ELECTROMAGNETIC STRUCTURE OF THE MASSIVE, DIRAC NEUTRINOS*

BY H. CZYŻ, K. KOŁODZIEJ AND M. ZRAŁEK

Department of Theoretical Physics, Silesian University, Katowice**

(Received May 21, 1986)

All the electromagnetic formfactors for the massive, Dirac neutrinos are calculated in the frame of the GSW model. Their dependence on the Kobayashi-Maskawa mixing and C, P, T properties are discussed. Three formfactors depend on gauge, the other ones are gauge independent. In the static limit $q^2 \to 0$, in one loop approximation the Dirac neutrino has two electromagnetic characteristics — the magnetic moment (which vanishes for massless neutrino) and anapole (which is different from zero in the massless limit). Present experimental data do not give possibility to check these static properties of the neutrinos.

PACS numbers: 12.15.-y

1. Introduction

One of the characteristic properties of neutrinos is their electromagnetic structure. Such electromagnetic structure was investigated [1, 2, 4, 5] in the frame of the Glashow-Salam-Weinberg (GSW) model [3]. In those papers usually unitary gauge was applied [1], not all possible diagrams were taken into account [2, 4] or only massless neutrinos were investigated [1, 5]. In spite of lack of the experimental verification (up to now) of the neutrino oscillation phenomena [6] the problem of massive neutrinos is still actual from experimental [6] and theoretical [7] point of view. In the case of massive neutrinos there is a problem what kind of mass they have, Dirac or Majorana? One of the features which will distinguish both cases in future experiments is the electromagnetic structure of the neutrinos. In this paper we would like to investigate in details the electromagnetic structure of massive Dirac neutrinos in the frame of GSW model. In the lepton sector of the model we use the Kobayashi-Maskawa mixing [8], like in the quark sector. In our calculations we apply full on-mass shell renormalization scheme [9] similar to [10]. In this renormalization scheme using linear 't Hooft-Feynman gauge we have obtained six different form-

* Presented at the IX Silesian School of Theoretical Physics, Szczyrk, Poland, 1985.

** Address: Zakład Fizyki Teoretycznej, Uniwersytet Śląski, Uniwersytecka 4, 40-007 Katowice, Poland.

factors $F_i(q^2)$ $i = 1, 2, \ldots 6$ for all neutrinos. We also obtain that the neutrino current is not conserved

$$q^\mu j_\mu \neq 0. \tag{1.1}$$

This unphysical result is caused by a gauge dependence. The current is calculated from the Green function which depends on gauge. Only the static limit $q^2 \to 0$ for formfactors is gauge independent and has physical interpretation. There were a few attempts [11] to give physical validity to formfactors in the case of $q^2 \neq 0$ so we will not consider this problem. Not all formfactors depend on gauge. To find which one depends on gauge we perform our calculations also in non-linear gauge [12, 13], where the Ward identities for the electromagnetic vertex remain the standard identities in quantum electrodynamics. It follows from this property that the neutrino current is conserved

$$q^\mu j_\mu^{\mathrm{NONLINEAR}} = 0. \tag{1.2}$$

From this calculation we find that three formfactors F_1, F_4, F_6 (see Chapter 3) depend on gauge. The other ones are gauge independent (in our gauges). In the one loop approximation the massive neutrino has a magnetic moment and an anapole moment [2]. The electric moment vanishes in this approximation. The magnetic moment is proportional to neutrino mass and vanishes for massless neutrino. The anapole moment depends on the KM mixing and is different from zero for massless neutrino.

In Chapter 2 we present the calculations of the neutrino current in the linear and the non-linear gauges. In Chapter 3 the properties of the neutrino current are discussed. Its static characteristics are presented in Chapter 4. In Chapter 5 we summarize our results.

2. *Electromagnetic current of the neutrinos in the frame of the Glashow-Salam-Weinberg model*

We assume that all neutrinos have non zero masses and there exists a mixing in the leptonic sector like one in the quark sector of GSW model with three generations of fermions. Thus we introduce the lepton Kobayashi-Maskawa (KM) matrix A_{cn} ($c, n = 1, 2, 3$ are flavour indices) which mixes flavours in the leptonic sector.

To investigate the structure of matrix elements of electromagnetic (EM) current between two one-particle neutrino states in momentum space we calculate $\nu\nu\gamma$ vertex within the GSW model to the one loop approximation. To check gauge dependence of the various formfactors we perform our calculations in two different gauges. Namely, in the linear 't Hooft-Feynman gauge, where the gauge fixing Lagrangian has the form

$$\mathscr{L}_{\mathrm{GF}}^{\mathrm{LIN}} = -|\partial_\mu W^{\mu\dagger} - iM_{\mathrm{W}}\varphi^\dagger|^2 - \tfrac{1}{2}(\partial_\mu Z^\mu - M_{\mathrm{Z}}\varphi_3')^2 - \tfrac{1}{2}(\partial_\mu A^\mu)^2, \tag{2.1}$$

and in the non-linear gauge with the gauge fixing term [12]

$$\mathscr{L}_{\mathrm{GF}}^{\mathrm{NONLIN}} = -\left|\left(\partial_\mu - ie\,\frac{M_{\mathrm{Z}}}{\sqrt{M_{\mathrm{Z}}^2 - M_{\mathrm{W}}^2}}\,A_\mu^3\right)W^{\mu\dagger} - iM_{\mathrm{W}}\varphi^\dagger\right|^2 - \tfrac{1}{2}(\partial_\mu Z^\mu - M_{\mathrm{Z}}\varphi_3')^2 - \tfrac{1}{2}(\partial_\mu A^\mu)^2. \tag{2.2}$$

In the last formula A_μ^3 denotes the third of the group components of SU(2) gauge field before transformation to the physical fields, which is done by the formula

$$A_\mu^3 = \frac{1}{M}(\sqrt{M_Z^2 - M_W^2}\, A_\mu - M_W Z_\mu). \tag{2.3}$$

We know that the GSW theory is renormalizable in both of these gauges [14]. It is worth to mention that we renormalize $\nu\nu\gamma$ vertex in a selfconsistent way. That means, we are able to remove divergences of all Green functions to an arbitrary order of perturbative calculations (see [10], [9]).

