Vol. B18 (1987) ACTA PHYSICA POLONICA No 3

EFFECTIVE LAGRANGIANS AND SOLITON PHYSICS I. DERIVATIVE EXPANSION, AND DECOUPLING*

BY I. J. R. AITCHISON

CERN, Geneva, Switzerland

(Received July 31, 1986)

This is a review of some recent work in which a derivative expansion technique is used to calculate terms in an effective Lagrangian, starting from some more fundamental Lagrangian.

PACS numbers: 11.15.Tk

1. Introduction

The development and application of non-perturbative methods in quantum field theory continues to be an important area of research, as indicated by its appearance among the subjects of this School. In this contribution, I want to review some recent work in this area, in which a derivative expansion technique is used to calculate terms in an "effective" Lagrangian, starting from some "more nearly fundamental" Lagrangian. The effective Lagrangian, it is hoped, should be relevant to non-perturbative aspects of the original field theory, in some low-energy regime.

The basic ideas and techniques are quite general, but for definiteness I shall mainly discuss the situation in which the "fundamental" Lagrangian is, ultimately, QCD. As we all know, QCD — though a theory of the strong interactions — is asymptotically free, which means that it becomes weakly interacting at high energies or momentum transfers, or equivalently at short distances. This allows one to use perturbation theory to calculate various quantities at high energies, with results that continue to be entirely consistent with experiment. But as the energy scale becomes smaller, the strength of the QCD interaction grows and perturbation theory becomes inapplicable. Of course, this had better be so, if QCD is to be a complete theory of the strong interactions: in jet phenomena, for example, we can virtually watch the evolution from the perturbative degrees of freedom (quarks and gluons), which are the initiators of the events at short distances, to the hadronic non-perturbative degrees of freedom which are observed at larger distances. The problem

* Presented at the XXVI Cracow School of Theoretical Physics, Zakopane, Poland, June 1–13, 1986.

is: given the validity of QCD in the perturbative regime, how do we deal with it in the non -perturbative one?

Prior to QCD, some progress was made in describing very low-energy hadronic phenomena by means of "phenomenological" Lagrangians, in which the hadrons were represented by elementary fields. Initially, these Lagrangians were constructed by requiring that they generate the same matrix elements for soft pion processes as those obtained by the methods of current algebra. Later, it was realized that the forms of such amplitudes, expanded in powers of momenta, were determined by the constraints of (spontaneously broken) chiral symmetry, independently of current algebra as such [1, 2]. Thus, at $O(p^2)$, the only possible Lagrangian for (massless) pions is

$$\mathscr{L}_2 = \tfrac{1}{2}\,\partial_\mu\phi_a\partial^\mu\phi_a, \tag{1}$$

where $\phi_a = (\sigma, \boldsymbol{\pi})$ and $\phi_a^2 = \sigma^2+\boldsymbol{\pi}^2 = f^2$, a constant ($f \cong 93$ MeV, the pion decay constant). Note that (1) is not quite as innocuous as it seems: replacing σ by $\sqrt{(f^2-\boldsymbol{\pi}^2)}$ it can be written as

$$\mathscr{L}_2 = \tfrac{1}{2}\,\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}+\tfrac{1}{2}\,(f^2-\pi^2)^{-1}(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})\,(\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}). \tag{2}$$

From (2) it is clear that $\mathscr{L}_2$ describes amplitudes for arbitrary numbers of pions, but only up to $O(p^2)$ in the momenta. Thus to this order in p^2, $\mathscr{L}_2$ need only be used in tree approximation. The second term in (2) then leads directly to the Weinberg (soft pion) predictions for the $\pi-\pi$ scattering lengths [3]. Equation (1) is the non-linear σ model appropriate to the chiral group $SU(2)_L\times SU(2)_R$, spontaneously broken to $SU(2)_V$.

Only one constant, f, is needed at $O(p^2)$. At $O(p^4)$, there are three possible terms we can form consistent with the assumed symmetry:

$$\mathscr{L}_4 = a\partial_\mu\phi_a\partial_\nu\phi_a\partial^\mu\phi_b\partial^\nu\phi_b+b\partial_\mu\phi_a\partial^\mu\phi_a\partial_\nu\phi_b\partial^\nu\phi_b+c\Box\phi_a\Box\phi_a. \tag{3}$$

Actually, if one terminates the phenomenological Lagrangian at fourth order in derivatives, one can show [4] that in the chiral limit, $(\Box\phi)^2$ can be replaced by $f^{-2}(\partial_\mu\phi_a\partial^\mu\phi_a)^2$, so that only two terms remain in (3). $O(p^4)$ corrections to the $O(p^2)$ results following from (2) can now be calculated using the vertices of $\mathscr{L}_4$ in tree approximation, or those of $\mathscr{L}_2$ in a single loop. Of course, the latter will contain ultra-violet divergences (and infra-red ones which are presumably regulated by a pion mass); but these can be absorbed into redefinitions of the arbitrary constants a and b. In this way one could envisage some sort of parametrization of low energy pion physics.

Naturally, as this procedure is extended to higher powers of p^2 we encounter more and more arbitrary constants (the number of invariants grows rapidly as the number of derivatives increases). One would obviously like to be able to *calculate* these coefficients from some "more nearly fundamental" theory. Can QCD help? A number of attempts have been made [5–15] to attack this problem directly, that is to derive a meson Lagrangian, or at least one suitable for physics below say 1 GeV, directly from $\mathscr{L}_{QCD}$. In my opinion, we cannot yet say that a rigorous calculation from QCD of the coefficients a, b and c in (3), for example, has been done. Nevertheles, theses coefficients *have* been calculated in various different ways, all of which give the same result — so we seem to getting close! I shall describe one such way in the next Section (another possibly related approach is described in Ref. [16]).

It is clear that our phenomenological pion Lagrangians (1) and (3) may be viewed as expansions of some action in powers of derivatives. Such an expansion is expected to be useful if the order of magnitude of the derivatives (or momenta) involved is significantly less than some mass scale. Now the pions we have been discussing are of course the lightest hadronic states, being the *massless* Goldstone modes of the spontaneously broken chiral symmetry, in the chiral limit. All other degrees of freedom are massive — even the quarks, which acquire constituent masses in the breaking of chiral symmetry. Thus we can regard derivative expansions of this type as arising via a process of "decoupling", whereby all massive degrees of freedom are eliminated leaving only the massless pions: the expansion parameter will be ~ (*pion momenta/mass of decoupled state*). This kind of separation between "heavy" or "short range" degrees of freedom, and "light" or "long range" degrees of freedom is reminiscent of similar situations in many body physics. I shall now describe one approach to such derivative expansions, in which the decoupled degrees of freedom are constituent quarks.

