T_0^{(n)}$ of mezo-atom in the n-th bound state. It is remarkable that bound states maintain the exponential decay law of the constituent muon, in spite of the fact that the bound muon has different Fermi momenta. This conflicts with the classical-like motion in L_4 and, results in an experimental test of the R_4 hypothesis, which will be discussed separately.

5. *Remarks on R_4-relationism*

The R_4-relationism shows that the state (4.1′) of "A_M+A_1" relaxes to zero if only one of its constituents is unstable. Indeed, a variation of one member of any relation modifies the whole relation! The R_4-relationism would then unravel the "mystery" of the "collapse of the wave packet" vastly discussed in connection with Bell's inequalities violation [15]. Of course, this requires a separate analysis, but the following oversimplified (since classical) example illustrates the "realism" of relations.

No classical example can illustrate the behaviour of a non-commuting observable and the difference between the R_4 and L_4 geometries, the more so that there is no "classical limit" of R_4. Nevertheless, the following example embraces some aspects of relations alien to events and therefore it can be helpful in understanding the very "relationism". Let the two parallelograms A_1 and A_2 be at the two ends of a large distance r_{12} and the relation "ψ" in question, the ratio of their hights h_1, h_2,

$$\psi = h_2/h_1. \tag{5.1}$$

Let the measurement of A_1 performed by an external device A_3 be connected with turning the A_1 about 90°, so that the new hight of A_1 becomes equal to its width $h_1' \neq h_1$ from before the measurement. Then the relation (5.1) suddenly changes into a new one

$$\psi' = h_2/h_1' \neq h_2/h_1 = \psi \tag{5.2}$$

without perturbing A_2. This shows the nonlocal nature of the relation ψ as opposed to the local character of the attributes h_1, h_1' of A_1 and h_2, h_2' of A_2. Note that the "local" attributes h_1, h_1' of A_1 as well, as h_2, h_2' of A_2 also express some relational attributes of the four points which determine a parallelogram. The sudden change of ψ into ψ' does not, however, question their realities in the realm of relations.

Bohr, who much like Einstein, recognized the L_4-framework was, after Einstein's arguments [3], forced to definitely give up the realistic interpretation of the quantum-potential relations described by the wave function. He recognized "ψ" as the maximal "information" to be obtained from all possible ("complementary") experiments perfomed with the help of classical (real!) devices [8]. However, this interpretation of "ψ" restricts unjustifiably the (NR) quantum mechanics to a subclass of relations referred to the "bases" when, as we know, R_4 converts into L_4. Whereas, the electron-proton relations of the hydrogen atom structure are hardly reducible to "informations". The atomic structure represents a relation-shape of the electron and proton, which, according to the R_4-relationism, exists in R_4 independently of any measurement ("information").

The "quantum-motion" described e.g. by (4.1′) provides us with another, nontrivial example of the relationism. Note that ψ from (4.1′) is multiplied by the relaxation factor $\exp(-t^*/2T^*)$, where no correlation occurs between the position $\boldsymbol{y} = \boldsymbol{X}^*$ (of A_M with regard to the "basis" A_1) with the time t^* of the relaxation factor. This shows that the quantum-motion does not represent any "transport" of A_M in spacetime. The classical-like correlation between the space and the time localizations of A_M in L_4 requires: (*i*) non--stationary states of a wave packet and (*ii*) the scattering states of "$A_M + A_1$" when its constituents are on their mass-shells. Then indeed,

$$\langle \boldsymbol{X}^* \rangle = \boldsymbol{V}_0^*(t^* - t_0^*); \quad \boldsymbol{V}_0^* = (\partial E^*/\partial \boldsymbol{P}^*)_0$$

and

$$\exp(-t^*/T^*) = C \exp[-\boldsymbol{n}\langle \boldsymbol{X}^* \rangle/(V_0^* T^*)]; \quad \boldsymbol{n} = \boldsymbol{V}_0^*/V_0^*. \tag{5.3}$$

Of course, (5.3) says nothing about the magnitude of the dispersion $\langle(\Delta \boldsymbol{X}^*)^2\rangle^{1/2}$ around $\langle \boldsymbol{X}^* \rangle$ which can be much larger than the mean wave length $\lambda\!\!\!^-_0 = \hbar/|\boldsymbol{P}_0^*|$, thus preserving the wave aspect of the quantum-motion.

Let us emphasize that the lack of correlation between the space and the time localizations of the constituents in stationary states of a composite system is of particular importance in understanding the quantum-motion inside a bound state when the constituents are off their mass-shells. Then, their Fermi momenta $\boldsymbol{q}$ do not determine the fixed value of internal energy level W_n. Consequently, in contrast to scattering states, the bound states are directly unobservable [1].