We found Feynman vertices in both gauges using our version of GSW Lagrangian [9]. The vertices in the non linear gauge which differ from those in the linear one and are used in this paper are listed in Appendix A. In the nonlinear gauge some three point couplings vanish and as a result smaller number of diagrams contribute to three or four--point Green functions (see for example Fig. 2). Unfortunately there appear some additional four-point vertices which make two-point Green functions a little bit more complicated (see, Appendix B).

The Feynman diagrams which contribute to the $\nu\nu\gamma$-vertex in linear and nonlinear gauges are shown in Fig. 1 and Fig. 2.

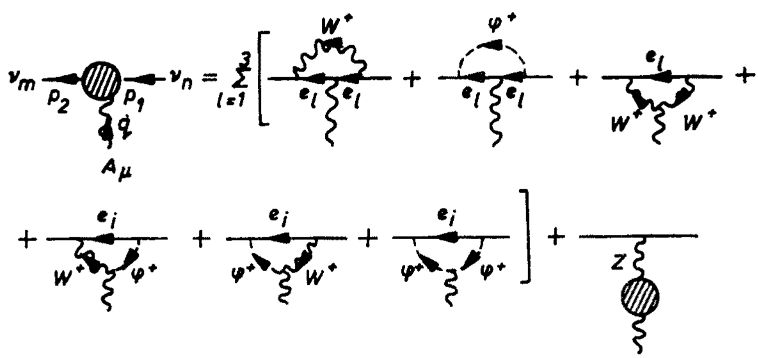


Fig. 1. Feynman diagram involved in vertex $\nu_n \nu_m \gamma$ in linear gauge

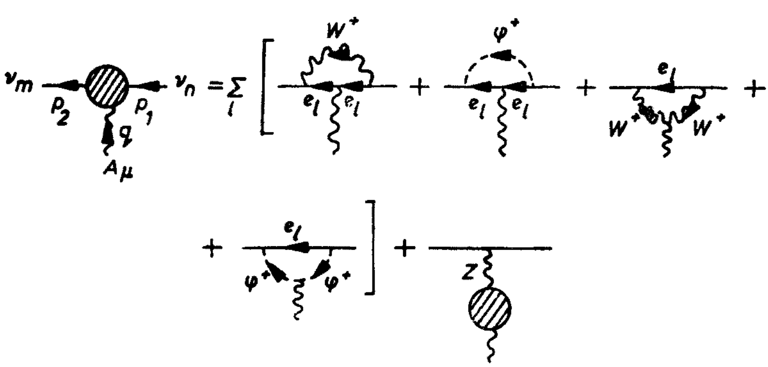
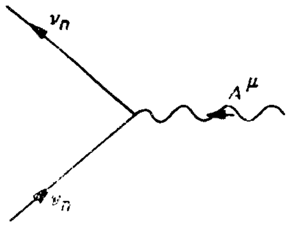


Fig. 2. Feynman diagram involved in vertex $\nu_n \nu_m \gamma$ in nonlinear gauge

Let us briefly describe our renormalization scheme [9]. We perform on-mass shell renormalization of GSW model up to the one loop approximation. In this case the counter-term to the $\nu\nu\gamma$ vertex, following from our Lagrangian, equals [9].



$$= \frac{ie}{4} \frac{M_Z^2}{M_W \sqrt{M_Z^2 - M_W^2}} \gamma^\mu (1-\gamma_5) \Delta_{ZA}, \tag{2.4}$$

where we introduced matrix renormalization of fields in $Z-A$ sector [10]

$$\begin{pmatrix} \overset{0}{Z}_\mu \\ \overset{0}{A}_\mu \end{pmatrix} = \begin{pmatrix} Z_{ZZ}^{1/2} & Z_{ZA}^{1/2} \\ Z_{AZ}^{1/2}, & Z_{AA}^{1/2} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \tag{2.5}$$

with

$$Z_{ZZ}^{1/2} = 1 - \Delta_{ZZ}, \quad Z_{AA}^{1/2} = 1 - \Delta_{AA}, \quad Z_{ZA}^{1/2} = -\Delta_{ZA}, Z_{AZ}^{1/2} = -\Delta_{AZ}.$$

We use on mass shell (OS) renormalization conditions in the form given in the papers [9, 10]. In our case it is enough to introduce only two conditions, for transverse part of $Z-A$ propagator

$$[\mathrm{Re}\, \Pi_{ZA}^{\mathrm{tr}}(q^2) - M_Z^2 \Delta_{ZA} + q^2(\Delta_{ZA} + \Delta_{AZ})]_{q^2=0} = 0,$$
$$[\mathrm{Re}\, \Pi_{ZA}^{\mathrm{tr}}(q^2) - M_Z^2 \Delta_{ZA} + q^2(\Delta_{ZA} + \Delta_{AZ})]_{q^2=M^2_Z} = 0. \tag{2.6}$$

Thus, to determine the renormalization constants Δ_{ZA} and Δ_{AZ}, we must calculate $Z-A$ propagator both in linear and non-linear gauges to the one loop approximation. Diagrams which contribute to these propagators and our final results for them are shown in Appendix B.

Substituting (B.1) and (B.2) into (2.6) one obtains the following values of Δ_{ZA} and Δ_{AZ}

$$\Delta_{ZA} = \frac{e^2}{8\pi^2} \frac{M_W}{\sqrt{M_Z^2 - M_W^2}} (C_{UV} - \ln M_W^2),$$

$$\Delta_{AZ} = \frac{e^2 M_W}{16\pi^2 \sqrt{M_Z^2 - M_W^2}} \Bigg[C_{UV} \left(-5 - \frac{M_Z^2}{6M_W^2} \right) - \frac{M_Z^2}{9M_W^2} + \left(-4 \frac{M_W^2}{M_Z^2} + \tfrac{2}{3} \right) \ln M_W^2$$
$$+ \tilde{F}_0(M_W, M_W, M_Z^2) \left(4 \frac{M_W^2}{M_Z^2} + \tfrac{13}{3} + \frac{M_Z^2}{6M_W^2} \right) - \frac{4}{M_W^2} \sum_i Q_i (2Q_i (M_Z^2 - M_W^2) - T_{3i} M_Z^2)$$
$$\times \left(\tfrac{1}{6} C_{UV} + \mathrm{Re}\, \tilde{F}_2(m_i, m_i, M_Z^2) - \mathrm{Re}\, \tilde{F}_1(m_i, m_i, M_Z^2) \right) \Bigg] \tag{2.7}$$