2. *Pion Lagrangian from decoupling quarks*

Ideally, we should like to begin with the generating functional

$$Z_{QCD} = \int \mathscr{D}(\text{quarks, gluons}) [\exp i \int \quad _{QCD} d^4x], \tag{4}$$

and by various manipulations of the functional integrals transform it into something involving hadronic degrees of freedom. I shall be much less ambitious. I shall *ignore* confinement, and *assume* chiral symmetry breaking. The simplest model with these ingredients, with the correct symmetries, and retaining the quark degrees of freedom, is one in which the quarks interact with the Goldstone modes of the broken chiral symmetry:

$$Z = \int \mathscr{D}\phi \mathscr{D}\psi \mathscr{D}\bar{\psi} [\exp i \int \mathscr{L}(\phi, \psi) d^4x], \tag{5}$$

where

$$\mathscr{L}(\phi, \psi) = \mathscr{L}(\phi) + \bar{\psi}[i\not{\partial} - g(\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}\gamma_5)]\psi. \tag{6}$$

We shall not, at this stage, be concerned with the precise form of the pure ϕ Lagrangian $\mathscr{L}(\phi)$ — except that it must contain a potential which gives a tree-level minimum at $\phi^2 = f^2$. Our interest is in the effect of decoupling the quarks. The expansion parameter will be (p/M_0), where M_0 is the dynamically generated quark mass $g\langle\sigma\rangle = gf$.

The integral over the quarks can be done formally, leading to

$$Z = \int \mathscr{D}\phi \{\exp i [\int \mathscr{L}(\phi) d^4x + \Gamma(\phi)]\}, \tag{7}$$

where the effective action $\Gamma(\phi)$ is

$$\Gamma(\phi) = -iN_c \operatorname{tr} \ln [i\not{\partial} - g\sigma(x) - ig\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)\gamma_5]. \tag{8}$$

N_c arises from summing over the colour index of the ψ's. Dyakonov and Eides [5] appear to have been the first to consider (8) as an effective chiral action. Although (8) is a very compact expression, it is purely formal, since it involves the non-commuting quantities $i\partial^\mu$ and functions of x, so that the functional operations indicated in (8) can be done neither in x-space nor in p-space. However, we are interested in a low-energy theory, for which a derivative expansion should be appropriate:

$$\Gamma(\phi) = \int d^4x[-V(\phi)+\tfrac{1}{2}\,T(\phi)\,(\partial\phi)^2+\,\ldots,]. \tag{9}$$

((9) is somewhat symbolic, since we have temporarily ignored the internal index on ϕ). We therefore want to evaluate the coefficient functions $V, T, \ldots$ in the derivative expansion of the effective action (fermion determinant) (8).

When my colleague Caroline Fraser and I became interested in this problem nearly three years ago, we could not find anywhere in the literature a convenient ready-made technique for calculating these coefficients. Fraser succeeded in inventing one [17] which is quite easy to explain (see also Ref. [18]). One sets $\phi(x) = \phi_0+\tilde{\phi}(x)$, where ϕ_0 is a constant field, and expands (8) and (9) in powers of $\tilde{\phi}$. By comparing the coefficients in these two expansions, the functions $V, T, \ldots,$ will be determined [17, 19, 20].

The expansion of (9) is of course straightforward. Consider that of (8). Thus far we have, by implication, been working in the co-ordinate representation, but in evaluating the traces which arise in the expansion of (8) it will be convenient to consider the momentum representation also. We introduce operators $\hat{x}^\mu$ and $\hat{p}^\mu$ such that $[\hat{p}^\mu, \hat{x}^\mu] = ig^{\mu\nu}$, with matrix elements $\langle x|\hat{x}^\mu|x'\rangle = x^\mu\delta^4(x-x')$ and $\langle x|\hat{p}^\mu|x'\rangle = i\partial^\mu_x\delta^4(x-x')$ in the coordinate representation, and $\langle p|\hat{x}^\mu|p'\rangle = -i\partial^\mu_p\delta^4(x-x')$ and $\langle p|\hat{p}^\mu|p'\rangle = p^\mu\delta^4(p-p')$ in the momentum representation. Expression (8) is now expanded as:

$$\begin{aligned}-iN_c\,\mathrm{tr}\ln[\hat{\not p}-M(\phi(\hat{x}))] &= -iN_c\,\mathrm{tr}\ln\{(\hat{\not p}-M_0)[1-(\hat{\not p}-M_0)^{-1}\tilde{M}]\}\\ &= -iN_c\,\mathrm{tr}\ln(\hat{\not p}-M_0)+iN_c\,\mathrm{tr}[(\hat{\not p}-M_0)^{-1}\tilde{M}]\\ &\quad+\frac{iN_c}{2}\,\mathrm{tr}[(\hat{\not p}-M_0)^{-1}\tilde{M}(\hat{\not p}-M_0)^{-1}\tilde{M}]+\,\ldots,\end{aligned} \tag{10}$$

where

$$M(\phi(\hat{x})) = g(\sigma(\hat{x})+i\tau\cdot\pi(\hat{x})\gamma_5),\qquad M_0 = M(\phi_0),\qquad\text{and}\qquad \tilde{M} = M(\phi)-M(\phi_0).$$

Since ϕ_0 is a constant, M_0 is independent of $\hat{x}$ and the first term $-iN_c\,\mathrm{tr}\ln(\hat{\not p}-M_0)$ can therefore easily be evaluated in momentum space; it may be identified with $-V(\phi_0)$, the $\tilde{\phi}$-independent term in the expansion of (9). The next term in (10) is evaluated as follows:

$$\begin{aligned}iN_c\,\mathrm{tr}[(\hat{\not p}-M_0)^{-1}\tilde{M}] &= iN_c\int d^4p\,\langle p|\,(\hat{\not p}-M_0)^{-1}\tilde{M}(\hat{x})\,|p\rangle\\ &= iN_c\int d^4p\,\langle p|\,(\hat{\not p}-M_0)^{-1}|p'\rangle\,\langle p'|x'\rangle\,\langle x'|\tilde{M}(\hat{x})\,|x\rangle\,\langle x|p\rangle d^4x d^4x' d^4p'\\ &= iN_c\int d^4p(\not p-M_0)^{-1}\langle p|x\rangle\tilde{M}(x)\,\langle x|p\rangle d^4x\\ &= iN_c\int\frac{d^4p}{(2\pi)^4}(\not p-M_0)^{-1}\cdot\int d^4x\tilde{M}(x).\end{aligned} \tag{11}$$

