NUMEROLOGY ON PION AND PROTON RAPIDITY

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The pseudo-rapidity of pion jets which were measured for 50 GeV and 150 GeV incident pions and protons on carbon, copper, and lead targets are analyzed. The shape of the rapidity distribution for a "fireball" which emits particles isotropically in its center of mass is a $\cosh^{-2} y$ distribution. It is possible to unfold all measured distributions into three groups which correspond to a low rapidity originating from the target fragmentation, a middle group which is a function of the center of mass of the projectile and target rapidity and a fast group which is due to the projectile.

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1. Introduction

The topic which I will present is called "Numerology" because the amount of analysis perhaps is not completely warranted by the quality of the data. However, it may entice others to improve on this work.

About 10 years ago Professor Povh, from the Max-Planck-Institut für Kernphysik at Heidelberg proposed at CERN an experiment to measure the fragmentation of several targets by high energy projectiles. The instrument which was built by his collaborators and students, the so-called "Igel" or Hedgehog consisted of several rings of Čerenkov--scintillation counters. Fig. 1 depicts a schematic diagram of the Igel [1]. The detectors covered about 50% of the total solid angle. Except for the most forward counters each one was able to discriminate the slow particles with $v/c < 0.7$ from the fast, mostly pions by a 3 mm thick layer of CsI(Tl) crystal cemented to the lucite Čerenkov radiator-lightpipe. The CsI scintillator produces a slow pulse whereas the fast rise-time pulse from the lucite radiator can be electronically separated. Forward angles of $<13°$ are viewed by four rings of hodoscopes of plastic scintillators. The incoming beam was analyzed by two gas Čeren-

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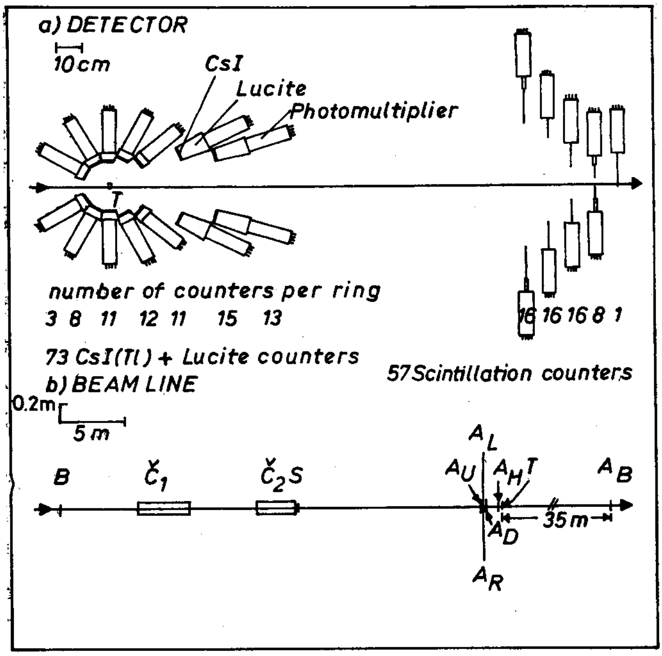


Fig. 1. The "Igel" detector; 7 rings of CsI(Tl)-Lucite scintillation-Čerenkov detectors and 4 rings of scintillation counters for the forward jets

kov counters enabling identification of the incoming particle. Additional counters defined the beam position and provided timing pulses.

The discrimination between fast and slow particles is in essence similar to the recognition of light and grey tracks in the earlier photographic plate work [2]. Results were obtained for pions, kaons, protons and anti-protons at several energies. We will discuss here only the π^+ and p data at 50 and 150 GeV. The results for the other projectiles and other energies had less statistics and were not as complete. The results of the multiplicity distributions have been published [3, 4, 5].