in the linear gauge, and

$$\Delta_{ZA} = 0,$$

$$\Delta_{AZ} = \frac{e^2 M_W}{16\pi^2 \sqrt{M_Z^2 - M_W^2}} \left[C_{UV} \left(-\tfrac{17}{3} - \frac{M_Z^2}{6M_W^2} \right) + \tfrac{8}{9} - \frac{M_Z^2}{9M_W^2} + \left(-\frac{28M_W^2}{3M_Z^2} + \tfrac{2}{3} \right) \ln M_W^2 \right.$$

$$+ \tilde{F}_0(M_W, M_W, M_Z^2) \left(\frac{28M_W^2}{3M_Z^2} + 5 + \frac{M_Z^2}{6M_W^2} \right) - \frac{4}{M_W^2} \sum_i Q_i(2Q_i(M_Z^2 - M_W^2) - T_{3i} M_Z^2)$$

$$\left. \times (\tfrac{1}{6} C_{UV} + \mathrm{Re}\, \tilde{F}_2(m_i, m_i, M_Z^2) - \mathrm{Re}\, \tilde{F}_1(m_i, m_i, M_Z^2)) \right] \tag{2.8}$$

in the non-linear one.

Renormalized neutrino EM current has the same structure in both considered gauges:

$$j^\mu(p, q) \equiv -ie\, \bar{u}_n(p_2) \Gamma^\mu u_n(p_1) = \frac{-ie^3 M_Z^2}{64\pi^2 (M_Z^2 - M_W^2)} \sum_l |A_{nl}|^2$$

$$\times \bar{u}_n(p_2)[A_l(q^2)\gamma^\mu + E_l(q^2)p^\mu + iG_l(q^2)q^\mu$$

$$+ \gamma_5(B_l(q^2)\gamma^\mu + iD_l(q^2)p^\mu + C_l(q^2)q^\mu)]u_n(p_1), \tag{2.9}$$

where $p = p_1 + p_2$, $q = p_2 - p_1$, A_{nl} — KM matrix element in leptonic sector, and we assumed that neutrino spinors $u_n(p_1)$, $\bar{u}_n(p_2)$ satisfy the Dirac equation.

Functions $A_l(q^2)$, $B_l(q^2)$, $C_l(q^2)$, $D_l(q^2)$, $E_l(q^2)$ and $G_l(q^2)$ are listed in Appendix C.

3. *Neutrino formfactors, current conservation*

The Hamiltonian of the neutrino interaction with electromagnetic field may be written in the form

$$H_I = j_\mu A^\mu, \tag{3.1}$$

where j_μ is, by definition, electromagnetic current and A^μ is the photon field. Matrix elements of this current have the form

$$j_\mu = -ie\bar{u}(p_2, \lambda_2)\Gamma_\mu u(p_1, \lambda_1), \tag{3.2}$$

where p_1, λ_1, (p_2, λ_2) are momentum and helicity of incoming (outgoing) particles (see Fig. 3).

Physical requirements, e.g. hermiticity of the hamiltonian and the fact that spinors u and $\bar{u}$ fulfil the Dirac equation, lead to the following form of the operator Γ_μ

$$\Gamma_\mu = F_1\gamma_\mu + F_2 P_\mu + iF_3 q_\mu$$

$$+ F_4\gamma_5(q^2\gamma_\mu + 2mq_\mu) + iF_5\gamma_5 P^\mu + F_6\gamma_5\gamma_\mu, \tag{3.3}$$

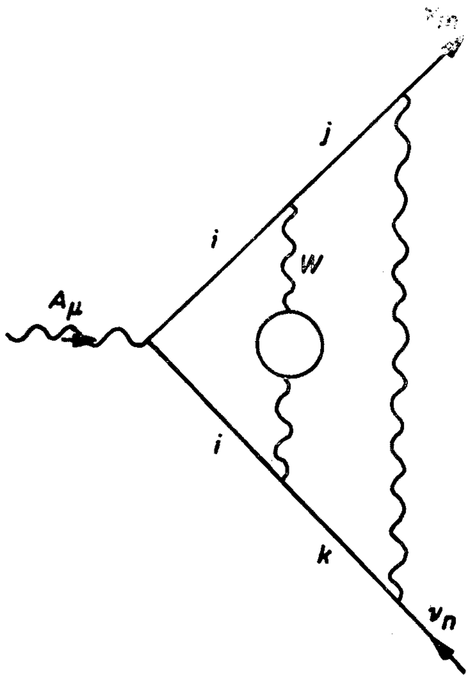


Fig. 3. Diagram which may contribute to electric dipole moment of the neutrino

where F_i ($i = 1, \ldots, 6$) are real scalars which depend only on q^2, and m is the mass of a particle. Our operator Γ_μ, which was obtained in straightforward calculations of neutrino current (see Chapter 2), has the same form and we may write F_i in terms of A_l, B_l, C_l, D_l, E_l, G_l (see (2.9))

$$F_1 = \frac{M_Z^2 e^2}{64\pi^2(M_Z^2 - M_W^2)} \sum_{l=1}^{3} A_{ln} A_{nl}^\dagger A_l,$$

$$F_2 = \frac{e^2 M_Z^2}{64\pi^2(M_Z^2 - M_W^2)} \sum_{l=1}^{3} A_{ln} A_{nl}^\dagger E_l,$$

$$F_3 = \frac{e^2 M_Z^2}{64\pi^2(M_Z^2 - M_W^2)} \sum_{l=1}^{3} A_{ln} A_{nl}^\dagger G_l,$$

$$F_4 = \frac{e^2 M_Z^2}{128\pi^2 m(M_Z^2 - M_W^2)} \sum_{l=1}^{3} A_{ln} A_{nl}^\dagger C_l,$$

$$F_5 = \frac{e^2 M_Z^2}{64\pi^2(M_Z^2 - M_W^2)} \sum_{l=1}^{3} A_{ln} A_{nl}^{\dagger} D_l,$$

$$F_6 = \frac{e^2 M_Z^2}{64\pi^2(M_Z^2 - M_W^2)} \sum_{l=1}^{3} A_{ln} A_{nl}^{\dagger} \left(B_l - \frac{C_l q^2}{2m} \right). \tag{3.4}$$