This corresponds to the term of order $\tilde{\phi}$ which is obtained when $V(\phi)$ in (9) is expanded about $\phi = \phi_0$. We note the separation of (11) into two factors, one an integral over p and the other an integral over x. If all terms in (10) had this product form, comparison with the $\tilde{\phi}$ expansion of (9) would be very simple, since the coefficients in the latter would be essentially just the p-integrals in such products. But it is clear that the product form of the last step in (11) originates in the fact that the operator whose trace is being evaluated is itself a product of one $\hat{p}$-dependent operator and one $\hat{x}$-dependent one. The operator in the third term in (10) is not of this form, and so its trace cannot be immediately written in the factorized form (11). However, this difficulty can be overcome by repeated use of the identity

$$\tilde{\phi}(\hat{x})(\hat{p}^2-M_0^2)^{-1} = (\hat{p}^2-M_0^2)^{-1}+(\hat{p}^2-M_0^2)^{-2}[\hat{p}^2,\tilde{\phi}]+(\hat{p}^2-M_0^2)^{-3}[\hat{p}^2,[\hat{p}^2,\tilde{\phi}]]+\ldots \tag{12}$$

(which can be employed after "rationalizing" the fermion propagator) together with

$$[\hat{p}^\mu,\tilde{\phi}(\hat{x})] = i\frac{\partial}{\partial\hat{x}_\mu}\tilde{\phi}(\hat{x}), \quad [\hat{p}^2,\tilde{\phi}(\hat{x})] = \frac{\partial}{\partial\hat{x}_\mu}\frac{\partial}{\partial\hat{x}^\mu}\tilde{\phi}(\hat{x})+2i\hat{p}_\mu\frac{\partial}{\partial\hat{x}_\mu}\tilde{\phi}(\hat{x}), \tag{13}$$

and similar expressions for higher commutators. In this way each operator being "traced" in (10) can be written as a sum of products of $\hat{p}$'s on the left, and functions of $\tilde{\phi}(\hat{x})$ and $(\partial/\partial\hat{x}^\mu)\tilde{\phi}(\hat{x})$ on the right. The traces can then be evaluated as in (11), the result being a p-integral times an x-integral involving $\tilde{\phi}(x)$ and $\partial\tilde{\phi}/\partial x^\mu$'s. In this way the coefficient functions in (9) are found.

It is clear that this method is perfectly general, and can be applied to boson determinants, for example (see Fraser [17] and Aitchison and Fraser [21]). We have also shown [20] that it provides a very efficient way of calculating anomaly-induced vertices, and Goldstone-Wilczek currents. A number of other methods for doing derivative expansions have also been proposed [22–26], notably in a series of papers by Zuk [27–29], and by Ball and Osborn [30, 31] and Ball [32, 33], using the powerful proper time or heat kernel technique [34, 35].

Returning to the problem at hand, one finds that the coefficient T is divergent; the cut off is adjusted to yield the properly normalized $\mathscr{L}_2$ of (1). At fourth order in derivatives, we obtained [19, 20] for the coefficients in (3)

$$a = N_c/(48\pi^2 f^4), \quad b = -N_c/(32\pi^2 f^4); \quad c = N_c/(48\pi^2 f^4). \tag{14}$$

The same result was derived using a graphical technique by Mackenzie, Wilczek and Zee [22a], and has also been found by many other authors using a variety of different initial Lagrangians, and methods [6–9, 16, 28, 30, 31, 36–39]. This is a sound reason for believing it to be more general than any of the individual models or derivations.

Perhaps the most interesting qualitative point to notice about the result (14) is that it is *independent* of the quark mass — though the expansion is in powers of p/M_0, where $M_0 = gf$. The coupling strength g has disappeared from (14), and only the symmetry-breaking parameter f sets the scale of this term. Higher order terms do involve powers of g in the denominator; for example, the sixth-order ones go like $(\partial\phi)^6/(f^8 g^2)$. We shall return to this point in Section 5.

How does the prediction (14) compare with the experimental values of the $\pi-\pi$ amplitudes at $O(p^4)$? To pursue the comparison we must first include an explicit pion mass term [1]

$$-(m_\pi^2/8f^2)(\pi^2)^2, \tag{15}$$

since it is a poor approximation to set $m_\pi = 0$ near threshold. We must next face the old problem [30] that, away from threshold, the $\pi-\pi$ amplitude is of course complex, due to unitarity, whereas the terms in $\mathscr{L}_2$ and $\mathscr{L}_4$ are purely real. One simple "unitarization" scheme is to interpret the matrix elements generated by $\mathscr{L}_2$ and $\mathscr{L}_4$ as "K-matrix" elements, and to construct the phase shifts δ_l^I via

$$(k_l^I)^{-1} = \varrho_l \cot \delta_l^I, \tag{16}$$

where ϱ_l is the two-body phase space factor in the l^{th} partial wave, and I is the isospin channel index. The $\pi-\pi$ phase shifts calculated this way [41] are shown in Table I and Fig. 1.

The most significant $O(s^2)$ corrections are in the $I = l = 0$ and $I = 2$, $l = 0$ channels. In the former, the correction contributes some 10° at 700 MeV, in the direction of increasing the soft pion phase shift, and thus tending to improve agreement with experiment [40, 42]. This is equivalent to the effect of a finite mass m_σ in this channel (our non-linear model corresponding originally to the limit $m_\sigma \to \infty$); we interpret this as having been generated by the quark dynamics. In the $I = 2$ $l = 0$ channel, the $O(s^2)$ term contributes 12° at 700 MeV, usefully correcting the tendency of the soft pion term to become too large in magnitude, relative to the admittedly poorly determined data.

