EFFECTIVE ACTION IN THE DE SITTER SPACE WITH TORSION

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Effective action for the $\lambda\varphi^4$ theory and scalar electrodynamics interacting in nonminimal way with the curvature and torsion in the de Sitter space is calculated. It is shown that torsion which was absent at the classical level is induced as a result of quantum corrections. The possibility of a first-order phase transition induced by curvature and torsion in scalar electrodynamics is investigated.

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1. Recently attention is directed at the different aspects of gravity with torsion (see, e.g. [1–3]). The present paper is devoted to some questions of quantum field theory in curved space-time with torsion. The properties of free quantum fields in curved space-time with torsion have been investigated in [4–12]. Interacting fields in space-time with torsion have been investigated in [13–14]. Let us discuss the results of papers [13–14] in details. It was found there that for multiplicative renormalizability of theory it is necessary to include in action the terms corresponding to nonminimal interaction of the matter with curvature and torsion. Indeed, consider the theory of scalar φ and spinor Ψ fields with Yukawa coupling. (In the present paper we assume for simplicity the torsion tensor $T_{\alpha\beta\gamma}$ to be entirely antisymmetric so that its independent components are determined by the pseudovector $S_\mu = \varepsilon_{\alpha\beta\gamma\mu}T^{\alpha\beta\gamma}$). Direct calculation of divergence index shows that it is necessary to introduce additional counterterms $R\varphi^2$, $S_\mu S^\mu\varphi^2$ and $\bar{\Psi}\gamma_5\gamma^\mu S_\mu\Psi$, where R is the curvature without torsion. Then, we should introduce in the initial Lagrangian the terms $(\xi_1^{(0)}R + \xi_2^{(0)}S_\mu S^\mu)\varphi^2 + i\xi_3^{(0)}\bar{\psi}\gamma_5\gamma^\mu S_\mu\psi$ with bare dimensionless parameters $\xi_1^{(0)}$, $\xi_2^{(0)}$, $\xi_3^{(0)}$ in order to provide multiplicative renormalizability. It leads to a theory which contains nonminimal interaction of matter with curvature and torsion. The most amazing fact is that it is necessary to introduce nonminimal interaction with torsion for the scalar field which does not interact minimally with torsion at all.

Consider now asymptotically free theory including scalars, spinors and gauge fields. The behaviour of such parameters as masses and couplings in strong gravitational fields

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is determined by the renormalization group (RG) equations (see, e.g. [13–16]). We will limit ourselves to the case of asymptotically free theories where effective charges and effective masses tend to zero in strong gravitational fields (at high energies). Effective charges corresponding to the dimensionless parameters ξ_1, ξ_2, ξ_3 do not appear in the flat space. Analysis which was performed in [13–15] shows that as energy grows, effective charges ξ_2, ξ_3 (and in some theories ξ_1) do not tend to zero but increase indefinitely. As a consequence, in the early Universe characterized by strong gravitational fields quantum fields (including scalars) should necessarily contain nonminimal interaction with torsion and curvature.

In the present paper effective action (EA) is calculated for the $\lambda\varphi^4$ theory and scalar electrodynamics (SE), which nonminimally interact with curvature and torsion in the de Sitter space. Effective equations, which lead to dynamical appearance of torsion as a result of quantum corrections, are constructed. The possibility of a first-order phase transition induced by curvature and torsion in SE is investigated.

2. Using the ζ-function regularization in the form proposed in [17] we have obtained EA of $\lambda\varphi^4$-theory in the de Sitter space with torsion. Let us choose background geometry in the following form:

$$\mathscr{L} = -\kappa^{-2}(R+hS^2-2\Lambda_0), \quad \kappa^2 = 16\pi G, \tag{1}$$

where $R = 4\Lambda$, $S^2 = S_\mu S^\mu$, $\int d^4x\sqrt{g} = \dfrac{24\pi^2}{\Lambda^2}$ (if $h = -\dfrac{1}{24}$ *Lagrangian* (1) corresponds to the Einstein-Cartan theory). Classical equations of motion for action (1) lead to the following solution: $\Lambda = \Lambda_0$, $S^2 = 0$ which corresponds to Euclidean de Sitter space. As it will be shown below, after taking into account quantum corrections, solutions for Λ and S^2 are changed.