We have here six independent functions, instead of four as for example in papers [2, 15] because we do not require up to now the current conservation

$$q_\mu j^\mu = 0. \tag{3.5}$$

If we define the electromagnetic current in terms of renormalizable gauge fields [11], the current conservation will be a result of Ward-Takahashi identity

$$q^\mu V_\mu^{ff\gamma} = e[S_F^{-1}(p-q) - S_F^{-1}(p)], \tag{3.6}$$

where $V^{ff\gamma}$ — fermion-fermion-photon vertex, $S_F(p)$ — fermion propagator.
It is fulfilled for QED [16]. In the frame of GSW model property (3.6) depends on gauge. It is satisfied e.g. in unitary gauge and in nonlinear gauge [13]. In linear 't Hooft-Feynman gauge (3.6) is not valid and we cannot expect the current conservation in the form (3.5). Broken current conservation means that some parts of the operator depend on a choice of the gauge and then they cannot be physical observables. However for $q^2 = 0$ the current conservation has still the form (3.5) and additionally we can interpret most of the formfactors. It is easy to see that $2mF_2(0) + F_1(0) =$ *charge* Q,
$\frac{F_1(0)}{2m} =$ *dipole magnetic moment* M
(nonrelativistic interaction has the form — $M\vec{\sigma}\vec{B}$),
$-F_5(0) =$ *dipole electric moment* D
(nonrelativistic interaction has the form — $D\vec{\sigma}\vec{E}$),
and $F_4(0) =$ *dipole anapole moment* A
(nonrelativistic interaction has the form $A\vec{\sigma}\,\mathrm{rot}\,\vec{B}$).
$F_3(0)$ and $F_6(0)$ are not physical observables and in nonrelativistic approximation we cannot write their interactions in form of coupling with electromagnetic fields (or derivatives of the fields).

It is interesting to see what kinds of properties the formfactors have when we assume that the theory has some discrete symmetries, like C, P, T. All these properties which are valid for any momentum transfer q^2 are given in Table I. We can see that when CP is not broken, F_5 must be equal to zero. In GSW model CP is broken only due to phase factor in the KM matrix, but in one loop approximation only absolute values of the elements of KM matrix go to the electromagnetic current and then F_5 must vanish in this approximation (what can be checked in straightforward calculations). At higher orders, where the

TABLE I

The properties of formfactors at discrete symmetries. "$-$" denote that formfactors are equal to zero when symmetry is conserved

Symmetry \ Formfactor	F_1	F_2	F_3	F_4	F_5	F_6
C	+	+	−	−	+	−
P	+	+	+	−	−	−
T(CP)	+	+	−	+	−	+

phase of the KM matrix enters the diagram, the formfactor F_5 (and from this the electric dipole moment) can be different from 0. An example of this diagram is given in Fig. 3.

Let us return to our results for electromagnetic current of neutrinos. After easy but rather long calculations we can obtain

$$q_\mu j^\mu = 0 \tag{3.7}$$

for nonlinear gauge, and

$$q^\mu j_\mu = \frac{-e^2 m M_Z^2 q^2}{32\pi^2 (M_Z^2 - M_W^2)(q^2 - M_Z^2)} (\ln M_W^2 - \tilde{F}_0(M_W, M_W, q^2)) \tag{3.8}$$

for linear gauge.

To obtain the results (3.7) and (3.8) we use reduction formulae (D.3–D.6) from Appendix D.

We see that for our linear gauge the electromagnetic current is conserved only for $q^2 = 0$, as expected.

4. *Static properties of neutrinos*

Let us see now what is the physical contents of electromagnetic current of neutrinos. It is worth to mention that our results are exact, we took into account all diagrams in one loop approximation and calculated without any approximation like $O\left(\frac{m_l^2}{M_W^2}\right)$. Our results are

$$G_l(q^2) = 0, \quad D_l(q^2) = 0, \tag{4.1}$$

what is the consequence of the CP violation only by phase in the KM matrix. This means that dipole electric moment of neutrino vanishes in one loop approximation as expected (see Chapter 3).

$$(2mE_l + A_l)(q^2 = 0) = 0, \tag{4.2}$$

where m is neutrino's mass. This means of course that neutrino's charge is equal to zero.

$$E_l(q^2)_{q^2=0} = \frac{1}{m}\left[-2 - \frac{m_l^2}{M_W^2} - \frac{m^2}{M_W^2}\right]$$

$$+\frac{1}{m^3}\ln\left(\frac{m_l^2}{M_W^2}\right)\left[\left(\tfrac{3}{2}-\frac{m_l^2}{2M_W^2}\right)m^2+\frac{m_l^2}{2}-M_W^2+\frac{m_l^4}{M_W^2}\right]$$

$$+\frac{2}{\sqrt{\Delta}}\ln\left(\frac{\sqrt{\Delta}-m^2+m_l^2+M_W^2}{2m_lM_W}\right)\cdot\frac{1}{m^3}$$

$$\times\left[\left(-\tfrac{3}{2}-\frac{m_l^2}{2M_W^2}\right)m^4+\left(-\frac{m_l^2}{2}+\frac{5M_W^2}{2}+\frac{m_l^4}{M_W^2}\right)m^2+\left(-M_W^4+\tfrac{3}{2}M_W^2m_l^2-\frac{m_l^6}{2M_W^2}\right)\right], \tag{4.3}$$

where

$$\Delta = m^4-2m^2(m_l^2+M_W^2)+(m_l^2-M_W^2)^2. \tag{4.4}$$

E_l is proportional to the magnetic moment of neutrino but it is rather difficult to see its behaviour using the above formula. When we left only first term in series in powers of $\frac{m}{M_W}$, we obtained for the magnetic moment of neutrino (see Chapter 3)

$$M \simeq 10^{-19}\mu_B\left(\frac{m}{1\ \text{eV}}\right) \tag{4.5}$$

so we get the same result as e.g. in [2].

