DUFFIN-KEMMER-PETIAU PARTICLE WITH INTERNAL STRUCTURE

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The Duffin-Kemmer-Petiau equation for a spin-0 or spin-1 particle is interpreted as the point-like limiting case of a new relativistic wave equation for a tight system of two Dirac particles. In the new wave equation, masses of two Dirac particles appear additively, in contrast to the familiar Breit equation where their kinetic energies are additive. From such a two-body relativistic wave equation an equivalent set of radial equations is derived when the internal interaction is described by central potentials. Then it is observed that for a Coulombic internal potential the new wave equation admits no physical solutions corresponding to $^1S_0$ states. A possible advantage of this fact for composite models of W and Z bosons is pointed out.

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As is well known, the Duffin–Kemmer–Petiau equation for a free spin-0 or spin-1 particle can be written in the form [1]

$$[\tfrac{1}{2}(\gamma_1+\gamma_2)\cdot P-M]\psi(X) = 0, \tag{1}$$

because the Duffin-Kemmer-Petiau 16×16 matrices may be represented as $\beta^\mu = \frac{1}{2}(\gamma_1^\mu+\gamma_2^\mu)$, where $(\gamma_i^\mu) = (\beta_i, \beta_i\vec{\alpha}_i)$, $i = 1, 2$, are two commuting sets of the usual Dirac matrices. In the quark model, when the point-like limit is applied to quark-antiquark states, Eq. (1) can be used to describe the external motion of a free pseudoscalar or vector meson. In this case M is an effective mass matrix. Thus, in the quark model, it is natural to consider Eq. (1) as the point-like limiting case of a relativistic wave equation for a tight system of two Dirac particles (a quark and an antiquark). Writing down such an equation as

$$[\gamma_1\cdot p_1+\gamma_2\cdot p_2-2m-S(x_1-x_2)]\psi(x_1, x_2) = 0, \tag{2}$$

where $x_1^0 = x_2^0$ and $p_1^0 = p_2^0$ (for equal times), and introducing $X = \frac{1}{2}(x_1+x_2)$, $x = x_1-x_2$ and $P = p_1+p_2$, $p = \frac{1}{2}(p_1-p_2)$ we obtain

$$[\tfrac{1}{2}(\gamma_1+\gamma_2)\cdot P+(\gamma_1-\gamma_2)\cdot p-2m-S(x)]\psi(X, x) = 0, \tag{3}$$

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where $x^0 = 0$ and $p^0 = 0$ (for equal times). Here we assumed that $p^0\psi(X, x)|_{x^0=0} = 0$. This new two-body relativistic wave equation reduces to the DKP equation (1) when $-(\gamma_1 - \gamma_2) \cdot p + 2m + S(x) \to M$ and $\psi(X, x) \to \psi(X)$. In the centre-of-mass frame, where $P = (E, 0)$, we get from Eq. (3) the following stationary wave equation for the internal motion:

$$[\tfrac{1}{2}(\beta_1 + \beta_2)E - (\vec{\gamma}_1 - \vec{\gamma}_2) \cdot \vec{p} - 2m - S(\vec{x})]\psi(\vec{x}) = 0. \tag{4}$$

Note that the familiar Breit equation [2] gives in the centre-of-mass frame a different internal motion equation which has the form

$$\{E - (\beta_1\vec{\gamma}_1 - \beta_2\vec{\gamma}_2) \cdot \vec{p} - (\beta_1 + \beta_2)[m + \tfrac{1}{2}S(\vec{x})]\}\psi(\vec{x}) = 0 \tag{5}$$

if there is only an internal scalar potential $S(\vec{x})$ additive to mass. If beside the scalar potential $S(\vec{x})$ there is also an internal vector potential $V(\vec{x})$ additive to energy, we substitute $E \to E - V(\vec{x})$ in Eqs (4) and (5). In contrast to Eq. (2), the Breit equation is not relativistically covariant, even in the free case. Hence, in relativistic considerations it is usually replaced by the covariant Bethe-Salpeter equation [2] which, however, even in its reduced one-time form [3] is hard to handle for relativistic bound-state problems [4][1]. In the nonrelativistic limit Eq. (2) and the Breit equation go over into the same two-body Schrödinger equation.