The p-wave phase shift is small, but rising. Of course, we see no ρ resonance amplitude explicitly in the coefficients (14), but this phase shift can certainly represent the tail of the ρ. We also note that the d-wave scattering lengths — not predicted by the soft pion theory — are given (in units of m_π^{-4}) by

$$a_2^0 = (160\pi^3)^{-1}(m_\pi/f)^4 \simeq 9.9\times10^{-4} \qquad a_2^2 = 0, \tag{17}$$

TABLE I

π-π phase shifts calculated using Eq. (16)

$E(=\sqrt{s})$ (MeV)	δ_0^0	δ_0^2	δ_1^1
360	11°	−3.6°	1°
440	21°	−7°	2.7°
520	31°	−10°	5°
600	41°	−12°	8.7°
680	50°	−12°	13°
760	58°	−11°	18.6°

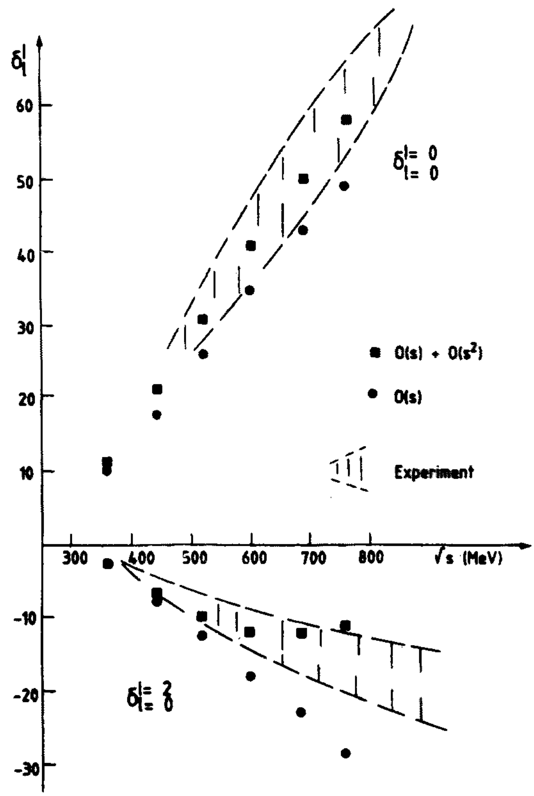

Fig. 1. δ_0^0 and δ_0^2 from Eq. (16) vs $\sqrt{s}$

which are to be compared with the quoted experimental values [42]

$$a_2^0 = (17 \pm 3) \times 10^{-4} \qquad a_2^2 = (1.3 \pm 3) \times 10^{-4}. \tag{18}$$

In summary, integration over fermion degrees of freedom seems to provide, at $O(p^4)$, qualitatively useful corrections to the $O(p^2)$ results in the $\pi - \pi$ sector.

Although the above calculation of phase shifts and low-energy parameters is one way of comparing (14) with data, there is another, perhaps more physically instructive way. The largest phase shifts are those in the ρ ($I = l = 1$) and "σ" ($I = l = 0$) channels: it would be interesting if we could relate the ρ and "σ" parameters (masses and couplings) directly to (14). We can do this with the help of the $1/N_c$ expansion of QCD.

3. *QCD and the $1/N_c$ expansion: pion Lagrangian from decoupling heavy mesons*

Assuming confinement, the asymptotic states of QCD are not the coloured quarks and gluons, but rather the observed colour singlet hadrons. In view of this, one might wonder whether in some way QCD itself could not be equivalently formulated in terms of these observed asymptotic degrees of freedom. Quite remarkably, the work of 't Hooft

[43] and Witten [44] shows that QCD is indeed equivalent — in the full field theory sense — to a theory of mesons and glueballs. It is unfortunate — though hardly surprising — that the arguments for this proposition do not enable us to construct the actual equivalent meson theory. We can certainly write down phenomenological meson Lagrangians, incorporating the known meson states, and their properties and symmetries, up to say about 1 GeV. But at first sight, this is not very predictive (though see Section 4!)

With one more ingredient, however, it becomes so. This is the notion that $1/N_c$ can be treated as an expansion parameter [43, 44]. 't Hooft and Witten showed that trilinear meson-meson coupling constants are of order $1/\sqrt{N_c}$, so that if this is treated as "small", we have a novel kind of perturbation theory. In fact, the phenomenological meson Lagrangian just mentioned can — to the extent that $1/\sqrt{N_c}$ is small — be treated semi-clasically: that is, we neglect all loops and use it in tree approximation only. We can now easily imagine writing down the resultant coupled field equation for π, σ, ρ, ω, A_1 etc. and solving them in powers of (*derivatives/masses*), thereby expressing all fields *except* the pion in terms of the pion fields and their derivatives. Thus we are here "decoupling" the heavy mesons, but at tree-level only.

Fraser, Miron and I carried out this procedure [41] up to $O(\partial\pi)^4$ for a number of possible meson Lagrangians incorporating the π, σ, ρ and A_1 fields (the ω contributes first at $O(\partial\pi)^6$). By comparing the result with the "quark loop" terms of (14), we will be able to *predict* the values of certain meson parameters in terms of f alone. The comparison is simpler if we first make use of the replacement $(\phi)^2 \to f^{-2}(\partial_\mu\phi_a\partial^\mu\phi_a)^2$, valid up to $O(\partial^4)$ (cf. remarks following (3)). Then the quark decoupling result was

$$\mathscr{L}_4^q = \frac{N_c}{48\pi^2 f^4}\{([\partial_\mu\phi_a\partial_\nu\phi_a)^2-(\partial_\mu\phi_a\partial^\mu\phi_a)^2]+\tfrac{1}{2}(\partial_\mu\phi_a\partial^\mu\phi_a)^2\} \tag{19}$$

(the reason for writing it this way will become clear immediately). To $O(\boldsymbol{\pi}^4)$ we may replace ϕ_a by $(0, \boldsymbol{\pi})$ in (19).

The result of the $O(\partial\boldsymbol{\pi})^4$ meson decoupling calculation is

$$\mathscr{L}_4^\rho = \frac{1}{2m_\rho^2 f^2}\cdot[(\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi})^2-(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})^2], \tag{20}$$

$$\mathscr{L}_4^\sigma \simeq \frac{1}{2m_\sigma^2 f^2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})^2, \tag{21}$$

where, in arriving at (20), we imposed in all our meson Lagrangians the KSRF relation [45]

$$m_\rho^2 = 2f^2 g_{\rho\pi\pi} \tag{22}$$

between the ρ mass, the $\rho\pi\pi$ coupling constant, and f. Equation (20) held in all three meson Lagrangians we considered [41]; the numerical factor in (21) differed somewhat among the models. The conclusion is very striking, and had in fact been established earlier by Pham and Truong [46]: namely, the ρ meson induces a term of the form (20) which involves the *difference* of the two possible invariants, while the σ meson induces a purely "symmetric" term. Comparing (19) with (20) we deduce

$$m_\varrho^2 = 24\pi^2 f^2/N_c \tag{23}$$

or, using (22),

$$g_{\varrho\pi\pi}^2 = 12\pi^2/N_c. \tag{24}$$

For $N_c = 3$, (23) gives $m_\varrho = 826$ MeV, while (24) gives $g_{\varrho\pi\pi} = 2\pi$. These values are encouragingly realistic. Similarly, from (19) and (21) we obtain

$$m_\sigma^2 \simeq 48\pi^2 f^2/N_c \simeq 2m_\varrho^2, \tag{25}$$

which gives $m_\sigma \approx 1200$ MeV.