We have used here the following values of the masses and fine structure constant taken from [17]

$$M_W = 80.8\ \text{GeV},\quad M_Z = 92.9\ \text{GeV},\quad m_e = 0.5110034\ \text{MeV}$$

$$m_\mu = 105.65932\ \text{MeV},\quad m_\tau = 1.7842\ \text{GeV},$$

$$\alpha = 1/137.03604,\quad e = 1.6021892\,.\,10^{-19}\text{C}.$$

It is worth to mention that M does not depend on the KM mixing because A_l does not depend on l in this approximation.

$$C_l(q^2=0) = \frac{1}{m}\left[1-\frac{m_l^2}{2M_W^2}+\frac{m^2}{2M_W^2}\right]\ln\left(\frac{m_l^2}{M_W^2}\right)$$

$$+\frac{2}{m\sqrt{\Delta}}\ln\left(\frac{\sqrt{\Delta}+m_l^2-m^2+M_W^2}{2m_lM_W}\right)$$

$$\times\left[\left(-\tfrac{11}{6}-\frac{2m_l^2}{3M_W^2}\right)m^2+\frac{m^4}{6M_W^2}+\left(-\frac{3m_l^2}{2}+M_W^2+\frac{m_l^4}{2M_W^2}\right)\right]. \tag{4.6}$$

Anapole moment is dependent on C_l (see Chapter 3) but as for E_l it is difficult to see its behavior using (4.6). In the same approximation as for E_l the C_l depends however on l

$$C_l(q^2 = 0) \simeq \frac{m}{M_W^2}\left(\tfrac{4}{3}\ln\frac{m_l^2}{M_W^2} - 2\right), \tag{4.7}$$

and in consequence the anapole moment is dependent on the KM mixing and may vary by 2 orders of magnitude when the KM matrix is changed:

$$A \simeq (10^{-18} \div 10^{-16})\mu_B \frac{1}{1\ \text{eV}}. \tag{4.8}$$

Both values, magnetic and anapole moments are, however, experimentally unaccessible. Antineutrino gives reactor experiments only upper bound for the magnetic moments of electron and muon neutrinos [17]

$$M_{\nu_e} < 1.4 \cdot 10^{-9}\mu_B, \quad M_{\nu_\mu} < 8.1 \cdot 10^{-9}\mu_B$$

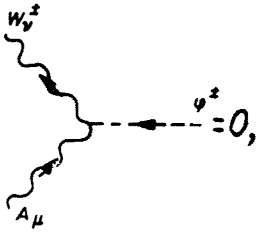
what is 9 order of magnitude bigger than reasonable theoretical values (with small neutrino's masses). For the anapole moment of the neutrinos there is no experimental evidence up to now.

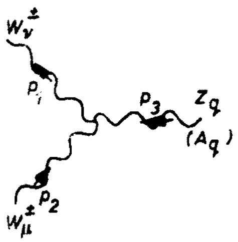
5. *Conclusions*

The massive Dirac neutrino possesses the magnetic moment and the anapole moment. The magnetic moment vanishes for the massless neutrino but the anapole moment is different from zero in this limit. In the second loop approximation, the electric dipole moment also appears. Unfortunately all these characteristics are few orders of magnitude out of experimental power. The static ($q^2 \to 0$) properties of neutrino are well defined, gauge independent values. But there still remains the question whether we can define formfactors as physical observables. In the paper [11] Jegerlehner and Fleisher have tried to define formfactors in a gauge independent manner in terms of "Abelian fields". They have concluded that formfactors in their definition are really gauge independent but they have bad behaviour for $q^2 \to \infty$. Formfactors in terms of fields in renormalizable gauge have good behaviour for big q^2 but they may be gauge dependent. We have calculated matrix elements of electromagnetic current of neutrinos in two gauges, in t'Hooft-Feynman linear gauge and in nonlinear gauge. It is worth to mention that electromagnetic current conservation is in fact a consequence of Ward-Takahashi identities. Then in some gauges we may have nonconservation of this current (for example in linear t'Hooft-Feynman gauge). In spite of gauge dependence of the current nonconservation, some of the formfactors may be gauge independent. We have only results for two gauges and we cannot conclude whether the formfactors are really independent. In our calculation three formfactors (F_1, F_4, and F_6) are gauge dependent and we may expect that some of the formfactors are really gauge independent.

APPENDIX A

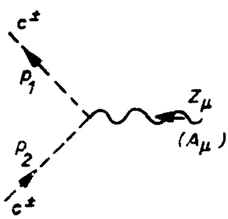
The Feynman vertices in the nonlinear gauge (2.2) which differ from vertices in the linear gauge used in this paper.





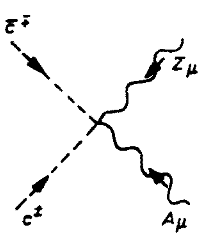
$$= \pm ie \frac{M_W}{\sqrt{M_Z^2-M_W^2}} [(p_1+p_2)^q g^{\mu\nu} + 2p_3^\nu g^{\mu q} - 2p_3^\mu g^{\nu q}],$$

$(\mp ie)$,



$$= \mp e \frac{M_W}{\sqrt{M_Z^2-M_W^2}} (p_1+p_2)^\mu,$$

$(\pm e)$,



$$= -e^2 \frac{2M_W}{\sqrt{M_Z^2-M_W^2}} g^{\mu\nu}$$

APPENDIX B

We present here the results for $Z-A$ propagator both in the linear and the nonlinear gauge. The diagrams which contribute to them are shown in Fig. B1 and Fig. B2 respectively.

$$\Pi_{ZA}^{\mu\nu}(k)^{\mathrm{LIN}} = \frac{1}{16\pi^2} \frac{e^2 M_W^2}{\sqrt{M_Z^2-M_W^2}} \left\{ \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) k^2 \left[C_{\mathrm{UV}} \left(3 + \tfrac{1}{6} \frac{M_Z^2}{M_W^2} \right.\right.\right.$$

$$\left. + 2\frac{M_Z^2}{k^2} \right) - \tfrac{1}{9} \frac{M_Z^2}{M_W^2} - \tfrac{2}{3} \frac{M_Z^2 - 6M_W^2}{k^2} \ln M_W^2 - \left(3 + \tfrac{1}{6} \frac{M_Z^2}{M_W^2} + \tfrac{4}{3} \frac{M_Z^2+3M_W^2}{k^2} \right)$$

$$\times \int_0^1 dx \ln (D_2(x, M_W)) + 4 \sum_i Q_i \left(2Q_i \left(\frac{M_Z^2}{M_W^2} - 1 \right) - T_{3i} \frac{M_Z^2}{M_W^2} \right) \cdot \left(\tfrac{1}{6} C_{UV} \right.$$

