THE OBSERVER'S DEFINITION OF THE QUANTUM MASSLESS PARTICLE IN TWO DIMENSIONS

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The influence of the observer's world-line on his concept of the particle is discussed. For a real massless scalar field the creation and annihilation operators are introduced for an observer in a two dimensional curved space-time. The vacuum expectation values of the stress-energy tensor are computed.

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1. *Introduction*

One of the most important problems in the quantum field theory in a curved space-time is that related to the concept of particle [1, 2]. Although a general solution of this problem seems to be impossible [2], one can ask, as it is the intention of the present paper, what influence has the observer's world-line on his definition of the particle. The consideration was here limited to two dimensions.

The paper is organized as follows. In Section 2 we introduce the definition of the creation and annihilation operator for the massless real scalar field in a two dimensional curved space-time. Our analysis is based on the concept of a detector proposed by Unruh [3, 4].In Section 3 we calculate the vacuum expectation values of the renormalized stress-energy tensor. Section 4 is a summary.

2. *Definition of the creation and annihilation operators*

We consider only a real massless scalar field ϕ. Let the interaction between the detector and the field be of the form [3, 4]

$$S_{\rm int} = \int\limits_{-\infty}^{+\infty} L_{\rm int}(s)ds = c\int\limits_{-\infty}^{+\infty} m(s)\phi[x(s)]ds, \tag{1}$$

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where $x(s)$ is a world-line of the detector (and simultaneously of the observer), the operator $m(s)$ is the detector's monopole moment taken in the proper time s along the world-line of the observer, ϕ is the field operator, and c is a small coupling constant.

Let $|\psi, E\rangle$ be a state of the system, where $|\psi\rangle$ is a field state, and $|E\rangle$ is a detector state of energy E. The amplitude K of transition from an initial state $|\psi_0, E_0\rangle$ to the final state $|\psi, E\rangle$ is proportional to the following quantity (in the first approximation):

$$K = i\langle E, \psi|S_{\text{int}}|\psi_0, E_0\rangle = ic\langle E, \psi| \int_{-\infty}^{+\infty} m(s)\phi[x(s)]ds|\psi_0, E_0\rangle. \tag{2}$$

Taking into account that due to Schrödinger equation

$$m(s) = e^{iHs}m(0)e^{-iHs}, \tag{3}$$

where $H|E\rangle = E|E\rangle$, we can write

$$K = ic\langle E|m(0)\,|E_0\rangle\,\langle\psi|\int_{-\infty}^{+\infty} e^{i(E-E_0)s}\cdot\phi[x(s)]ds|\psi_0\rangle. \tag{4}$$

Let us assume that $|E_0\rangle$ is the ground state of the detector. If K given by (4) is for a certain state $|\psi_0\rangle$ equal to zero for every $|\psi\rangle$ and $E > E_0$, then we can say that *such a state* $|\psi_0\rangle$ *is a vacuum state relative to the observer*. This vacuum state we denote further by $|0\rangle$. If K is, however, not equal to zero for some values $E > E_0$, we can say that there are in the field some particles with energies $E - E_0$. We see that the term

$$\int_{-\infty}^{+\infty} e^{iEs}\phi[x(s)]ds, \tag{5}$$

where $E > 0$, behaves like the annihilation operator. But there is a serious objection: the term (5) cannot be proportional to the annihilation operator, because this term does not distinguish particles of the same energy and different directions of motion. It is a result of the fact that our detector has only one degree of freedom: it is measuring only the energy of particles. In order to use this term as a definition of the annihilation operator we should restrict ourselves to the case in which the field has also one degree of freedom.