1.1. Slow tracks

The multiplicity distribution of the slow particles is treated separately from the fast ones because these particles seem to be produced in different processes. Figs 2a, b, c show the observed and the calculated distributions for three elements and for incident pions and protons. The distributions are not dependent on the energy of the projectile between 50 GeV and 150 GeV incident energy. The calculated distributions are based on the average number of collisions between the projectile and the nuclear constituents in a Woods-Saxon nucleus for a given reaction cross section ($\sigma_{\pi N} = 2.45$ Fermi^2 and $\sigma_{pN} = 4$ Fermi^2). The calculated value for $\langle v \rangle$ is close to that obtained from [6]

$$\langle v \rangle = \frac{A\sigma_{hp}}{\sigma_{hA}} \tag{1}$$

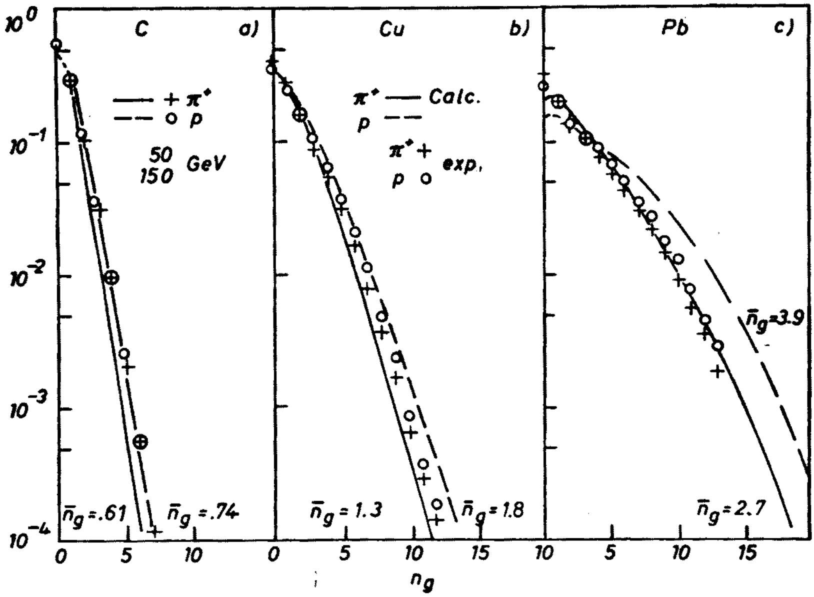


Fig. 2a, b, c. The multiplicities of slow secondaries [$v/c < 0.7$] from 50 and 150 GeV π^+ and p on C, Cu and Pb targets. The curves are calculated by the average number of collisions of the incident particle with nucleons in a Woods Saxon nucleus, $\langle\nu\rangle$. This number defines a Poisson distribution. A fraction of the hits initiates a nuclear cascade. The latter is calculated with a Monte Carlo calculation

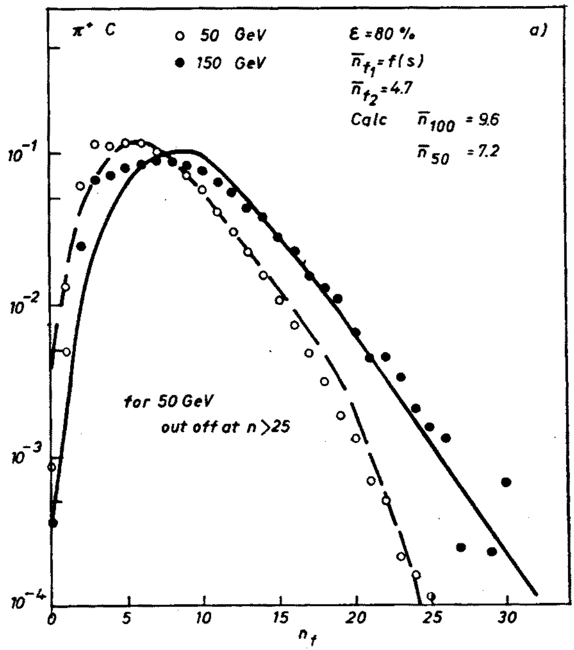
Of the struck nuclei a constant fraction (to be determined by the experimental fit) was assumed to produce a nucleonic cascade inside the nucleus to give rise to the observed slow particle distribution. The cascade was obtained by the use of a Monte Carlo calculation and folded with Poisson distributions. the same fraction as before was used to produce the nucleons which continued in the cascade. Also, a correction accounted for the experimental efficiency of the detectors (the experimental results were not corrected for this). The agreement is good which shows that the production of slow tracks seems to be understood and that one can use the average number of projectile nucleon collisions $\langle\nu\rangle$ with some confidence.