There are certainly a number of queries one might raise at this point. For example, is 1/3 really so small that meson loop contributions at $O(p^4)$ can be neglected (supposing they could be calculated)? How de we know, on the other hand, that the $O(\partial\boldsymbol{\pi})^4$ terms are reliably given by (19)? Could there not be additional quark interactions beyond those in (6), involving the heavier mesons for example, which might induce corrections to (19)? Perhaps somehow (19), (20), (21) are *consistently* relatable to one another, to the same order in $1/N_c$. At all events, (23), (24) and (25) seem to imply that the simple Lagrangian (6) already has quite a lot of ρ and σ physics in it.

4. *The soliton sector*

It will not have escaped the reader's attention that there seems to be one very large gap in the equivalence

confined QCD ≡ theory of mesons and glueballs;

namely, where are the baryons? It is here that the real interest, for me at any rate, of the $1/N_c$ idea lies. Witten showed [44] that for large N_c baryon masses scale like N_c. This is reminiscent of the behaviour of *solitons* in a theory in which the coupling constant is g: the soliton mass $\sim 1/g^2$, so that putting $g \sim 1/\sqrt{N_c}$, we find *mass* $\sim N_c$. The idea that baryons are solitons in a meson theory was put forward in remarkable papers by Skyrme [47] more than 25 years ago. After an interval of 20 years, two equally remarkable papers by Witten [48, 49] served to provoke renewed and active interest in this suggestion.

Skyrme's solitons are, in fact, precisely certain static field configurations of the non-linear σ-model we have been considering. These configurations have the form

$$\sigma = f\cos\theta(r), \tag{26}$$

$$\boldsymbol{\pi} = \hat{\boldsymbol{r}} f\sin\theta(r), \tag{27}$$

where $\theta(0) = n\pi$ and $\theta(\infty) = 0$. We note that (26) and (27) automatically satisfy $\sigma^2 + \boldsymbol{\pi}^2 = f^2$. Such a field configuration acts as a *mapping* between real three-dimensional space and the isospin space of SU(2) — as is most immediately seen from (27), in which the isospin vector $\boldsymbol{\pi}$ "points along" the real vector $\hat{\boldsymbol{r}}$. This mapping is characterized by a winding number N, which counts the number of times (compactified) real space is mapped into internal isospin space:

$$N = -(12\pi^2 f^4)^{-1} \varepsilon_{ijk}\varepsilon_{abcd} \int \phi_a \partial^i \phi_b \partial^j \phi_c \partial^k \phi_d d^3 \boldsymbol{r}. \tag{28}$$

Arguments can be given [49, 50, 48, 20] for identifying N with baryon number B. When $\theta(0) = \pi$ (i.e., $n = 1$), one finds $N = 1$.

What Lagrangian produces these soliton configurations? We would clearly start by considering the simplest possibility, which is just $\mathscr{L}_2$ of (1). Certainly, this must be included, since it has the right symmetries and is a standard "kinetic energy" piece. But it cannot be the whole story. It is easy to see that static energy arising from $\mathscr{L}_2$ alone cannot correspond to a soliton extended in space. Consider

$$E_{\text{static}} \sim \int (\nabla\phi)^2 d^3\boldsymbol{r} \sim \int \left(\frac{d\theta}{dr}\right)^2 r^2 dr. \tag{29}$$

Suppose θ has a spatial extension of order R, e.g., $\theta \sim e^{-r/R}$. Now set $r/R = t$. We find $E(R) = RE(t = 1) \to 0$ as $R \to 0$. So the minimum energy configuration of $\mathscr{L}_2$ alone will be point-like, and have zero energy — and this does not look very like a nucleon. Accordingly, Skyrme [47a] — not wishing to enlarge the number of basic fields in his model — introduced a term of *fourth* order in derivatives, namely

$$\mathscr{L}_{\text{SK}} = \frac{1}{4e^2 f^2} [(\partial_\mu \phi_a \partial_\nu \phi_a)^2 - (\partial_\mu \phi_a \partial^\mu \phi_a)^2], \tag{30}$$

where e (not the electronic charge) is a new parameter. Repeating the reasoning following (29), we can easily see that the static energy from (30) will scale as $1/R$. Hence a stable configuration with $R = 0$ can exist, by a balance between the R and R^{-1} contributions. This constitutes the dynamics of the Skyrme model.

It seemed natural to ask [19, 22a] whether the Skyrme parameter e^2 could be *predicted* from some underlying theory — interpreting $(\mathscr{L}_2 + \mathscr{L}_{\text{SK}})$ as a particular low-energy approximation to the meson Lagrangian which is equivalent to QCD. The result of the $O(\partial\phi)^4$ calculations have of course already been given in Sections 2 and 3, especially Eq. (19). From the latter we find at once

$$e = 2\pi [= g_{\varrho\pi\pi}, \text{ cf. Eq. (24)}], \tag{31}$$

for $N_c = 3$. Equation (31) is, in fact, not far from the value of e required in phenomenological applications of the model consisting of $(\mathscr{L}_2 + \mathscr{L}_{\text{SK}})$ to the nucleon's static properties [51–53]. However, there is a serious problem with (19), which is that the second term contributes *negatively* in the static energy, and can even be larger in magnitude than the first. Thus (19) — or more generally (14) — does not produce a dynamically stable soliton. (This was not clear to us in Ref. [19], but was to Mackenzie et al. [22a]).

Where can stability come from? The terms $\mathscr{L}_4^q$ have been calculated by expanding the non-local determinant (8) in powers of $\partial\phi$. We might wonder whether some higher order terms — e.g., $(\partial\phi)^6$ — in this expansion could do the trick. Unfortunately, as we shall see, this turns out not to be the case. Ripka and Kahana [39] have actually calculated the full contribution E^q of the (renormalized) determinant to the static energy, for the ansatz $\theta(r) = \pi \exp(-r/R)$. The divergent $O(\partial^2)$ piece has been subtracted from the determinant, so that $\varepsilon^q \equiv E_q/fgN_c$ represents the full effect of *all* terms of $O(\partial^4)$ and higher. ε^q is positive — and hence stabilizing — for all soilton sizes R. It is quite remarkable that we apparently do have here a fully three-dimensional and realistic example of a soliton stabilized by quantum loop corrections. Indeed, Ripka and Kahana find [39] that the contribution of ε^q, together with that from $\mathscr{L}_2$, gives a satisfactory nucleon *mass/size*, for reasonable values of g.