In the present note we discuss the new wave equation (4) in the case of central potentials $V(r)$ and $S(r)$ (where $r = |\vec{x}|$). We derive the corresponding set of radial equations and show that for a Coulombic potential $V(r) = -\alpha/r$ and a nonsingular $S(r)$ Eq. (4) (in contrast to Eq. (5)) has no physical solutions describing $^1S_0$ states. However, for $V(r)$ less singular at $r = 0$ than $1/r$ the $^1S_0$ physical solutions to Eq. (4) can exist. It implies that Eq. (4) cannot be valid in QED (say, for positronium), though this argument does not exclude the use of Eq. (4) in non-Abelian gauge theories where the asymptotic freedom causes a softening of the Coulombic singularity in $V(r)$.

In order to find the set of radial equations equivalent to Eq. (4) we follow the procedure used in Ref. [5] for the derivation of radial equations from the Breit equation (5).

As the first step, we split Eq. (4) into components in the double Dirac representation where we get $\psi = (\psi_{\beta_1\beta_2})$ with the indices $\beta_i = \pm 1$ being eigenvalues of the Dirac matrices β_i, $i = 1, 2$. Then, we go over to the wave-function components

$$\left.\begin{matrix}\phi \\ \phi^0\end{matrix}\right\} = P_0 \frac{i}{\sqrt{2}}(\psi_{++} \mp \psi_{--}), \quad \left.\begin{matrix}\vec{\phi} \\ \vec{\phi}^0\end{matrix}\right\} = \tfrac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)P_1 \frac{1}{\sqrt{2}}(\psi_{+-} \pm \psi_{-+}) \tag{6}$$

and

$$\left.\begin{matrix}\chi \\ \chi^0\end{matrix}\right\} = P_0 \frac{i}{\sqrt{2}}(\psi_{+-} \pm \psi_{-+}), \quad \left.\begin{matrix}\vec{\chi} \\ \vec{\chi}^0\end{matrix}\right\} = \tfrac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)P_1 \frac{1}{\sqrt{2}}(\psi_{++} \pm \psi_{--}), \tag{7}$$

where $\vec{\sigma}_i$, $i = 1, 2$, are two commuting sets of the usual Pauli spin matrices and

$$P_0 = \tfrac{1}{4}(1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2), \quad P_1 = \tfrac{1}{4}(3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2) \tag{8}$$

[1] Note that Eq. (2) can be derived from the Bethe-Salpeter equation by means of the last Ref. [3]. Then, in general, $S(\vec{x})$ is an integral operator describing two-body interaction in a relativistically covariant one-time way. This covariance is spoilt, however, if the perturbative series $S(\vec{x})$ (corresponding to a given Bethe-Salpeter kernel) is broken off.

denote the projection operators on states with total spin $s = 0$ and $s = 1$, respectively, $s(s+1)$ being the quantum number of $\vec{S}^2$ with $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$. Note that $(\vec{\sigma}_1 - \vec{\sigma}_2)P_{0,1} = P_{1,0}(\vec{\sigma}_1 - \vec{\sigma}_2)$. In terms of the components (6) and (7) Eq. (4) (containing $V(\vec{x})$ as well as $S(\vec{x})$) splits into three independent subsets of equations:

$$\begin{aligned} \tfrac{1}{2}(E-V)\phi^0 - i\vec{p}\cdot\vec{\phi} &= (m+\tfrac{1}{2}S)\phi, \\ \tfrac{1}{2}(E-V)\phi &= (m+\tfrac{1}{2}S)\phi^0, \\ -i\vec{p}\phi &= (m+\tfrac{1}{2}S)\vec{\phi} \end{aligned} \tag{9}$$

and

$$\begin{aligned} \tfrac{1}{2}(E-V)\vec{\chi} + i\vec{p}\chi^0 &= (m+\tfrac{1}{2}S)\vec{\chi}^0, \\ \tfrac{1}{2}(E-V)\vec{\chi}^0 + i\vec{p}\times\vec{\phi}^0 &= (m+\tfrac{1}{2}S)\vec{\chi}, \\ i\vec{p}\cdot\vec{\chi}^0 &= (m+\tfrac{1}{2}S)\chi^0, \\ -i\vec{p}\times\vec{\chi} &= (m+\tfrac{1}{2}S)\vec{\phi}^0 \end{aligned} \tag{10}$$

and finally $\chi = 0$. We can see that the "large-large" components ψ_{++} are included into the subset (9) when $s = 0$ and into the subset (10) when $s = 1$. Thus, these subsets of equations describe in a relativistic way a parasystem and an orthosystem of two Dirac particles, respectively.