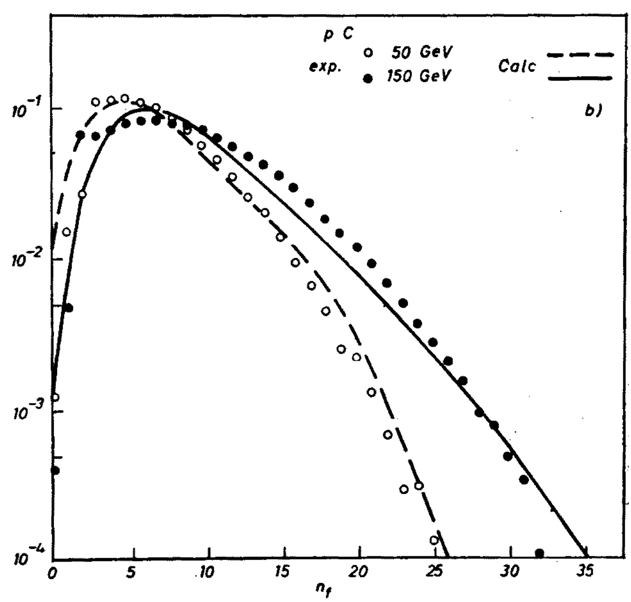
1.2. Fast tracks

The fast particles, preponderantly pions, are not produced in a cascade. In the primary collision of the projectile and the nucleons relativistic particles are produced which must remain close to or become part of the incident particle; there is no time for them to spread out. Perhaps, the incident hadron may be regarded as a highly excited structure which pionizes outside the nucleus. A similar structure may result from the struck nucleon. Poisson distributions are folded for the number of collisions and the number of fast nucleons produced in the primary collisions. The average number of fast tracks in a primary collision is [7]

$$\begin{matrix}\mathrm{p}\\ \pi\end{matrix}\langle n_f\rangle = \left.\begin{matrix}1.25\\ 1.54\end{matrix}\right\} \times [1.4+0.15\ln s+0.155\ln^2 s]\,. \tag{2}$$

A cut off for very large multiplicities has to be introduced because of the available energy. However, this simple assumption gives rise to multiplicities which are too large for protons at 150 GeV. It was then assumed that only the first one or two collisions occur with the normal proton nucleon cross-section and afterwards, the excited hadron ploughs through the nucleus interacting with a constant cross-section of about 2.45 Fermi[2] instead of about