Therefore we consider here the following case. In a two dimensional curved space-time with the metric (in the isotropic form)

$$ds^2 = C(u, v)dudv \tag{6}$$

the observer is moving along the world-line:

$$u = u(s), \tag{7a}$$

$$v = v(s). \tag{7b}$$

We have a massless real scalar field ϕ satisfying:

$$\Box\phi = \frac{4}{C}\frac{\partial}{\partial u}\frac{\partial}{\partial v}\phi = 0. \tag{8}$$

This field contains two parts: ϕ_+ (dependent on u) and ϕ_- (dependent on v), and each one taken separately has one degree of freedom. Consequently for each of the parts of the field operator ϕ we formulate the following definition of the creation and annihilation operator ($\lambda = +1$ or $\lambda = -1$, respectively):

$$a_\lambda(E) = \sqrt{\frac{E}{\pi}} \int_{-\infty}^{+\infty} e^{iES}\phi_\lambda[x(s)]ds \tag{9a}$$

$$a_\lambda^+(E) = \sqrt{\frac{E}{\pi}} \int_{-\infty}^{+\infty} e^{-iES}\phi_\lambda[x(s)]ds. \tag{9b}$$

Operators (9a) and (9b) enjoy the following properties:

1. They satisfy the relations

$$[a_\lambda(E), a_{\lambda'}^+(E')] = \delta_{\lambda\lambda'}\delta(E-E'). \tag{10}$$

2. For the flat space-time and an inertial observer they are equal to the usual annihilation and creation operators.

Making use of the fact that the one particle wave function $f_\lambda(x, E)$ can be obtained from the equality:

$$f_\lambda(x, E) = [\phi, a_\lambda^+(E)], \tag{11}$$

we have

$$f_\lambda(x, E) = \frac{1}{\sqrt{4\pi E}}\, e^{-iEs_\lambda(x)}, \tag{12}$$

where $s_+(x)$ and $s_-(x)$ are the r.h. sides of the inverse relations to (7a) and (7b), respectively.

Using (12), we obtain the relation:

$$u^\mu \frac{\partial f_\lambda(x, E)}{\partial x^\mu} = -iEf_\lambda(x, E), \tag{13}$$

where $u^\mu = \dfrac{dx^\mu(s)}{ds}$ is the fourvelocity of the observer. This relation is the requirement which is usually imposed on a one particle wave function [1, 2].

3. *The renormalized stress-energy tensor*

In this section, using the explicit formulae (12) we compute the renormalized vacuum expectation values of the stress-energy tensor. We use the point-splitting method proposed by Adler, Lieberman and Ng [5] and corrected by Wald [6].

The classical expression for the stress-energy tensor of the scalar massless field is

$$T_{\mu\nu_{\rm clas}} = \phi_{;\mu}\phi_{;\nu} - \tfrac{1}{2}\, g_{\mu\nu}\phi_{;\alpha}\phi^{;\alpha}. \tag{14}$$

The method which we use permits us to compute the expectation values $\langle 0|T_{\mu\nu}|0\rangle$ as a limit

$$\langle 0|T_{\mu\nu}|0\rangle = \lim_{x'\to x} T_{\mu\nu}(x, x'), \tag{15}$$

where

$$T_{\mu\nu}(x, x') = \tfrac{1}{4}\left(\delta_\nu^{\nu'}\nabla_\mu\nabla_{\nu'} + \delta_\mu^{\mu'}\nabla_{\mu'}\nabla_\nu - g_{\mu\nu}\delta_\alpha^{\beta'}\nabla_{\beta'}\nabla^\alpha\right)\langle 0|\phi(x)\phi(x') + \phi(x')\phi(x)\,|0\rangle, \tag{16}$$

and where the unprimed indices refer to tensors in the tangent space at x, the primed indices to tensors at x', and $\delta_\mu^{\mu'}$ is the bitensor of the parallel displacement.

We substitute the expressions (12) into (16) and performing the integration over energies we obtain

$$T_{\mu\nu}(x, x') = \sum_\lambda -\frac{1}{8\pi}\,\frac{\delta_\mu^{\mu'}\nabla_{\mu'}S_\lambda(x')\nabla_\nu S_\lambda(x) + \delta_\nu^{\nu'}\nabla_\mu S_\lambda(x)\nabla_{\nu'}S_\lambda(x') - g_{\mu\nu}\delta_\alpha^{\beta'}\nabla_{\beta'}S_\lambda(x')\nabla^\alpha S_\lambda(x)}{[S_\lambda(x) - S_\lambda(x')]^2}. \tag{17}$$