Let us write the Lagrangian of $\lambda\varphi^4$ theory:

$$\mathscr{L} = \kappa^{-2}(R+hS^2-2\Lambda_0)+\tfrac{1}{2}(\partial_\mu\varphi)^2+\frac{\lambda\varphi^4}{4!}+\tfrac{1}{2}\xi_1R\varphi^2+\tfrac{1}{2}\xi_2S^2\varphi^2. \tag{2}$$

This theory does not interact with torsion in the minimal way ($\xi_2 = 0$). However, it is interesting to investigate properties of matter interacting with torsion in the frame of such simple model before going to more complicated theories of matter interacting with torsion in a minimal way. We assume that the scalar field is the only quantum field. Then for the de Sitter space and constant scalar background we obtain the following one-loop effective action:

$$\Gamma_1 = \tfrac{1}{2}\ln\det\Delta_0(X), \quad X = \lambda\varphi^2/2+4\xi_1\Lambda+\xi_2S^2 \tag{3}$$

(notation as in [17]). Since the answer for Γ_1 is written in [17] for arbitrary X we write the final expression

$$\Gamma = I+\Gamma_1 = 24\pi^2[-2\kappa^{-2}(2y+hS^2y^2/2-\Lambda_0y^2)+\lambda x^2/24+2\xi_1x$$

$$+\xi_2 S^2 xy/2]+\tfrac{1}{2}B_4 \ln\frac{1}{3\mu^2 y}-\tfrac{1}{6}[\tfrac{1}{4}b_0^2-\tfrac{11}{24}b_0$$

$$-\tfrac{1}{2}\int_0^{b_0} dz(z-\tfrac{1}{4})\Psi(1,5\pm\sqrt{z})]+\text{const.} \tag{4}$$

Here

$$x=\frac{\varphi^2}{\lambda},\quad y=\Lambda^{-1},\quad b_0=\tfrac{9}{4}-3\lambda x/2-12\xi_1-3\xi_2 yS^2,$$

$$B_4=\tfrac{29}{90}+12\xi_1^2-4\xi_1+3\lambda^2x^2/16+(6\xi_1-1)\lambda x/2+\tfrac{3}{4}\xi_2^2S^4y^2$$

$$+\tfrac{3}{4}\lambda\xi_2 S^2xy+\xi_2S^2y(6\xi_1-1),\qquad \psi(x\pm y)=\psi(x+y)+\psi(x-y),$$

$\psi(x)$ is the logarithmic derivative of Euler's Γ function and μ^2 is the normalization point. Note, that when $S^2=0$ expression (4) coincides with the corresponding expression in [17]. Effective equations $\frac{\partial\Gamma}{\partial\Lambda}=\frac{\partial\Gamma}{\partial x}=\frac{\partial\Gamma}{\partial S^2}=0$ enable us to take into account back reaction of quantized matter on background geometry and to determine Λ, x^2, S^2.

Let $x=0$. Then, EA (4) leads to the following equations:

$$\begin{cases}\dfrac{\partial\Gamma}{\partial S^2}=-24\kappa^{-2}\pi^2hy+\hbar A=0\\[2ex] \dfrac{\partial\Gamma}{\partial y}=-48\pi^2\kappa^{-2}(2+hS^2y-2\Lambda_0y)-\dfrac{\hbar}{2y}B_4(x=0)+\hbar S^2A=0\end{cases} \tag{5}$$

$$A\equiv\tfrac{1}{2}[\tfrac{3}{2}\xi_2^2S^2y+\xi_2(6\xi_1-1)]\ln\frac{1}{3\mu^2y}+\xi_2/4[\tfrac{4}{3}-12\xi_1-3\xi_2yS^2$$