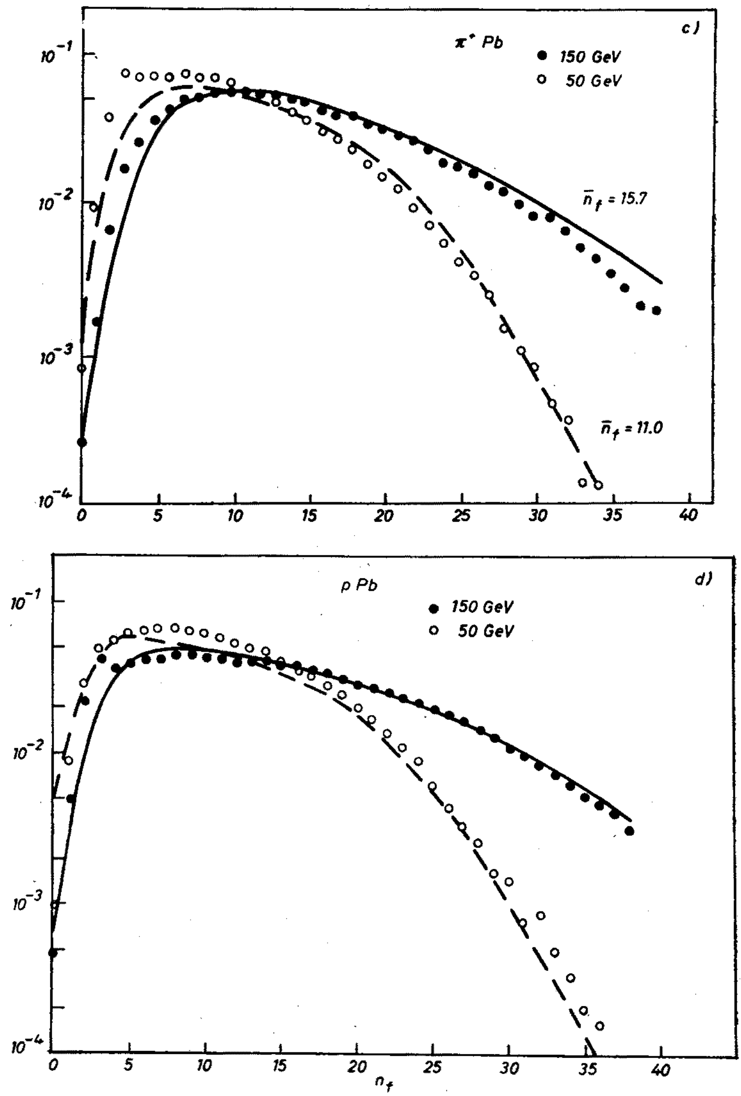


Fig. 3a, b, c, d. The multiplicity distribution of relativistic secondaries from 50 and 150 GeV π^+ and p on C and Pb. Each collision between the projectile and a nucleon produces an average number of pions n_f which defines a Poisson distribution. These distributions of each hit are folded

4 Fermi^2 for the elementary proton-nucleon reaction cross-section. This conclusion was based on the observation that the pion induced multiplicity distribution produced a better fit to all the data than those for protons. The pion nucleon cross-section is about 2.45 Fermi^2. It is possible that the interpretation of the rapidity distributions in the following chapter bears on this. Figs 3a, b, c, d show the experimental multiplicity distribution and the calculated results. The agreement is satisfactory. Again, no fundamentally new information about the reaction mechanism can be extracted from these multiplicity distributions.

2. *Rapidity distributions*

2.1. Unfolding procedure

The rapidity is defined by,

$$y = 0.5 \ln \frac{E+p_l}{E-p_l} \tag{3}$$

in which E is the total energy of the observed particles and p_l is their longitudinal momentum. In this experiment the magnitude of the momentum is not measured. If the mass of the particle is small compared to its transverse momentum the rapidity can be approximated by the so-called pseudo-rapidity which corresponds to an angular distribution,

$$y \approx \eta = -\ln [\tan \theta/2]. \tag{4}$$

For a Lorentz transformation of a frame moving with the velocity β, we obtain,

$$y' = y + \ln \frac{1+\beta}{1-\beta}. \tag{5}$$

When a moving object emits particles spherically isotropic the distribution of the particles as a function of the pseudo-rapidity is,

$$n = \frac{N}{\cosh^2 y}. \tag{6}$$

Therefore it is possible to unfold an experimental pseudo-rapidity distribution into a sum,

$$n = \sum_i \frac{N_i}{\cosh^2 (y-y_i)}. \tag{7}$$

The y_i are the rapidities of the moving objects (fireballs!).

Some of the representative experimental rapidity distributions are shown in Figs 4a, b, c, d. These curves are much wider than the function for a single moving decaying object. In fact, with the help of the CERN unfolding program we did not succeed to unfold these spectra into two components. A minimum of three distributions were required for $\chi^2 = 1.8$ per degree of freedom. An improvement was obtained by letting the width of the middle distribution free,

$$n = \frac{N_2}{\cosh^2 w(y-y_2)}. \tag{8}$$

The value for w should not deviate much from 1. Actually, it varied between 0.7 and 0.85 for all 12 analyzed spectra. A value different from 1 signifies that the distribution is not due to a single moving object or, that the emission of particles is not spherically isotropic.