APPENDIX I

Relation-shapes in R_3 and L_4

"The sameness" of c-number relation-shapes $F(\boldsymbol{y}^2)$ in R_3 and $G(x^2)$ in L_4—cf. (3.5)—is restricted by the convergence of the corresponding integrals which express the quantum non-commutative "$x-p$" algebra. Let us illustrate this by a few important examples.
(*i*) Let $G(x^2) = x^2$. Since the integral (3.6) is divergent, there is no relation-shape in R_3 which would be "the same" as the four-interval x^2 in L_4. At the same time, if $F(\boldsymbol{y}^2) = \boldsymbol{y}^2$, neither $\tilde{F}(\boldsymbol{q}^2)$ nor $\tilde{G}(p^2)$ exist and hence no relation-shape exists in L_4 "the same" as the distance square $\boldsymbol{y}^2$ in R_3. Perhaps, the quarks are confined because of the lack of isomorphy between the R_4 and L_4 relation-shapes.
(*ii*) Let $F(\boldsymbol{y}^2)$ be the Yukawa potential in R_3, $F = \exp(-\kappa r)/(4\pi r)$ ($r = |\boldsymbol{y}|$), that describes the action-at-a-distance in R_4. By analytically extending $\tilde{F}(\boldsymbol{q}^2 = p^2) = \tilde{G}(p^2) = (\kappa^2 + p^2)^{-1}$ to negative p^2, one obtains $G(x^2) = \Delta^C(x^2; \kappa)$, where "C" denotes the contour of integration

in the complex p_0-plane, while Δ^C is the corresponding Green's function (distribution) of a scalar particle of mass κ. Thus

$$\exp(-\kappa r)/(4\pi r) \doteq \Delta^C(x^2; \kappa). \tag{I.1}$$

(*iii*) Loosely bound systems, as e.g. atoms, are well described by the NR-framework in R_4^G embedded in G_4. However, according to the R_4-relationism, internal wave functions in R_4^G do not represent event-shapes, but relation-shapes. Therefore, since L and not G expresses the true symmetry of events, the relation-shapes obtained in R_4^G must be identified (in their analytic forms) with the relation-shapes in R_4. For example, when $\psi_0(r)$ represents the internal ground state wave function of the hydrogen atom ($\psi_0 = \exp(-r/2R)/(8\pi R^3)^{1/2}$) obtained from the NR Schroedinger equation, the distance r between the proton and electron must be identified with the L-absolute interval $|y|$ in R_3. Thus, the elastic form factor $F_0 = |\psi_0|^2$ becomes an L-absolute relation-shape, $F_0(y^2) = \exp(-r/R)/(8\pi R^3)$ and hence,

$$F_0(y^2) \doteq G_0(x^2) = -(2R^3)^{-1}\partial/\partial\beta(\Delta^C(x^2; \beta)); \quad \beta = 1/R. \tag{I.2}$$

The conclusion would be that for loosely bound states (e.g. atoms) when strong inequality $q^2 \ll m_e^2c^2$ (m_e — electron mass) is fulfilled by almost all Fermi momenta q, the NR-framework provides us with correct L-invariant atomic form factors [1]

$$G_{nk}(x^2) \doteq F_{nk}(y^2) = \psi_n^*(y^2)\psi_k(y^2).$$

APPENDIX II

Life-times of mezo-atom

The electromagnetic interaction between the muon and the nucleus in mezo-atom (with single muon) results in loosely bound systems, which justifies evaluating the proper life-times $T_0^{(n)}$ of mezo-atom in the n-th state following the formula (4.4). The internal energy levels W_n of such atoms can be taken from the NR Schroedinger equation completed by the energy-mass relation hence,

$$W_n = (M+m)c^2 - (1/2n^2)\alpha^2 Z^2 \mu c^2; \quad \mu = m/(1+m/M), \tag{II.1}$$

where m, M are the muon and the nucleus masses, respectively, $n = 1, 2, \ldots$ and $\alpha =$ "1/137". If Δm denotes the uncertainty of the muon mass then

$$T_0 = \hbar/\Delta mc^2 = (2.19703 \pm 0.00004) \times 10^6 \text{ sec.} \tag{II.2}$$

is the proper life-time of free muon. According to (II.1),

$$\Delta W_n = (\partial W_n/\partial m)\Delta m = [1-(1/2n^2)\alpha^2 Z^2(1+m/M)^{-2}]\Delta mc^2.$$