As the second step, in the case of central potential $V(r)$ and $S(r)$ we eliminate from the subsets (9) and (10) the angular coordinates by applying the multipole technique introduced in Ref. [5]. To this end we expand the vector component $\vec{\phi}$, $\vec{\phi}^0$, $\vec{\chi}$ and $\vec{\chi}^0$ involved in Eqs (9) and (10) into three parts: "electric", "longitudinal" and "magnetic" defined for $\vec{\phi}$ as

$$\begin{aligned} \phi_{\rm E}(\vec{x}) &= \hat{x}\cdot\vec{\phi}(\vec{x}) = \phi_{\rm E}(r)Y_{jm}(\hat{x}), \\ \phi_{\rm L}(\vec{x}) &= \left(\frac{\partial}{\partial\hat{x}} - 2\hat{x}\right)\cdot\vec{\phi}(\vec{x}) = \phi_{\rm L}(r)Y_{jm}(\hat{x}), \\ \phi_{\rm M}(\vec{x}) &= \left(\hat{x}\times\frac{\partial}{\partial\hat{x}}\right)\cdot\vec{\phi}(\vec{x}) = \phi_{\rm M}(r)Y_{jm}(\hat{x}) \end{aligned} \tag{11}$$

and analogically for $\vec{\phi}^0$, $\vec{\chi}$ and $\vec{\chi}^0$. At the same time the scalar components are presented as

$$\phi(\vec{x}) = \phi(r)Y_{jm}(\hat{x}) \tag{12}$$

and analogically for ϕ^0 and χ^0. Here, $\hat{x} = \vec{x}/r$ and $\partial/\partial\hat{x} = r\partial/\partial\vec{x} - \vec{x}\partial/\partial r$, implying $\hat{x}^2 = 1$, $\hat{x}\cdot\partial/\partial\hat{x} = 0$, $\partial/\partial\hat{x}\cdot\hat{x} = 2$, $(\partial/\partial\hat{x})^2 = -\vec{L}^2 = (\partial/\partial\hat{x} - 2\hat{x})^2$, $\hat{x}\times\partial/\partial\hat{x} = \vec{x}\times\partial/\partial\vec{x} = i\vec{L}$, $\hat{x}\times\vec{L} = i\partial/\partial\hat{x}$ and $\partial/\partial\hat{x}\times\partial/\partial\hat{x} = i\vec{L}$. Note that the Hermitian conjugate to $\partial/\partial\hat{x}$ is $-\partial/\partial\hat{x} + 2\hat{x}$. In terms of the multipole components (11) we get

$$\vec{\phi}(\vec{x}) = \hat{x}\phi_{\rm E}(\vec{x}) - \frac{\partial}{\partial\hat{x}}\frac{\phi_{\rm L}(\vec{x})}{j(j+1)} - \left(\hat{x}\times\frac{\partial}{\partial\hat{x}}\right)\frac{\phi_{\rm M}(\hat{x})}{i(i+1)} \tag{13}$$

and analogical expansions for $\vec{\phi}^0$, $\vec{\chi}$ and $\vec{\chi}^0$. Hence

$$\|\vec{\phi}\|^2 = \|\phi_E\|^2 + \frac{\|\phi_L\|^2 + \|\phi_M\|^2}{j(j+1)}, \tag{14}$$

where $\|\ \|$ denotes the norm in Hilbert space. Note that for $s = 1$ states the relation $J_k\vec{e} = \vec{e}L_k$ holds, where $\vec{e}$ is any of three vector operators $\hat{x}$, $\partial/\partial\hat{x}$ and $\hat{x}\times\partial/\partial\hat{x}$ appearing in Eq. (13) and $\vec{J} = \vec{L}+\vec{S}$ stands for the total angular momentum, while (for $s = 1$ states) the total spin $\vec{S}$ acts on $\vec{e}$ according to the formula $(S_k\vec{e})_l = -i\varepsilon_{klm}e_m$. Thus, the three terms in Eq. (13) are three different eigenstates of $\vec{J}^2$ and J_z with the same eigenvalues $j(j+1)$ and m, where $j = 0, 1, 2, \ldots$ and $m = j, \ldots, -j$. Then, making use of the formulae (11) and (12) as well as analogous ones for other components we obtain from Eqs (9) and (10) for any given j three independent subsets of $5+6+4$ radial equations (cf. Appendix), 7 of them being algebraic equations. Making use of the algebraic equations to eliminate 7 radial components we get finally for any given j the following three independent subsets of $2+4+2$ first-order differential equations:

(*i*) subset including $s = 0$ "large-large" components

$$\left[\left(\frac{E-V}{2}\right)^2 - \frac{j(j+1)}{r^2} - (m+\tfrac{1}{2}S)^2\right]\phi - (m+\tfrac{1}{2}S)\left(\frac{d}{dr}+\frac{2}{r}\right)\phi_E = 0,$$

$$-(m+\tfrac{1}{2}S)\phi_E - \frac{d}{dr}\phi = 0, \tag{15}$$

(*ii*) subset including $s = 1$ "large-large" components with negative parity

$$\left[\left(\frac{E-V}{2}\right)^2 - (m+\tfrac{1}{2}S)^2\right]\chi_E^0 + (m+\tfrac{1}{2}S)\frac{d}{dr}\chi^0 + \tfrac{1}{2}(E-V)\frac{1}{r}\phi_M^0 = 0,$$

$$\left[\left(\frac{E-V}{2}\right)^2 - (m+\tfrac{1}{2}S)^2\right]\chi_L - (m+\tfrac{1}{2}S)\left(\frac{d}{dr}+\frac{1}{r}\right)\phi_M^0 - \tfrac{1}{2}(E-V)\frac{j(j+1)}{r}\chi^0 = 0,$$

$$\left[-\frac{j(j+1)}{r^2} - (m+\tfrac{1}{2}S)^2\right]\chi^0 + (m+\tfrac{1}{2}S)\left(\frac{d}{dr}+\frac{2}{r}\right)\chi_E^0 + \tfrac{1}{2}(E-V)\frac{1}{r}\chi_L = 0,$$

$$\left[-\frac{j(j+1)}{r^2} - (m+\tfrac{1}{2}S)^2\right]\phi_M^0 - (m+\tfrac{1}{2}S)\left(\frac{d}{dr}+\frac{1}{r}\right)\chi_L - \tfrac{1}{2}(E-V)\frac{j(j+1)}{r}\chi_E^0 = 0, \tag{16}$$

(*iii*) subset including $s = 1$ "large-large" components with positive parity

$$\left[\left(\frac{E-V}{2}\right)^2 - \frac{j(j+1)}{r^2} - (m+\tfrac{1}{2}S)^2\right]\chi_M + (m+\tfrac{1}{2}S)\left(\frac{d}{dr}+\frac{1}{r}\right)\phi_L^0 = 0,$$

$$-(m+\tfrac{1}{2}S)\phi_L^0 + \left(\frac{d}{dr}+\frac{1}{r}\right)\chi_M = 0. \tag{17}$$

For all $s = 1$ components the triplet $\langle E|$, $\langle L|$, $\langle M|$ can be expressed through the triplet $\langle l = j-1|$, $\langle l = j+1|$, $\langle l = j|$ (with l being the orbital angular momentum) by means of the transformation

$$\langle \mathrm{E}| = \sqrt{\frac{j}{2j+1}}\langle l = j-1| + \sqrt{\frac{j+1}{2j+1}}\langle l = j+1|,$$

$$\frac{\langle \mathrm{L}\rangle}{\sqrt{j(j+1)}} = -\sqrt{\frac{j+1}{2j+1}}\langle l = j-1| + \sqrt{\frac{j}{2j+1}}\langle l = j+1|,$$