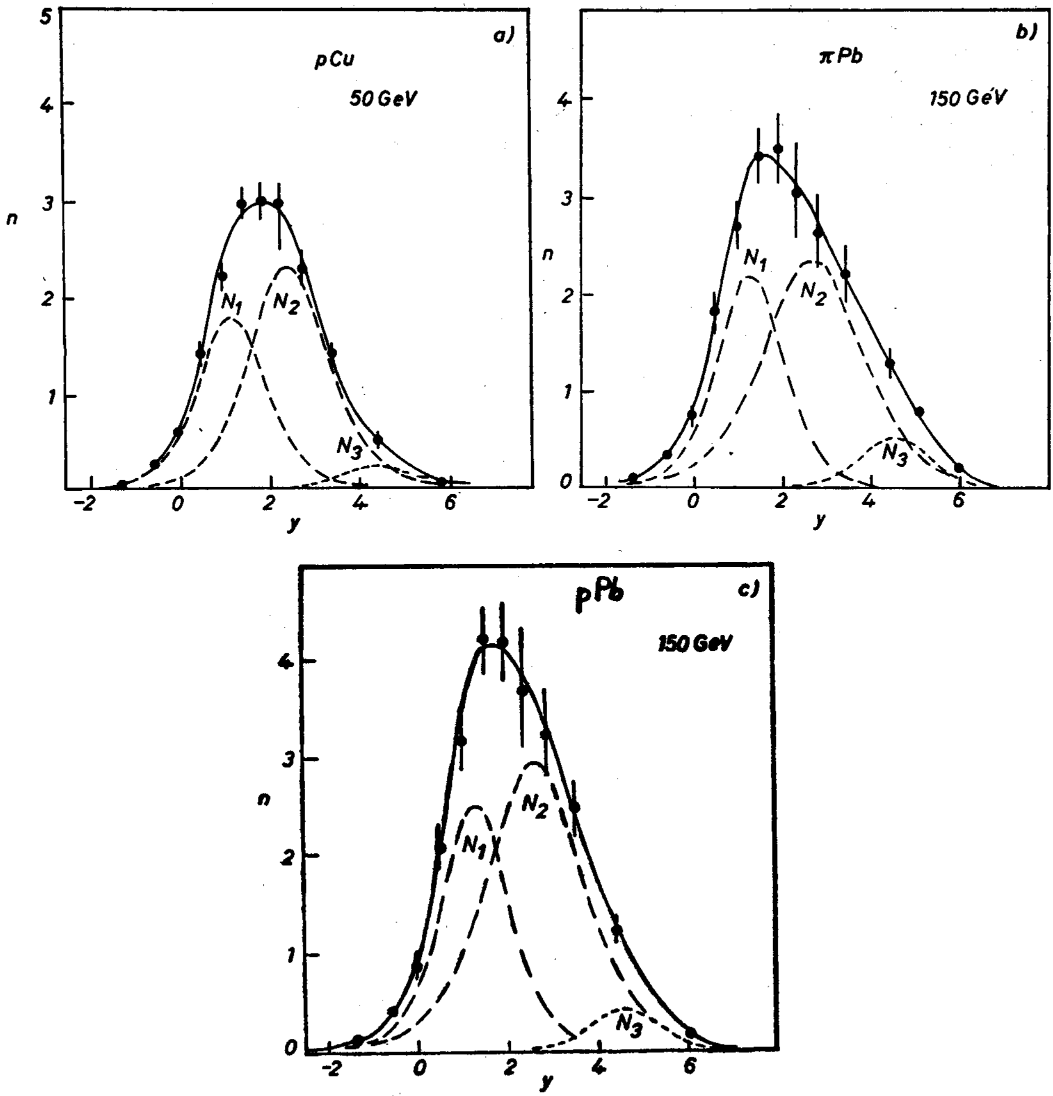


Fig. 4a, b, c. Some representative cases of the experimental pseudo-rapidity distributions. With the help of a CERN unfolding routine the measured data are unfolded to determine, N_i, y_i and w.

Besides, the experimental uncertainties will result in unfolding errors. Fig. 4 shows also the unfolded distributions. The curve drawn through the experimental points is the reconstructed sum of the three unfolded spectra.

2.2. Interpretation

Provided that the data are sufficiently good to allow the analysis described above one may try to interpret the three bumps. The variables for N_i and for y_i are the incident beam energy, the type of projectile, the number of collisions inside the nucleus and perhaps, the total number of decay products, though the latter is itself a function of the former variables. Therefore, we only consider the variables, number of collisions, incident energy and projectile type.

Each bump will be discussed in succession for the 50 GeV and 150 GeV pion and proton data from low to high rapidity. The values for y_i and for N_i are presented in Figs 5a, b, 6a, b and 7a, b vs the most important variable.