$$\left. + \int_0^1 dx (x^2 - x) \ln (D_2(x, m_i)) \right) \Bigg] + \frac{k^\mu k^\nu}{k^2} 2M_Z^2 \left[C_{UV} - \int_0^1 dx \ln (D_2(x, M_W)) \right] \Bigg\},$$

$$\Pi_{ZA}^{\mu\nu}(k)^{\text{NONLIN}} = \frac{1}{16\pi^2} \frac{e^2 M_W^2}{\sqrt{M_Z^2 - M_W^2}} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) k^2 \Bigg[C_{UV} \left(\tfrac{17}{3} + \tfrac{1}{6} \frac{M_Z^2}{M_W^2} \right)$$

$$+ \tfrac{1}{9} \frac{M_Z^2}{M_W^2} - \tfrac{8}{9} - \tfrac{2}{3} \frac{M_Z^2 - 14 M_W^2}{k^2} \ln M_W^2 - \left(\tfrac{17}{3} + \tfrac{1}{6} \frac{M_Z^2}{M_W^2} \right.$$

$$\left. - \tfrac{2}{3} \frac{M_Z^2 - 14 M_W^2}{k^2} \right) \int_0^1 dx \ln (D_2(x, M_W)) + 4 \sum_i Q_i \left(2Q_i \left(\frac{M_Z^2}{M_W^2} - 1 \right) \right.$$

$$\left. - T_{3i} \frac{M_Z^2}{M_W^2} \right) \cdot \left(\tfrac{1}{6} C_{UV} + \int_0^1 dx (x^2 - x) \ln (D_2(x, m_i)) \right) \Bigg],$$

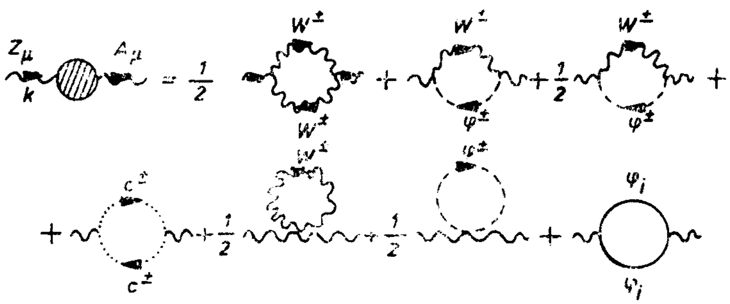


Fig. B1. The diagrams which contribute the $Z-A$ propagator in the linear gauge. (We should sum over all indicated particles with proper combinatorial factors.)

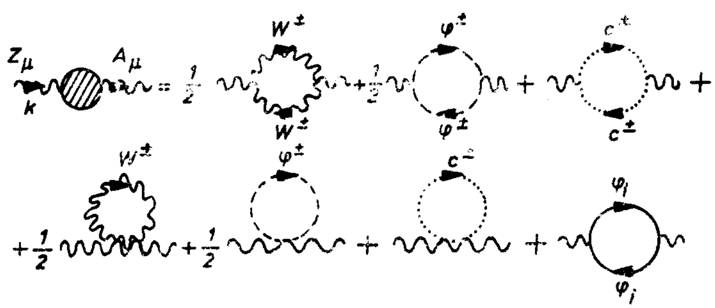


Fig. B2. The diagrams which contribute to the $Z-A$ propagator in the nonlinear gauge. (We should sum over all indicated particles with proper combinatorial factors.)

where Q_i — electrical charge of i-th fermion, T_{3i} — 3-rd component of weak isospin of i-th fermion and $C_{UV} = \frac{2}{\varepsilon} - \gamma_E + \ln(4\pi\mu^2)$, ($\gamma_E$ — Euler's constant), $D_2(x, m) = m^2 - k^2 \times x(1-x)$.

APPENDIX C

We present here the list of the coefficients A_l, B_l, E_l, D_l, F_l, and C_l which were defined in Eq. (2.9) both in linear and nonlinear gauge.

a) The linear gauge:

$$A_l^{\text{LIN}}(q^2) = 2\ln M_W^2 + \tfrac{1}{2}\kappa_l - 2 - 2\int_0^1 dx \int^1 dy y \{[-2m_n^2 + \tfrac{1}{2} g_l(m_l^2 - m_n^2)$$

$$+ q^2 + y(4m_n^2 - q^2) + \tfrac{1}{2} y^2 \kappa_l(-m_n^2 + \tfrac{1}{4} q^2) - \tfrac{1}{8} \bar{x}^2 y^2 \kappa_l q^2] \frac{1}{D_{3l}}$$

$$+ \frac{1}{\bar{D}_{3l}} [-m_l^2 + y(-4m_n^2 + q^2) + y^2(m_n^2 - \tfrac{1}{4} q^2) + \tfrac{1}{4} \bar{x}^2 y^2 q^2]$$

$$-\tfrac{1}{2}\kappa_l \ln D_{3l} + (2 + \tfrac{1}{2}\kappa_l) \ln \bar{D}_{3l} - \frac{q^2}{q^2 - M_Z^2} [X^{\text{LIN}}(q^2) - \text{Re}\, X^{\text{LIN}}(M_Z^2)],$$

$$B_l^{\text{LIN}}(q^2) = 2\ln M_W^2 + \tfrac{1}{2} g_l - 2 - 2\int_0^1 dx \int_0^1 dy y \left\{ \frac{1}{D_{3l}} [\tfrac{1}{2} \varrho_l(m_l^2 - m_n^2) \right.$$

$$+ q^2 + y(m_n^2 \varrho_l - q^2) + \tfrac{1}{2} y^2 \varrho_l(-m_n^2 + \tfrac{1}{4} q^2) - \tfrac{1}{8} \bar{x}^2 y^2 q^2 \varrho_l]$$

$$+ \frac{1}{\bar{D}_{3l}} [-m_l^2 + yq^2 + y^2(m_n^2 - \tfrac{1}{4} q^2) + \tfrac{1}{4} \bar{x}^2 y^2 q^2] - \tfrac{1}{2} \varrho_l \ln D_{3l}$$