But these calculations are very lengthy, and one would like to be able to deal with a local effective Lagrangian rather than the non-local determinant. Consequently, the $O(\partial\phi)^6$ terms in the expansion of (8) were calculated [33, 54, 55] to see if they would provide a fair approximation to the full E^q, at least for some realistic R's. The contribution ε_6^q of these sixth order terms was calculated in [55] for the same ansatz $\theta(r) = \pi \exp(-r/R)$. However, ε_6^q does not provide a useful approximation to ε^q in the region of interest, which is $X \sim 1$ to 2. Though stabilizing, ε_6^q is not large enough in magnitude to prevent the soliton collapsing to too small a size. Indeed, ε^q does not appear to scale as *any* simple power of R: all derivatives are contributing. Thus the derivative expansion of (8) fails for the purpose of predicting the terms in the meson Lagrangian which are responsible for realistic nucleon stability.

The same conclusion can also be drawn [41] as regards the attempt to obtain an effective pion Lagrangian — for use in the soliton sector — by decoupling the heavy mesons (Section 3). One is therefore left with the alternatives (a) numerical evaluations of (8), and other approaches to the π-quark model of (6) (see also [56]); (b) solution of the meson field equation *without* decoupling the heavy mesons. Of these, the latter is certainly directly in line with the $1/N_c$ ideas, and is likely to be easier numerically. The most recent work in this direction, by Lacombe, Loiseau, Vinh Mau and Cottingham [57] is encouraging: solving the classical theory of π, ρ, ε, ω and A_1 mesons in the soliton sector, they obtain significantly better agreement with experimental data than in the original one-parameter Skyrme model. It should be noted that the meson Lagrangian used here is highly constrained both by symmetry requirements and by the need to fit the observed meson parameters (e.g., m_ρ, m_ω, $g_{\rho\pi\pi}$, g_ω, ...).

5. *Decoupling, and the possibility of quantizing anomalous gauge theories*

We end by mentioning briefly some speculations [33] on an apparently rather different matter, having to do with the problem of quantizing gauge theories that have anomalies in gauged currents. The conventional view has, for a long time, been that such anomalies cannot be tolerated because they spoil renormalizability and unitarity — hence one requires typically an *anomaly-cancellation* mechanism. For example, the lepton and quark anomalies in the standard $SU(2)\times U(1)$ model do cancel. Note that the anomalies are independent of the fermion masses, and so such a cancellation between different fermion doublets is possible. These anomaly matching conditions are important constraints on possible models.

Recently, however, it has been suggested that it may after all be possible to quantize gauge theories consistently, even in the presence of an anomaly which is not cancelled by the conventional mechanism. Examples in $1+1$ dimension have been discussed by Jackiw and Rajaraman [58] and Faddeev and co-workers have considered the $3+1$ problem. In particular, Faddeev and Shatashvili [59, 60] propose that by adding a (gauged) *Wess--Zumino* term to the action for a Weyl fermion interacting with Yang-Mills fields (which, by itself, would be anomalous), a consistent quantum theory is obtained. They interpret the inclusion of such a term somewhat along the lines of the usual "ghost" terms, in that the scalar field in their $W-Z$ action is only present in closed loops (and indeed they introduce no kinetic piece for it). The coefficient of the $W-Z$ term is fixed by topological considerations [48], and one may therefore hope that renormalizability is not lost.

The $1+1$ dimensional theories considered by Jackiw and Rajaraman seem to provide explicit illustrations of quantizable, though anomalous models. For example, the (exactly soluble) chiral Schwinger model, in which a $U(1)$ gauge field interacts with a Weyl fermion in $1+1$, can apparently yield a consistent, unitary and Lorentz-invariant theory with a spectrum which includes a single massive vector meson [58]. Rajaraman interprets this phenomenon as "mass generation by anomaly, as distinct from the Higgs mechanism" [61]. If this interpretation could be generally true in $3+1$ dimensions, clearly our expectations about the Higgs sector (such as they are) in the standard model would be drastically altered. The non-Abelian $1+1$ case was also studied by Rajaraman [62] and by him and Lott [63]. In this case it also seems that consistent quantization may be possible — but one must note that these claims are not uncontroversial, even in the Abelian case [64].

We could, however, take a somewhat different point of view [33] — namely, that perhaps the scalar field in the $W-Z$ term introduced "by hand" by Faddeev is actually that of a (very strongly interacting) Higgs field. Suppose we start from a theory with one fermion multiplet of given chirality and mass m, and a second fermion multiplet of opposite chirality and different mass M, and we arrange for the chiral gauge anomalies to cancel. The fermion masses are generated via the Higgs field vacuum expectation value, as in (6) for the ungauged chiral flavour symmetry case. Consider what happens as we send $M \to \infty$ (which means the corresponding coupling $g_M \to \infty$, for fixed vacuum expectation value). This is precisely a decoupling problem, as in the earlier sections, but now we are interested only in the terms which survive as $g_M \to \infty$. The work of d'Hoker and Farhi [36, 65] and Ball [33] shows that in this limit the heavy fermion leaves a residue behind, which consists of two parts: (a) a $W-Z$ term in the Higgs fields, (b) terms of the form (3), of order $(\partial\phi)^4$ (recall that higher order derivatives go down like inverse powers of g_M). In addition, since we are by implication considering the "non-linear" limit $\sigma^2+\boldsymbol{\pi}^2 = F^2$ for the Higgs fields, we need to examine possible terms remaining in the Higgs sector, in this limit ($m_H \to \infty$). We must also remember to include gauge fields.

The suggestion here is, therefore, that the extra $W-Z$ term introduced by Faddeev and Shatashvili arises precisely from such a decoupling mechanism, while the vector meson mass discovered by Jackiw and Rajaraman is a manifestation of a Higgs mechanism after all, but with $m_H \to \infty$. In this case, the unconventional procedure of Faddeev and Shatashvili may be equivalent to the conventional procedure (i.e., of *anomaly cancellation + Higgs mechanism*), in a certain limit. Certainly, this would seem to indicate that it would *not* be possible to quantize an anomalous gauge theory and come out with a *massless* vector meson. The Higgs sector has to be there in order to give the "heavy" fermion a mass, and it then generates a vector meson mass as usual.