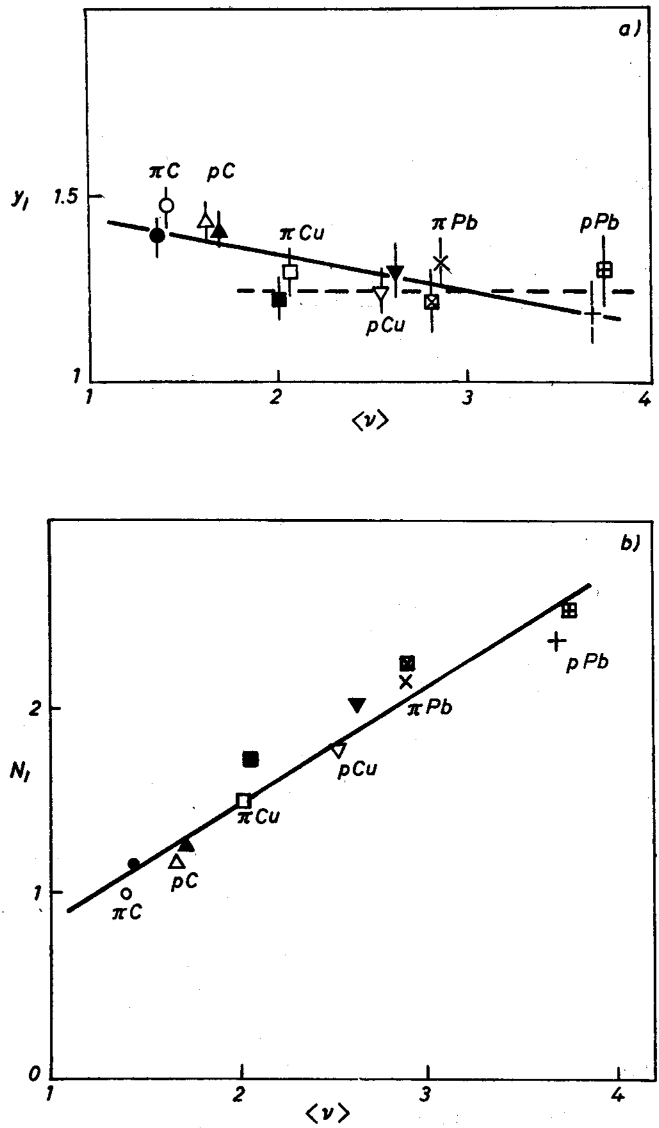


Fig. 5a, b. The quantities y_1 and N_1 as a function of $\langle \nu \rangle$, the average number of internal collisions. The open and unframed symbols are for 50 GeV, the solid and framed ones for 150 GeV incident energy

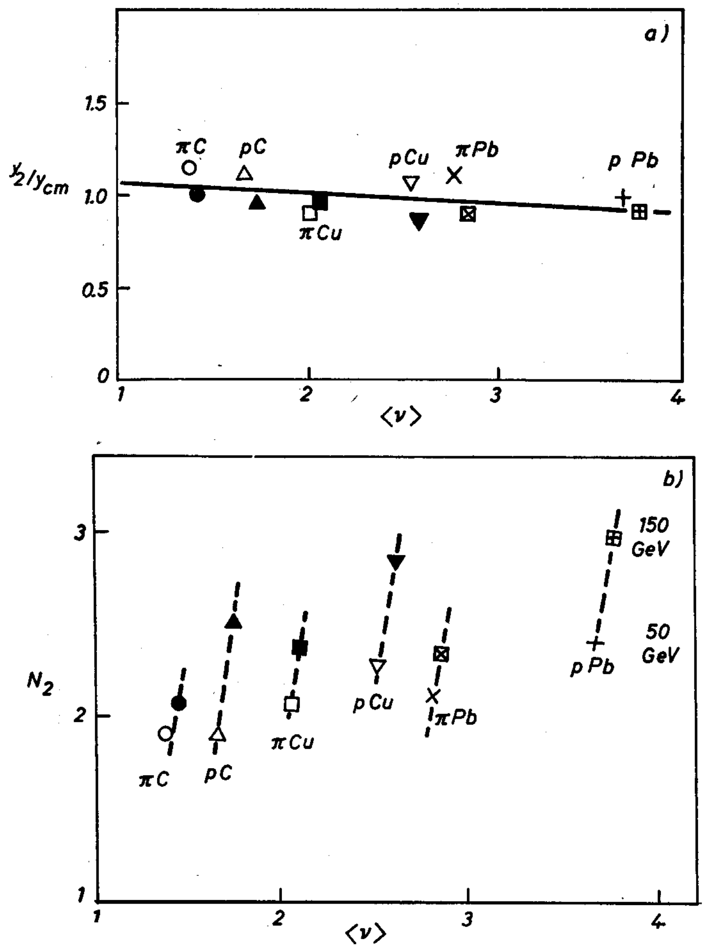


Fig. 6a, b. The quantities y_2/y_{cm} and N_2 as a function of $\langle\nu\rangle$. y_{cm} is the center of mass rapidity of the projectile and the target nucleus