Consequently, in the same accuracy up te α^2, the proper life-times $T_0^{(n)} = \hbar/\Delta W_n$ amount to

$$T_0^{(n)} = [1+(1/2n^2)\alpha^2 Z^2(1+m/M)^{-2}]T_0. \tag{II.3}$$

Of course, the largest dilatation occurs for $n = 1$ when

$$T_0^{(1)} = [1+(\alpha^2 Z^2/2)(1+m/M)^{-2}]T_0.$$

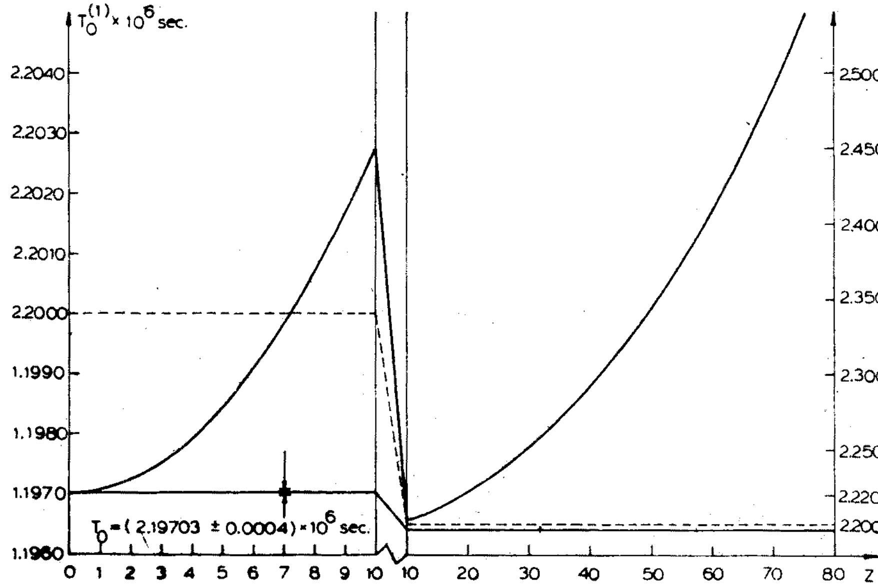

Fig. 1

Assuming that $M = AM_N$, where M_N is the nucleon mass and $A = 2Z-1$ (in order to contain the hydrogen mezo-atom when $A = Z = 1$), we get:

$$T_0^{(1)}(Z) = \{1+(\alpha^2 Z^2/2)\,[1+m/(2Z-1)M_N]^{-2}\}T_0 \tag{II.4}$$

which, as a function of Z, is plotted in Fig. 1.

REFERENCES

[1] Z. Chyliński, *Phys. Rev.* **A32**, 764 (1985); Raport IFJ 1300/PM, Kraków 1985.
[2] W. Heisenberg, in: *Quantum Theory and Its Interpretation* in: *Niels Bohr*, North-Holland, Wiley, New York 1967.
[3] A. Einstein, B. Podolsky, N. Rosen, *Phys. Rev.* **47**, 777 (1935).
[4] A. Aspect, P. Grangier, G. Roger, *Phys. Rev. Lett.* **49**, 2 (1982); **49**, 25 (1982).
[5] J. S. Bell, *Rev. Mod. Phys.* **38**, 447 (1966); *Physics* **1**, 195 (1964).
[6] P. W. Bridgman, in: *The Nature of Physical Theory*, Dover Publ., New York, 1936, pp. 85–89, 95.
[7] L. Landau, R. E. Peierls, *Z. Phys.* **69**, 56 (1931).
[8] N. Bohr, *Phys. Rev.* **48**, 696 (1935); *Dialectica* **1**, 312 (1948).
[9] A. S. Eddington, *Proc. Camb. Phil. Soc.* **35**, 196 (1939).
[10] P. A. M. Dirac, R. E. Peierls, M. H. L. Pryce, *Proc. Camb. Phil. Soc.* **38**, 193 (1942).
[11] L. L. Foldy, *Phys. Rev.* **122**, 275 (1961).
[12] R. P. Feynman, in: *Theory of Fundamental Processes*, W. A. Benjamin Inc., New York 1961, p. 145.
[13] D. G. Currie, T. F. Jordan, E. C. G. Sudarshan, *Rev. Mod. Phys.* **35**, 350 (1963).
[14] Symposium on the Foundations of Modern Physics, Joensu, Finland 1985, ed. Peter Mittelstaedt, Univ. of Cologne.
[15] G. C. Wick, *Phys. Rev.* **96**, 1124 (1954); R. E. Cutkosky, *Phys. Rev.* **96**, 1135 (1954).