As yet, this is only a suggestion. One difficulty is that although the W-Z (Higgs) term, being of topological origin, may not affect renormalizability, the $O(\partial\phi)^4$ terms of (3) certainly seem to. However, Braaten has recently claimed [66] that a "generalized Skyrme" model, with all three invariants (3) present, is both renormalizable and asymptotically free! His Lagrangian has the form

$$\mathscr{L} = \mathscr{L}_2 + \frac{1}{4e^2f^2}\left[(\partial_\mu\phi_a\partial_\nu\phi_a)^2 - (\partial_\mu\phi_a\partial^\mu\phi_a)^2\right] - \frac{\delta^2}{4e^2f^2}(\partial_\mu\phi_a\partial^\mu\phi_a)^2 - \frac{1}{2M^2}(\Box\phi)^2, \tag{32}$$

where e^2, δ^2 and M^2 are all positive. Referring to (3) and (14), we see that our $(\partial\phi)^4$ terms arising from heavy fermion decoupling give

$$\mathscr{L} = \mathscr{L}_2 + \frac{N_{TC}}{48\pi^2F^2}\left[(\partial_\mu\phi_a\partial_\nu\phi_a)^2 - (\partial_\mu\phi_a\partial^\mu\phi_a)^2\right]$$
$$- \frac{N_{TC}}{96\pi^2F^2}(\partial_\mu\phi_a\partial^\mu\phi_a)^2 + \frac{N_{TC}}{48\pi^2F^2}(\Box\phi)^2. \tag{33}$$

F and N_{TC} could presumably now have the significance of "technicolour" parameters. Thus the coefficients of the first two $O(\partial\phi)^4$ terms are the same sign as in Braaten's model, but that of the last term has the opposite sign. For Braaten, this last term is interpreted as a Pauli-Villars cut-off, since it modifies the pion propagator from the $(p^2)^{-1}$ arising from $\mathscr{L}_2$ to $(p^2 - p^4/M^2)^{-1} = (p^2)^{-1} - (p^2 - M^2)^{-1}$.

Indeed, it is responsible — via the increased powers of p^2 in the propagator — for rendering the theory renormalizable (see also Slavnov [67]). The price to be paid, of course, is that the theory is not unitary at energies above $M(= \pi F\sqrt{24/N_{TC}})$. On the other hand, the sign of the $(\Box\phi)^2$ term predicted by (33) corresponds to a tachyonic pole in the pion propagator. We also recall that the solitonic sector was unstable, if that sign is as in (33) — modulo the effect of the gauge fields. This may be a signal that the true ground state is not to be found by minimizing the $\mathscr{L}$ of (33), but that further quantum effects — such as the infra-red summations required in some similar solid state systems — must be taken into account [68]. Perhaps when the true ground state is found, the theory will remain renormalizable and asymptotically free, and will also be unitary due to the disappearance (somehow!) of the M^2 pole.

Another open question at this stage is the nature of the terms left in the Higgs sector as $m_H \to \infty$. Fraser and I did a conventional one-loop calculation of the effective action in the linear $SU(2) \times SU(2)$ σ-model, and investigated the limit $m_H \to \infty$ [21]. We found, as expected [69], terms of the form a and b in (3), which were proportional to ln m_H^2 (as well as other finite terms of this form, and other "non-invariant" terms (69a)). The presence of these terms would of course substantially modify (33). On the other hand, Chan has claimed [70] that a different approach to the limit $m_H \to \infty$ of the one-loop action yields only a *finite* term of the form b in (3), a single renormalization sufficing for this and for $\mathscr{L}_2$. If true, this would significantly affect the decoupling idea, suggested above.

I am very grateful to Caroline Fraser for much helpful correspondence.

REFERENCES

[1] B. W. Lee, *Chiral Dynamics*, Gordon and Breach, New York 1972.
[2] S. Weinberg, *Physica* **96A**, 328 (1979).
[3] S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966).
[4] E. Brezin, S. Hikami, J. Zinn-Justin, *Nucl. Phys.* **B165**, 528 (1980).
[5] D. I. Dyakonov, M. I. Eides, *Pisma v Zh. Eksp. Teor. Fiz.* **38**, 358 (1983) (*J.E.T.P. Lett.* **38**, 433 (1984)).
[6] J. Balog, *Phys. Lett.* **149B**, 197 (1984); see also *Nucl. Phys.* **B258**, 361 (1985).
[7] K. Sao, Gifu preprint GWJC-1 (1984).
[8] P. Simic, *Phys. Rev. Lett.* **55**, 40 (1985); Rockefeller Univ. preprint RU/85/124 (1985).
[9] A. Dhar, S. R. Wadia, *Phys. Rev. Lett.* **52**, 959 (1984); A. Dhar, R. Shankar, S. R. Wadia, *Phys. Rev.* **D31**, 3256 (1985).
[10] A. Andrianov, Yu. Novozhilov, *Phys. Lett.* **153B**, 422 (1985).
[11] A. Andrianov, *Phys. Lett.* **157B**, 425 (1985).
[12] R. T. Cahill, C. D. Roberts, *Phys. Rev.* **32**, 2419 (1985); R. T. Cahill, C. D. Roberts, Flinders University preprints FIAS-R-158 (June 1985) and FIAS-R-163 (September 1985).
[13] N. I. Karchev, A. A. Slavnov, Imperial College preprint Imperial/TP/84-85/28 (1985).
[14] J. Balog, P. Vecsernyés, *Phys. Lett.* **163B**, 217 (1985).
[15] D. I. Dyakonov, V. Yu. Petrov, Leningrad preprint 1162 (February 1986).
[16] D. Ebert, H. Reinhardt, Berlin preprint PHE 86-03 (1986).
[17] C. M. Fraser, *Z. Phys.* **C28**, 101 (1985).
[18] A. Vainshtein, V. Zakharov, V. Novikov, M. Shifman, *Sov. J. Nucl. Phys.* **39**, 77 (1984).
[19] I. J. R. Aitchison, C. M. Fraser, *Phys. Lett.* **146B**, 63 (1984).
[20] I. J. R. Aitchison, C. M. Fraser, *Phys. Rev.* **D31**, 2605 (1985).
[21] I. J. R. Aitchison, C. M. Fraser, *Phys. Rev.* **D32**, 2190 (1985).
[22] (a) R. Mackenzie, F. Wilczek, A. Zee, *Phys. Rev. Lett.* **53**, 2203 (1984); (b) R. Mackenzie, F. Wilczek, *Phys. Rev.* **D30**, 2194, 2260 (1984).
[23] L.-H. Chan, *Phys. Rev. Lett.* **54**, 1222 (1985); **55**, 21 (1985); **56**, 404E (1986).
[24] O. Cheyette, *Phys. Rev. Lett.* **55**, 2394 (1985).
[25] M. K. Gaillard, Berkeley preprint LBL-20308, UCB-PTH-85/41 (1985).
[26] I. Moss, D. Toms, W. Wright, Newcastle preprint 85 TP-7 (1985).
[27] J. A. Zuk, *J. Phys.* **A18**, 1795 (1985).
[28] J. A. Zuk, *Z. Phys.* **C29**, 303 (1985).
[29] J. A. Zuk, *Phys. Rev.* **D32**, 2653 (1985).
[30] R. D. Ball, H. Osborn, *Phys. Lett.* **165B**, 410 (1985).
[31] R. D. Ball, H. Osborn, *Nucl. Phys.* **B263**, 245 (1986).
[32] R. D. Ball, *Phys. Lett.* **171B**, 435 (1986).
[33] R. D. Ball, Cambridge Thesis (1986).
[34] J. S. Schwinger, *Phys. Rev.* **82**, 664 (1951).
[35] B. S. de Witt, *Phys. Rep.* **19C**, 295 (1975).