$$\frac{\langle \mathrm{M}|}{\sqrt{j(j+1)}} = \langle l = j|. \tag{18}$$

So, we can see from Eqs (15), (16) and (17) that the "large-large" components contained in the subsets (*i*), (*ii*) and (*iii*) correspond to: (*i*) $l = j \geqslant 0$ with $s = 0$ (${}^1j_j$ states), (*ii*) a mixture of $l = j-1$ and $l = j+1$ with $s = 1$ (${}^3(j\mp 1)_j$ states) and (*iii*) $l = j \geqslant 1$ with $s = 1$ (${}^3j_j$ states), respectively. All components contained in the subsets (*i*), (*ii*) and (*iii*) have, respectively, the total parity: (*i*) $P = \eta(-1)^j$, (*ii*) $P = \eta(-1)^{j+1}$ and (*iii*) $P = \eta(-1)^j$, where $\eta = +1$ for a fermion-fermion system and $\eta = -1$ for a fermion-antifermion system (called sometimes the fermionium). Note that $\langle \mathrm{E}| = \langle l = 1|$, $\langle \mathrm{L}| = 0$ and $\langle \mathrm{M}| = 0$ if $j = 0$, thus $\phi_{\mathrm{E}} = \phi_{l=1}$, $\phi_{\mathrm{L}} = 0$ and $\phi_{\mathrm{M}} = 0$, etc. for $j = 0$

Eliminating ϕ_{E} from the subset (*i*), Eq. (15), one obtains the following second-order differential equation for ϕ:

$$\left[\left(\frac{E-V}{2}\right)^2 + \frac{1}{r}\frac{d^2}{dr^2}r - \frac{j(j+1)}{r^2} - (m+\tfrac{1}{2}S)^2 - \frac{1}{2m+S}\frac{dS}{dr}\frac{d}{dr}\right]\phi = 0. \tag{19}$$

If $V(r) = -\alpha/r$ and $S(r)$ is nonsingular at $r = 0$, Eq. (19) implies that $r\phi \sim r^\gamma \to 0$ for $r \to 0$, where $\gamma = \frac{1}{2} + \sqrt{(j+\frac{1}{2})^2 - \left(\frac{\alpha}{2}\right)^2}$ with $0 < \alpha < 1$. Then, the second equation (15) gives $r\phi_{\mathrm{E}} \sim r^{\gamma-1}$ for $r \to 0$, so ϕ_{E} with $j = 0$ does not satisfy the regularity condition $r\phi_{\mathrm{E}} = 0$ at $r = 0$. In this case, therefore, no S_0 states exist.

Similarly, eliminating ϕ_{L}^0 from the subset (*iii*), Eq. (17), one gets the second-order differential equation for χ_{M}:

$$\left[\left(\frac{E-V}{2}\right)^2 + \frac{1}{r}\frac{d^2}{dr^2}r - \frac{j(j+1)}{r^2} - (m+\tfrac{1}{2}S)^2 - \frac{1}{2m+S}\frac{dS}{dr}\left(\frac{d}{dr} + \frac{1}{r}\right)\right]\chi_{\mathrm{M}} = 0. \tag{20}$$

If again $V(r) = -\alpha/r$ and $S(r)$ is nonsingular at $r = 0$, Eq. (20) gives $r\chi_{\mathrm{M}} \sim r^\gamma \to 0$ for $r \to 0$ and then the second equation (17) shows that $r\phi_{\mathrm{L}}^0 \sim r^{\gamma-1}$ for $r \to 0$. Here, however, ϕ_{L}^0 does not violate the regularity condition $r\phi_{\mathrm{L}}^0 = 0$ at $r = 0$ because $j \geqslant 1$.

Finally, the subset (*ii*), Eq. (16), leads in the particular case of $j = 0$ to the second order differential equation for χ_E^0:

$$\left[\left(\frac{E-V}{2}\right)^2+\frac{1}{r}\frac{d^2}{dr^2}r-\frac{2}{r^2}-(m+\tfrac{1}{2}S)^2-\frac{1}{2m+S}\frac{dS}{dr}\left(\frac{d}{dr}+\frac{2}{r}\right)\right]\chi_E^0=0 \tag{21}$$

and to the relationship between χ_E^0 and χ^0:

$$\left(\frac{d}{dr}+\frac{2}{r}\right)\chi_E^0=(m+\tfrac{1}{2}S)\chi^0. \tag{22}$$

For the same $V(r)$ and $S(r)$ as before, Eq. (21) shows that $r\chi_E^0 \sim r^\gamma$ for $r \to 0$, where $\gamma = \frac{1}{2} + \frac{1}{2}\sqrt{9-\alpha^2}$. Then, Eq. (22) gives $r\chi^0 \sim r^{\gamma-1}$ for $r \to 0$, so χ^0 with $j = 0$ satisfies the regularity condition $r\chi^0 = 0$ at $r = 0$. Hence, in contrast to $^1S_0$ states, $^3P_0$ states exist (as well as $^3S_1$, $^3P_1$, $^3P_2$ and all other states with $j \geqslant 1$).