$$+(12\xi_1+3\xi_2yS^2-2)\Psi(\tfrac{3}{2}\pm\sqrt{9/4-12\xi_1-3\xi_2yS^2})].$$

Excluding A from Eq. (5), we get equation for S^2, solution for which is:

$$S_{1,2}^2=(-A_1\pm[A_1^2-3\hbar\xi_2^2y^2(96\pi^2y\kappa^{-2}(2-2\Lambda_0y)+\hbar(\tfrac{29}{90}-4\xi_1$$

$$+12\xi_1^2))]^{1/2})(1.5\hbar\xi_2^2y)^{-1},\quad A_1=48\pi^2h\kappa^{-2}y^2+\hbar\xi_2y(6\xi_1-1). \tag{6}$$

Therefore, quantum corrections lead to the dynamically induced torsion which did not exist at the classical level. Substituting (6), for example, into the first equation of (5) we can find expression for y.

Let $\Lambda_0=0$, $\xi_1=\frac{1}{6}$ and suppose that terms $\sim\hbar$ in expression (6) could be neglected. Then we get

$$S^2\approx-192\pi^2h/3\hbar\kappa^2\xi_2^2\qquad(h<0). \tag{7}$$

Substituting (7) into the first equation of (5) and assuming that $S^2y\ll1$, $|\xi_2|\sim1$, $\ln\frac{1}{3\mu^2y}\sim1$, we obtain

$$\Lambda\approx-144\pi^2/\hbar\xi_2\kappa^2\qquad(\xi_2<0). \tag{8}$$

Condition $S^2 y \ll 1$ is here equivalent to the condition $|h| < |\xi_2|$. Thus, there appears a pure quantum solution for Λ, S^2. This quantum solution is determined by parameters of torsion h, ξ_2. From the viewpoint of RG, parameter ξ_2 increases in strong gravitational fields [13]. The estimations show that this growth within the evolution of the Universe from Planck's scales is negligible. That is why, in accordance with (8), cosmological constant becomes extremely large. In a similar way one can write effective equations for $x \neq 0$ and get small Λ by fine tuning of parameters Λ_0, ξ_2, h. In this case expression for Λ is the same as in the theory without torsion [17].

Note that EA can be found in a similar way for SE with the Lagrangian

$$L = -\kappa^{-2}(R - hS^2 - 2\Lambda_0) + F_{\mu\nu}^2/4 + (D_\mu \varphi^a)^2/2 + \frac{\lambda(\varphi^a\varphi_a)^2}{4!}$$

$$+\xi_1 R\varphi^a\varphi_a/2 + \xi_2 S^2 \varphi^a \varphi_a/2, \qquad a = 1, 2$$

$$L_g = (2\alpha)^{-1}(\nabla_\mu A^\mu + \alpha g \varepsilon_{ab}\phi_a\varphi_b)^2, \tag{9}$$

where α is a gauge parameter, ϕ_a is the background field and φ_b is the quantum field.

3. Let us investigate now phase transitions induced by curvature and torsion in SE. We consider the situation when $\varphi^2 \gg R$, $\varphi^2 \gg S^2$, $R = 4\Lambda$. In this case, considering only leading terms in expansion of EA for SE (or getting EA by the direct solution of RG equations [18]) we obtain approximate expression for EA of SE in the de Sitter space in the form:

$$\Gamma = \int d^4x \sqrt{g}\, V, \qquad V = \frac{\lambda}{4!}\varphi^4 + \xi_1 R\varphi^2/2 + \xi_2 S^2 \varphi^2/2$$

$$+A_3\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \tfrac{25}{6}\right) + (B_1 R + B_2 S^2)\varphi^2(\ln \varphi^2/\mu^2 - 3) \tag{10}$$

where

$$A_3 = \frac{1}{(16\pi)}(12g^4 + 10\lambda^2/9 - 4\alpha\lambda g^2/3), \qquad B_1 = (4\pi)^{-2}(\tfrac{1}{4}g^2 + \alpha\xi g^2/2 + \lambda(\xi - \tfrac{1}{6})/3),$$