Separating the divergent part from (17) and putting the remainder into (15), we get

$$\langle 0|T_{\mu\nu}|0\rangle_{\text{ren}} = \begin{bmatrix} T_{uu} & T_{uv} \\ T_{vu} & T_{vv}\end{bmatrix}$$

$$= -\frac{1}{24\pi}\begin{bmatrix} \dfrac{(S_+)_{,uuu}}{(S_+)_{,u}} - \frac{3}{2}\left[\dfrac{(S_+)_{,uu}}{(S_+)_{,u}}\right]^2; & 0 \\ 0; & \dfrac{(S_-)_{,vvv}}{(S_-)_{,v}} - \frac{3}{2}\left[\dfrac{(S_-)_{,vv}}{(S_-)_{,v}}\right]^2 \end{bmatrix} + \frac{1}{24\pi}\begin{bmatrix} \dfrac{C_{,uu}}{C} - \frac{3}{2}\left(\dfrac{C_{,u}}{C}\right)^2, & 0 \\ 0, & \dfrac{C_{,vv}}{C} - \frac{3}{2}\left(\dfrac{C_{,v}}{C}\right)^2 \end{bmatrix}. \tag{18}$$

(In the meantime an averaging over all directions has been performed — details in [5].) The quantity (18), however, does not satisfy the conservation law:

$$\langle 0|T^{\mu\nu}|0\rangle_{\text{ren};\mu} = 0 \tag{19}$$

and, therefore, we should add to (18) a term which would restore the conservation [6] (it is so-called trace anomaly). Finally, we derive that:

$$\langle 0|T_{\mu\nu}|0\rangle_{\text{ren}}^{\text{final}} = (18) - \frac{1}{48\pi}g_{\mu\nu}R$$

$$= -\frac{1}{24\pi}\begin{bmatrix} \dfrac{(S_+)_{,uuu}}{(S_+)_{,u}} - \frac{3}{2}\left[\dfrac{(S_+)_{,uu}}{(S_+)_{,u}}\right]^2; & 0 \\ 0; & \dfrac{(S_-)_{,vvv}}{(S_-)_{,v}} - \frac{3}{2}\left[\dfrac{(S_-)_{,vv}}{(S_-)_{,v}}\right] \end{bmatrix} + \frac{1}{24\pi}\begin{bmatrix} \dfrac{C_{,uu}}{C} - \frac{3}{2}\dfrac{C_{,u}^2}{C^2}; & -\dfrac{C_{,uv}}{C} + \dfrac{C_{,u}C_{,v}}{C^2} \\ -\dfrac{C_{,uv}}{C} + \dfrac{C_{,u}C_{,v}}{C^2}; & \dfrac{C_{,vv}}{C} - \frac{3}{2}\dfrac{C_{,v}^2}{C^2} \end{bmatrix}, \tag{20}$$

where R is the Ricci scalar.

The renormalized stress-energy tensor has two parts: the first one depends on the world-line of the detector can be interpreted as the detector response on its noninertial motion, whereas the second one depends on the structure of the space-time and can be interpreted as a polarization of vacuum by the gravitational field. However, this partition is not an absolute one, because in different isotropic coordinate systems we can obtain different partitions, whereas the total tensor (20) is independent of a coordinate system.

4. *Summary*

One of the conclusions of this paper is that a definition of a particle should be based on an analysis of the functioning of detectors of one kind or another. Only in such a case we can obtain a reasonable picture of the particle.

Furthermore the form of the renormalized stress-energy tensor derived here indicates that we cannot consider the two parts of this tensor independently of one another. Meaningful statements can be only formulated about the total tensor partition of it into the "polarization of vacuum" and the response on noninertial motion of the detector parts is accidental.

REFERENCES

[1] N. D. Birrell, P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge 1982, Chapter 3.
[2] B. S. DeWitt, *Phys. Rep.* **19C**, 297 (1975).
[3] B. S. DeWitt, in *General Relativity*, eds. S. W. Hawking, W. Israel, Cambridge University Press, Cambridge 1979.
[4] W. G. Unruh, *Phys. Rev.* **D14**, 870 (1976).
[5] S. L. Adler, J. Lieberman, Y. J. Ng, *Ann. Phys.* (USA) **106**, 279 (1977).
[6] R. M. Wald, *Phys. Rev.* **D17**, 1477 (1978).