Thus, in the case of a Coulombic $V(r)$ and a nonsingular $S(r)$ the use of Eq. (4) is not allowed, at least in a physical situation where an $^1S_0$ state exists. However, if $V(r)$ has a weaker singularity at $r = 0$ than $1/r$, as is the case in non-Abelian gauge theories, our argument no longer justifies such a conclusion.

Note that the counterparts of Eqs (19) and (20) in the case of Breit equation (5) with central potentials $V(r)$ and $S(r)$ have the form [5]

$$\left[\left(\frac{E-V}{2}\right)^2+\frac{1}{r}\frac{d^2}{dr^2}r-\frac{j(j+1)}{r^2}-(m+\tfrac{1}{2}S)^2+\frac{1}{E-V}\frac{dV}{dr}\frac{d}{dr}\right]\phi^0=0 \tag{23}$$

and

$$\left[\left(\frac{E-V}{2}\right)^2+\frac{1}{r}\frac{d^2}{dr^2}r-\frac{j(j+1)}{r^2}-(m+\tfrac{1}{2}S)^2+\frac{1}{E-V}\frac{dV}{dr}\left(\frac{d}{dr}+\frac{1}{r}\right)\right]\chi_M^0=0, \tag{24}$$

respectively, while the counterparts of the second Eqs (15) and (17) are, respectively,

$$\tfrac{1}{2}(E-V)\phi_E+\frac{d}{dr}\phi^0=0 \tag{25}$$

and

$$\tfrac{1}{2}(E-V)\phi_L^0-\left(\frac{d}{dr}+\frac{1}{r}\right)\chi_M^0=0 \tag{26}$$

(for the complete set of radial equations following from the Breit equation cf. the last reference [5], where the same components (6) and (7) are used but without factor i in the case of ϕ, ϕ^0, χ and χ^0). Here, the regularity condition at $r = 0$ is satisfied both by ϕ^0, χ_M^0 and ϕ_E, ϕ_L (even when $j = 0$), if our case of a Coulombic $V(r)$ and a nonsingular $S(r)$ is considered. The essential disadvantage of Breit equation is its relativistically non-covariant form, even in the free case.

Of course, we should remember that in our radial equations only central (thus static) potentials $V(r)$ and $S(r)$ are taken into account. Then, the nonstatic (i.e. "magnetic" and/or retardation) corrections [6] are to be considered perturbatively. Note that the static term $\beta_1\beta_2 V_S(r)$ added to $E-V(r)$ does not change the form of Eq. (4) with $V(r)$, in contrast to Eq. (5), because $(\beta_1+\beta_2)\beta_1\beta_2 = \beta_1+\beta_2$.

In conclusion, one may formulate the conjecture that in non-Abelian gauge theories the new equation (4) (with potentials V and S) is *relevant* as being a natural two-body extension of the relativistic DKP equation for a spin-0 or spin-1 point particle. An alternative conjecture one may make is that the relevance of the equation (4) is not necessarily tied to non-Abelian gauge theories, but rather to the subelementary level of matter where leptons and quarks and/or W and Z bosons are built up of hypothetical preons [7] carrying (at least some of them) spin 1/2. In the case of this alternative conjecture Eq. (4) is not valid in QCD (say, for quarkonia), though it can be used in preon models.

Then, considering in particular a preon model where the W and Z bosons are composites [7] of two spin-1/2 preons, one might speculate that the puzzling experimental absence of the corresponding spin-0 composites we still witness [8] is connected with the option that in such a preon model the internal potential V in Eq. (4) is Coulombic or *nearly* Coulombic. In this way $^1S_0$ states may be excluded (while $^3P_0$ states are lying higher than $^3S_1$ states).