[36] E. D'Hoker, E. Farhi, *Nucl. Phys.* **B248**, 59 (1984).
[37] R. I. Nepomachie, *Ann. Phys.* **158**, 67 (1984).
[38] M. Gaudin, Saclay preprint SPh-T/84-006 (1984).
[39] G. Ripka, S. Kahana, *Phys. Lett.* **155B**, 327 (1985).
[40] See, e.g., B. R. Martin, D. Morgan, G. Shaw; *π-π Interactions in Particle Physics*, Academic Press, New York 1976.
[41] I. J. R. Aitchison, C. M. Fraser, P. J. Miron, *Phys. Rev.* **D33**, 1994 (1986).
[42] Particle Data Group, *Rev. Mod. Phys.* **56**, S1 (1984); N. M. Cason et al., *Phys. Rev.* **D28**, 1586 (1983); L. Rosselet et al., *Phys. Rev.* **15**, 572 (1977).
[43] G. 't Hooft, *Nucl. Phys.* **B72**, 461 (1974); **B75**, 461 (1974).
[44] E. Witten, *Nucl. Phys.* **B160**, 57 (1979).
[45] K. Kawarabayashi, M. Suzuki, *Phys. Rev. Lett.* **16**, 255 (1966); Riazuddin, Fayyazuddin, *Phys. Rev.* **147**, 1071 (1966).
[46] T. N. Pham, T. N. Truong, *Phys. Rev.* **D31**, 3027 (1985).
[47] (a) T. H. R. Skyrme, *Proc. Roy. Soc.* **A260**, 127 (1961); (b) *Nucl. Phys.* **31**, 556 (1962); see also *Proc. Roy. Soc.* **A247**, 260 (1958) and *Proc. Roy. Soc.* **252**, 236 (1959).
[48] E. Witten, *Nucl. Phys.* **B223**, 422, 433 (1983).
[49] J. Goldstone, F. Wilczek, *Phys. Rev. Lett.* **47**, 986 (1981).
[50] A. P. Balachandran, V. P. Nair, S. G. Rajeev, A. Stern, *Phys. Rev. Lett.* **49**, 1124 (1982).
[51] G. S. Adkins; C. R. Nappi, E. Witten, *Nucl. Phys.* **B228**, 552 (1983).
[52] A. D. Jackson, M. Rho, *Phys. Rev. Lett.* **51**, 751 (1983).
[53] C. R. Nappi, Proc. Santa Barbara Workshop on Nuclear Chromodynamics, 1985, to be published by World Scientific.
[54] J. A. Zuk, Ref. [28].
[55] I. J. R. Aitchison, C. M. Fraser, K. E. Tudor, J. A. Zuk, *Phys. Lett.* **165B**, 162 (1985).
[56] M. C. Birse, M. Banerjee, *Phys. Lett.* **135B**, 284 (1984); *Phys. Rev.* **D31**, 118 (1985).
[57] M. Lacombe, B. Loiseau, R. Vinh Mau, W. N. Cottingham, Orsay preprint IPNO-TH-86-36 (1986).
[58] R. Jackiw, R. Rajaraman, *Phys. Rev. Lett.* **54**, 1219 (1985).
[59] L. D. Faddeev, S. L. Shatashvili, *Theor. Math. Phys.* **60**, 206 (1984).
[60] L. D. Faddeev, S. L. Shatashvili, *Phys. Lett.* **167B**, 225 (1986).
[61] R. Rajaraman, Bangalore preprint, talk presented at the Second Asia Pacific Physics Conference, Bangalore (1986).
[62] R. Rajaraman, *Phys. Lett.* **162B**, 148 (1985).
[63] J. Lott, R. Rajaraman, *Phys. Lett.* **165B**, 321 (1985).
[64] A. Das, *Phys. Rev. Lett.* **55**, 2126 (1985); C. R. Hagen, *Phys. Rev. Lett.* **55**, 2223C (1985); R. Jackiw, R. Rajaraman, *Phys. Rev. Lett.* **55**, 2224C (1985); R. Banerjee, *Phys. Rev. Lett.* **56**, 1889 (1986); K. Harada, K. Kubota, T. Tsutsui, *Phys. Lett.* **173**, 77 (1986); I. G. Halliday, E. Rabinovici, A. Schwimmer, *Nucl. Phys.* **B268**, 413 (1986).
[65] E. D'Hoker, E. Farhi, *Nucl. Phys.* **B248**, 77 (1984).
[66] E. Braaten, Northwestern preprint (1986).
[67] A. A. Slavnov, *Nucl. Phys.* **B31**, 301 (1971).
[68] R. D. Ball, private communication.
[69] (a) T. Appelquist, C. Bernard, *Phys. Rev.* **D23**, 425 (1981); (b) *Phys. Rev.* **D22**, 200 (1980); T. Appelquist in *Gauge Teories and Experiments at High Energies*, Proc. 21st Scottish Universities Summer School, St. Andrews, Scotland 1980, edited by K. C. Bowler and D. G. Sutherland, SUSSP, Edinburgh, Scotland 1981; A. C. Longhitano, *Phys. Rev.* **D22**, 1166 (1980).
[70] L.-H. Chan, Louisiana State University preprint, DOE/ERO5490-67 (1985).