2.2.1. The low rapidity distribution

Fig. 5a depicts y_1 as a function of the average number of collisions $\langle\nu\rangle$. The open and unframed symbols are those for 50 GeV incident energy, the filled and framed ones are those for 150 GeV. The solid line is a least square fit through all the data points; the dashed line is the fit without the carbon points. A weak linear dependence of the rapidity on the number of collisions is well established. Perhaps, one could make a case that for medium- and heavy nuclei y_1 is even constant, while for a light nucleus the "fireball" is not yet completely formed and therefore, more kinetic energy is available. However, the number of emitted pions N_1 is a good linear function of $\langle\nu\rangle$ and carbon does not fall off the line. Both quantities y_1 and N_1 are functions of the number of nucleonic collisions which makes plausible that they originate from the target nucleons or target quarks; the dependence on the particle type and on the incident energy is small.

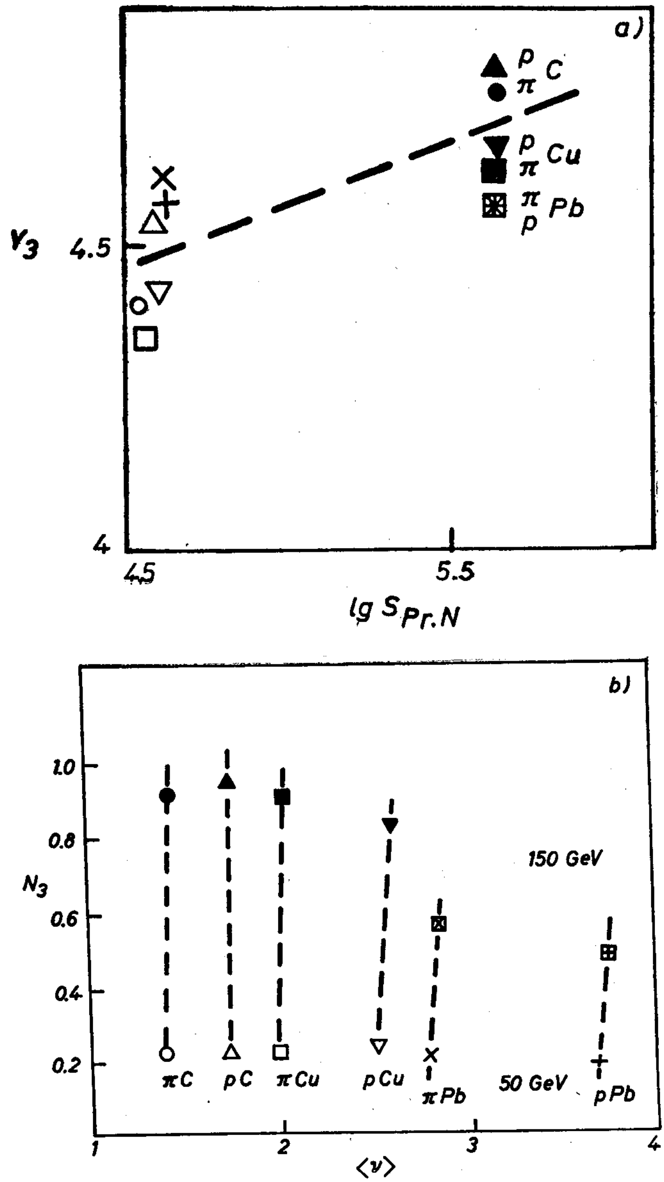


Fig. 7a. The rapidity y_3 as a function of $[p_{in}+p_N]^2$ in which p_{in} is the 4-vector of the projectile momentum and p_N that of the nucleon inside the nucleus; b. N_3 as a function of $\langle \nu \rangle$

2.2.2. The middle rapidity distribution

Fig. 6a presents the ratio of the rapidity y_2 to the rapidity of the center of mass of projectile and target vs. the average number of collisions. Again the line is a least square fit. It is apparent that y_2/y_{cm} is practically constant and even more surprising, it is close to 1. On the other hand, N_2 is a pronounced function of the energy and of the size of the target. Perhaps we see here the effect of the strings [8].