$$\left. + (2 + \tfrac{1}{2}\varrho_l) \ln \bar{D}_{3l} \right\} - \frac{q^2}{q^2 - M_Z^2} [X^{\text{LIN}}(q^2) - \text{Re}\, X^{\text{LIN}}(M_Z^2)],$$

$$E^{\text{LIN}}(q^2) = -2m_n \int_0^1 dx \int_0^1 dy y \left\{ \frac{1}{D_{3l}} [2 + y(-4 + \tfrac{1}{2}\varrho_l) + \tfrac{1}{2} y^2 \kappa_l] \right.$$

$$\left. + \frac{1}{\bar{D}_{3l}} \left[\frac{m_l^2}{M_W^2} + y\left(\tfrac{1}{2}\varrho_l - 2\frac{m_l^2}{M_W^2}\right) + \tfrac{1}{2} y^2 \kappa_l \right] \right\},$$

$$D_l^{\text{LIN}}(q^2) = i m_n \varrho_l \int_0^1 dx \int_0^1 dy y^2 (1-y) \bar{x} \left(\frac{1}{D_{3l}} - \frac{1}{\bar{D}_{3l}} \right),$$

$$F_l^{\mathrm{LIN}}(q^2) = i2m_n \int_0^1 dx \int_0^1 dy y^2 \bar{x} \left[\frac{1}{D_{3l}} (-2 + \tfrac{1}{2} \varrho_l + \tfrac{1}{2} y\kappa_l) + \frac{1}{\bar{D}_{3l}} \left(-\frac{m_l^2}{M_W^2} + \tfrac{1}{2} y\kappa_l \right) \right],$$

$$C_l^{\mathrm{LIN}}(q^2) = -2m_n \int_0^1 dx \int_0^1 dy y \left\{ \frac{1}{D_{3l}} [2 + y(-2 + \tfrac{1}{2} \varrho_l) - \tfrac{1}{2} \bar{x}^2 y^2 \varrho_l] + \frac{1}{\bar{D}_{3l}} [2y - \tfrac{1}{2} \bar{x}^2 y^2 \varrho_l] \right\} - \frac{2m_n}{q^2 - M_Z^2} \left[X^{\mathrm{LIN}}(q^2) - \mathrm{Re}\, X^{\mathrm{LIN}}(M_Z^2) - Y^{\mathrm{LIN}}(q^2) + \frac{2M_Z^2}{q^2} \ln M_W^2 \right],$$

where

$$X^{\mathrm{LIN}}(k^2) = \tfrac{2}{3} \left(\frac{M_Z^2}{M_W^2} - 6 \right) \frac{M_W^2}{k^2} \ln M_W^2 - 2 \frac{M_Z^2}{k^2} \ln M_W^2 + \left(3 + \tfrac{1}{6} \frac{M_Z^2}{M_W^2} + \tfrac{4}{3} \frac{M_Z^2 + 3M_W^2}{k^2} \right) \int_0^1 dx \ln D_2(x, M_W) - 4 \sum_i Q_i \left[2Q_i \left(\frac{M_Z^2}{M_W^2} - 1 \right) - T_{3i} \frac{M_Z^2}{M_W^2} \right] \int_0^1 dx x(x-1) \ln D_2(x, m_i),$$

$$Y^{\mathrm{LIN}}(k^2) = \frac{2M_Z^2}{k^2} \int_0^1 dx \ln D_2(x, M_W).$$

b) The nonlinear gauge:

$$A_l^{\mathrm{NL}}(q^2) = \tfrac{1}{2} \kappa_l - 2 \int_0^1 dx \int_0^1 dy y \left\{ \frac{1}{D_{3l}} [-2m_n^2 + \tfrac{1}{2} \varrho_l (m_l^2 - m_n^2) + q^2 + y(4m_n^2 - q^2) + \tfrac{1}{2} y^2 \kappa_l (-m_n^2 + \tfrac{1}{4} q^2) - \tfrac{1}{8} \bar{x}^2 y^2 q^2 \kappa_l] + \frac{1}{\bar{D}_{3l}} y(-4m_n^2 + q^2) + \tfrac{1}{2} \kappa_l \ln \bar{D}_{3l} - \tfrac{1}{2} \kappa_l \ln D_{3l} \right\} - \frac{q^2}{q^2 - M_Z^2} [X^{\mathrm{NL}}(q^2) - \mathrm{Re}\, X^{\mathrm{NL}}(M_Z^2)],$$

$$B_l^{\rm NL}(q^2) = \tfrac{1}{2}\,\varrho_l - 2\int_0^1 dx \int_0^1 dy\, y \left\{ \frac{1}{D_{3l}}\,[\tfrac{1}{2}\,\varrho_l(m_l^2 - m_n^2) + q^2 \right.$$

$$+ y(m_n^2\varrho_l - q^2) + \tfrac{1}{2}\,y^2\varrho_l(-m_n^2 + \tfrac{1}{4}\,q^2) - \tfrac{1}{8}\,\bar{x}^2 y^2 q^2 \varrho_l]$$

$$\left. + \frac{1}{\bar{D}_{3l}}\,yq^2 + \tfrac{1}{2}\,\varrho_l \ln \bar{D}_{3l} - \tfrac{1}{2}\,\varrho_l \ln D_{3l} \right\} - \frac{q^2}{q^2 - M_Z^2}\,[X^{\rm NL}(q^2) - {\rm Re}\, X^{\rm NL}(M_Z^2)],$$

$$E_l^{\rm NL}(q^2) = E_l^{\rm LIN}(q^2),$$

$$D_l^{\rm NL}(q^2) = D_l^{\rm LIN}(q^2),$$

$$F_l^{\rm NL}(q^2) = F_l^{\rm LIN}(q^2),$$

$$C_l^{\rm NL}(q^2) = -2m_n \int_0^1 dx \int_0^1 dy\, y \left\{ \frac{1}{D_{3l}}\,[2 + y(-2 + \tfrac{1}{2}\,\varrho_l) - \tfrac{1}{2}\,\bar{x}^2 y^2 \varrho_l] \right.$$

$$\left. + \frac{1}{\bar{D}_{3l}}\,(2y - \tfrac{1}{2}\,\bar{x}^2 y^2 \varrho_l) \right\} - \frac{2m_n}{q^2 - M_Z^2}\,[X^{\rm NL}(q^2) - {\rm Re}\, X^{\rm NL}(M_Z^2)],$$