$$B_2 = \xi_2(4\pi)^{-2}(\alpha g^2/2 + \lambda/3)$$

and normalization of [18] is used. We are only interested in the first-order phase transitions when the order parameter φ^2 at some critical R_c and S_c^2 is quickly changed. Let $z = \varphi^2/\mu^2$, $y_1 = R/\mu^2$, $y_2 = S^2/\mu^2$. Critical parameters, corresponding to first-order phase transition are found from the conditions:

$$V(z_c, y_{ci}) = 0, \qquad \left.\frac{\partial V}{\partial z}\right|_{z_c, y_{ci}} = 0, \qquad \left.\frac{\partial^2 V}{\partial x^2}\right|_{x_c, y_{ci}} > 0. \tag{11}$$

These conditions lead to the equations

$$2A_3^2 = \sum_{i=1}^{2} D_i q_i \pm [(\sum_{i,j} (D_i D_j - 4A_3^2 B_i B_j) q_i q_j]^{1/2}$$

$$D_i = A_3 \xi_i/2 - 5A_3 B_i/6 - \lambda B_i/24, \qquad q_i = y_{ci}/z$$

$$z = \exp [(\lambda/24 - 25A_3/6 + \sum_i (\xi_i/2 - 3B_i) q_i)(\sum_i B_i q_i - A_3)^{-1}]$$

$$\lambda/24 - 8A_3/3 + A_3 \ln z + \sum_i B_i q_i/2 > 0. \tag{12}$$

The evident conditions $q_i > 0$ and $q_i \ll 1$ should be added to expressions (12) (otherwise the obtained results would contradict the initial assumptions $\varphi^2 \gg R$ and $\varphi^2 \gg S^2$). Moreover, in expressions (12) we must leave only the terms which are not beyond the scope of the one-loop approximation.

Assume that as in the flat space $\lambda/24 = 11A_3/3$ [21] and hence $\lambda \sim g^4$. Let $|\xi_i| \gg g^2$. Then $D_i \simeq \dfrac{A_3\xi_i}{2}$ and from the first expression (12) we obtain $2A_3 = \sum_i \xi_i q_i$ and $q_i \approx 2A_3/\xi_i$. In the same approximation $z_c \approx e^{-1/2}$, $y_{ci} \approx 2e^{-1/2} A_3/\xi_i$. Condition $q_i \ll 1$ leads to $|\xi_i| \ll g^{-4}$. The third condition (12) is also fulfilled. Since $A_3 > 0$, then for $g^2 \ll \xi_i \ll g^{-4}$ the theory admits first-order phase transition induced by curvature and torsion. It is evident that when $\lambda \sim g^4$, z_c, y_{ci} do not depend on the gauge parameter α. It is easy to see that speculations given above are valid in the case $R = 0$, too. In this case phase transition is induced only by torsion in flat space. Note that without torsion phase transition induced by curvature in SE have been investigated in [19, 20].

Now let us investigate the analogue of dimensional transmutation [21] in the de Sitter space. Let V have minimum at $\varphi = \varphi_c \neq 0$ and let us choose $\mu = \varphi_c$. Then, the condition $\left.\dfrac{\partial V}{\partial \varphi}\right|_{\varphi=\varphi_c} = 0$ results in

$$\varphi_c^2 = \sum_i (\xi_i/2 - 2B_i) P_i/(22A_3/3 - \lambda/12), \qquad P_1 = R, \quad P_2 = S^2. \tag{13}$$

The minimum of V is determined easily (if the right-hand part of (13) is negative then minimum does not exist). The dimensional transmutation does not take place. Let $\lambda/24 = 11A_3/3$. Then, from the condition $\left.\dfrac{\partial V}{\partial \varphi}\right|_{\varphi=\varphi_c} = 0$ we obtain $\sum_i (\xi_i/2 - 2B_i) P_i = 0$ and finally $\xi_i = 4B_i$. Thus, when $\lambda/24 = 11A_3/3$, dimensionless parameters ξ_1, ξ_2 are expressed through other parameters, and at the same time the independent dimensional parameter φ_c^2 appears. Dimensional transmutation does take place.

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