Eventually, we would like to point out that the essential difference between the new wave equation (2) and the Breit equation is the additivity of "masses" $\gamma_i \cdot p_i$ in the former case versus the additivity of kinetic energies $\beta_i\vec{\gamma}_i \cdot \vec{p}_i$ in the latter. As conjectured in this note, such an unconventional property of the new wave equation may be connected with non-Abelian gauge theories (and so with the confinement) or alternatively with preon models. It is worthwhile to mention that a similar unconventional feature appears in the spinless relativistic wave equation proposed several years ago by Feynman, Kislinger and Ravndal [9] for a confined quark-antiquark pair,

$$[p_1^2+p_2^2-2m^2+\mu^4(x_1-x_2)^2]\psi(x_1, x_2) = 0, \tag{27}$$

where "masses" squared p_i^2 are additive. It was observed recently [10] that, if one wanted to carry out in the FKR equation (27) the Dirac-type squared-root operation

$$\sqrt{p_1^2+p_2^2} \to \gamma_1 \cdot p_1+\gamma_2 \cdot p_2, \tag{28}$$

one should introduce two *anticommuting* sets of Dirac matrices $(\gamma_i^\mu) = (\beta_i, \beta_i\vec{\alpha}_i)$:

$$\{\gamma_i^\mu, \gamma_j^\nu\} = 2\delta_{ij}g^{\mu\nu}. \tag{29}$$

Then, one would obtain the wave equation of the form (2) *but* with γ's satisfying the Clifford-algebra relations (29) (and with $2m \to \sqrt{2}m$). The unconventional spin-1/2 particles whose Dirac matrices would obey the anticommutation relations (29), $i, j = 1, 2, \ldots, N$, were called the *non-Abelian* Dirac particles [10]. In the present note we do not take into account such an option, considering Eq. (2) with γ's commuting for $i \neq j$, what corresponds to the conventional Dirac particles.

APPENDIX

Complete set of radial equations

(*i*) subset including $s = 0$ "large-large" components

$$\begin{aligned}
&\tfrac{1}{2}(E-V)\phi^0 - \left(\frac{d}{dr} + \frac{2}{r}\right)\phi_E - \frac{1}{r}\phi_L = (m+\tfrac{1}{2}S)\phi,\\
&\tfrac{1}{2}(E-V)\phi = (m+\tfrac{1}{2}S)\phi^0,\\
&-\frac{d}{dr}\phi = (m+\tfrac{1}{2}S)\phi_E,\\
&\frac{j(j+1)}{r}\phi = (m+\tfrac{1}{2}S)\phi_L,\\
&0 = (m+\tfrac{1}{2}S)\phi_M,
\end{aligned} \tag{A1}$$

(*ii*) subset including $s = 1$ "large-large" components with negative parity

$$\begin{aligned}
&\tfrac{1}{2}(E-V)\chi_E + \frac{d}{dr}\chi^0 = (m+\tfrac{1}{2}S)\chi_E^0,\\
&\tfrac{1}{2}(E-V)\chi_L - \frac{j(j+1)}{r}\chi^0 = (m+\tfrac{1}{2}S)\chi_L^0,\\
&\tfrac{1}{2}(E-V)\chi_E^0 + \frac{1}{r}\phi_M^0 = (m+\tfrac{1}{2}S)\chi_E,\\
&\tfrac{1}{2}(E-V)\chi_L^0 - \left(\frac{d}{dr} + \frac{1}{r}\right)\phi_M^0 = (m+\tfrac{1}{2}S)\chi_L,\\
&\left(\frac{d}{dr} + \frac{2}{r}\right)\chi_E^0 + \frac{1}{r}\chi_L^0 = (m+\tfrac{1}{2}S)\chi^0,\\
&-\frac{j(j+1)}{r}\chi_E - \left(\frac{d}{dr} + \frac{1}{r}\right)\chi_L = (m+\tfrac{1}{2}S)\phi_M^0,
\end{aligned} \tag{A2}$$

(*iii*) subset including $s = 1$ "large-large" components with positive parity

$$\begin{aligned}
&\tfrac{1}{2}(E-V)\chi_M = (m+\tfrac{1}{2}S)\chi_M^0,\\
&\tfrac{1}{2}(E-V)\chi_M^0 + \frac{j(j+1)}{r}\phi_E^0 + \left(\frac{d}{dr} + \frac{1}{r}\right)\phi_L^0 = (m+\tfrac{1}{2}S)\chi_M,
\end{aligned}$$

$$-\frac{1}{r}\chi_M = (m+\tfrac{1}{2}S)\phi_E^0,$$

$$\left(\frac{d}{dr}+\frac{1}{r}\right)\chi_M = (m+\tfrac{1}{2}S)\phi_L^0. \tag{A3}$$

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