where

$$X^{\rm NL}(k^2) = \tfrac{2}{3}\left(\frac{M_Z^2}{M_W^2} - 14\right)\frac{M_W^2}{k^2}\ln M_W^2$$

$$+ \left[\tfrac{1}{6}\left(\frac{M_Z^2}{M_W^2} + 34\right) - \tfrac{2}{3}\,\frac{M_W^2}{k^2}\left(\frac{M_Z^2}{M_W^2} - 14\right)\right]\int_0^1 dx \ln D_2(x, M_W)$$

$$-4\sum_i Q_i\left[2Q_i\left(\frac{M_Z^2}{M_W^2} - 1\right) - T_{3i}\,\frac{M_Z^2}{M_W^2}\right]\int_0^1 dx\, x(x-1)\ln D_2(x, m_i),$$

and in both gauges we introduced abbreviations

$$D_{3l} \equiv D_{3l}(x, y) \equiv (1-y)M_W^2 + y[m_l^2 - q^2 x(1-x)] - y(1-y)\bar{p}^2,$$

$$\bar{D}_{3l} \equiv \bar{D}_{3l}(x, y) \equiv (1-y)m_l^2 + y[M_W^2 - q^2 x(1-x)] - y(1-y)\bar{p}^2,$$

$$\bar{p} \equiv (1-x)p_1 + x p_2,$$

$$\bar{x} \equiv 2x - 1,$$

$$\kappa_l \equiv \frac{2M_W^2 + m_l^2 + m_n^2}{M_W^2},$$

$$\varrho_l \equiv \frac{2M_W^2 + m_l^2 - m_n^2}{M_W^2}.$$

APPENDIX D

It is easy to see that these are connections between integrals over Feynman parameters which appeared in Appendix C which are necessary for obtaining for example the equalities (3.7) and (3.8). The method of obtaining reduction formulae, which are listed below, is described in [18]. We define first a few values

$$C_{n,m} = \int_0^1 dx \int_0^1 dy \frac{x^n y^m}{D_3},$$

$$C_{ln} = \int_0^1 dx \int_0^1 dy y \ln D_3, \tag{D.1}$$

where

$$D_3 = (1-y)M + y[m^2 - q^2 x(1-x)] - y(1-y)\bar{p}^2,$$

and

$$\bar{p} = (1-x)p_1 + xp_2, \qquad q = p_2 - p_1.$$

p_1, p_2 — fourvectors which have the following property $p_1^2 = p_2^2$,

$$\tilde{F}_k(m_1, m_2, q^2) = \int_0^1 dx x^k \ln(q^2 x^2 + x(m_2^2 - m_1^2 - q^2) + m_1^2 - i\varepsilon) \tag{D.2}$$

ε — positive infinitesimal, and m_1, M — masses.

The reduction formulae read

$$2C_{11} = C_{02} = 2C_{12} = \frac{1}{2(p_1^2 - q^2/4)}$$

$$\times [\tilde{F}_0(m, m, q^2) - \tilde{F}_0(M, m, p_1^2) - (M^2 + p_1^2 - m^2)C_{01}] \tag{D.3}$$

$$C_{ln} = \tfrac{1}{2}\tilde{F}_0(m, m, q^2) + M^2 C_{01} - C_{12}(p_1^2 - m^2 + M^2) - \tfrac{1}{2} \tag{D.4}$$

$$C_{23} = -\frac{1}{4q^2(p_1^2 - q^2/4)}\{\tilde{F}_0(m, m, q^2)(p_1^2 - q^2/2)$$

$$+ \tilde{F}_1(M, m, p_1^2)(-2p_1^2 + q^2) + 2M^2 p_1^2 C_{01}$$

$$+ 2C_{12}(p_1^2 - m^2 + M^2)(-p_1^2 - q^2/2) - p_1^2\}, \tag{D.5}$$

$$C_{03} = 2C_{13} = -\frac{1}{4(p_1^2 - q^2/4)}\{-\tilde{F}_0(m, m, q^2) + 2\tilde{F}_1(M, m, p_1^2)$$

$$+ 2M^2 C_{01} - 1 - 6(p_1^2 - m_1^2 + M^2)C_{12}\}. \tag{D.6}$$

REFERENCES

[1] W. A. Bardeen, R. Gastmans, B. Lautrop, *Nucl. Phys.* **B46**, 319 (1972).
[2] B. W. Lee, R. E. Shrock, *Phys. Rev.* **D16**, 1444 (1977).
[3] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, J. C. Ward, *Phys. Lett.* **13**, 168 (1964); S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961).
[4] N. M. Monyonko, J. H. Reid, *Prog. Theor. Phys.* **73**, 734 (1985).
[5] J. L. L. Martinez, A. Rosado, A. Zepeda, *Phys. Rev.* **D29**, 1539 (1984).
[6] P. Frampton, P. Vogel, *Phys. Rep.* **82**, 33 (1982).
[7] B. Kayser, *Phys. Rev.* **D24**, 110 (1981).
[8] M. Kobayashi, T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
[9] M. Abdelhafiz, H. Czyż, K. Kołodziej, J. Sładkowski, M. Zrałek, *On mass-shell renormalization of the electroweak theory (collection of one-loop approximation formulae)* — in preparation.
[10] K. I. Aoki, Z. Hioki, R. Kawabe, M. Konuma, T. Muta, *Suppl. Prog. Theor. Phys.* **73**, 1 (1982).
[11] F. Jegerlehner, J. Fleischer, *Phys. Lett.* **151B**, 65 (1985), see also references therein.
[12] M. B. Gavela, G. Girardi, C. Malleville, P. Sorba, *Nucl. Phys.* **B193**, 257 (1981).
[13] N. G. Deshpande, M. Nazerimonfared, *Nucl. Phys.* **B213**, 390 (1983).
[14] G. 't Hooft, *Nucl. Phys.* **B33**, 173 (1971).
[15] B. Kayser, *Phys. Rev.* **D26**, 1662 (1982).
[16] C. Itzykson, J. B. Zuber, *Quantum Field Theory*, Mc Graw Hill, Inc. 1980.
[17] Review of Particle Properties, *Rev. Mod. Phys.* **56**, no 2, part II (1984).
[18] G. Passarino, M. Veltman, *Nucl. Phys.* **B160**, 151 (1979).