2.2.3. The high rapidity distribution

The value of y_3, Fig. 7a, is more of a function of $s = (p_{proj}+p_{nucleon})^2$ than of $\langle\nu\rangle$. Also N_3 increases rapidly with s. Most likely, the fireball originates from the incident particle. Still several questions remain. The rapidity of a proton at 50 GeV is 4.668 which is not very different from the value of $y_3 = 4.5$. At 150 GeV the proton rapidity is 5.767 a difference of 1.1, whereas y_3 for that energy is only about 4.8, an increase of 0.3. It may be possible that y_3 cannot increase as rapidly anymore because the multiplicity in the elementary interaction increases; it went up by a factor 1.5, leaving less energy for the increase of the fireballs rapidity. Neither N_3 nor y_3 seems to depend strongly on the type of projectile; the number of quarks in the incident particle does not have much influence.

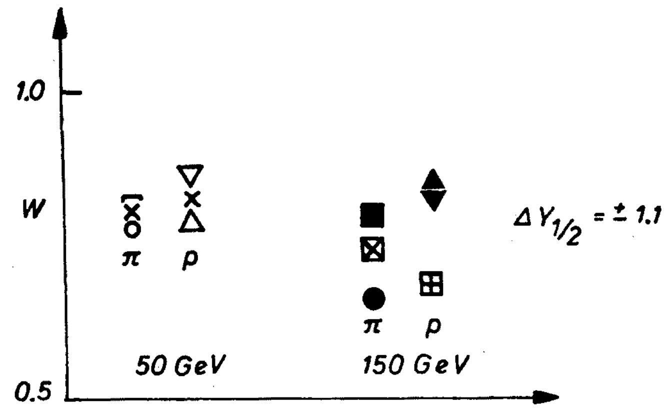


Fig. 8. The quantity w which determines the width of the central distribution. The abscissa has no significance. Apparently, w is not a function of the three chosen variables

Finely, Fig. 8 presents the width of the middle distribution, it varies around 0.8 which makes the width of the middle spectrum larger than that of a $\cosh^{-2}(y-y_i)$ distribution. We cannot decide whether this is the consequence of a non-spherical emission, a failure of the model of three "fireballs", or just the result of the unfolding procedure. Of course, a rapidity distribution of the prescribed form can be generated by several objects all with the same, or almost the same, rapidity y_i instead of by a single fireball. We will not be able to distinguish between such multiple incoherent sources and a single highly excited state.

3. *Discussion*

The results of this analysis have been anticipated before and several models have been proposed [9]. In the first place, one has to point out that the variation of the variables is very limited; $\langle\nu\rangle$ varies only by a factor of 2.5, for the energy variation one has to consider the logarithm of s, which changes from 4.7 to 5.8. Consequently, our conclusions may have only limited validity. An attempt to fit the UA5 results by a similar analysis were unsuccessful [10]. The central part is too wide, though not as much as some models predict. However, the collective effects in hadron nucleus interactions are missing in nucleon nucleon or anti-nucleon nucleon collisions and one might expect different results.

The central rapidity region has recently been discussed by Kerman and Svetitsky [11]. They consider an exchange of a single soft gluon in the primary interaction. The two colliding nucleons are then connected by a color flux tube with high color fields. The particle production is incoherent and takes place analogous to pair production in strong fields. They arrive at the result that,

$$N_2 = \sqrt{\langle \nu \rangle}\, N_{pp}. \tag{9}$$

The ratio of pions produced at 150 and 50 GeV in the primary collisions is about 1.5. This factor would bring the 150 GeV and the 50 GeV points for N_2 fairly close together (Fig. 6b). Also, the $\langle \nu \rangle^{1/2}$ dependence for N_2 is within the limits of uncertainty.

4. *Conclusions*

In summary, the unfolding procedure is supporting the model of three rapidity regions, originating from wounded target nucleons, from the projectile, and a central distribution from a strong color field tube. The low rapidity region seems to be well established. The high rapidity region shows some unexpected behavior. The following argument may make this plausible. The target nucleon is hit and wounded only once. However, the incident particle is hurt in every collision by its passage through the nucleus. As has already been hypothesized in the multiplicity calculation, the projectile might behave like a common hadron during the first couple of collisions whereafter it becomes a highly excited structure with a different reaction cross-section. In this limited range of number of collisions and energies one cannot decide whether the three regions of rapidities are decoupled or, that the field in the flux tube gives rise to creation probabilities which are correlated. None of the different incident particles, kaons, anti-protons, which were not discussed here gave vastly different results, neither in the multiplicity nor in the rapidity data. The hadrons and nucleons in the nucleus could have lost their identity.

It is worthwhile to continue with this type of experiments with greater precision over as large a range of energies as possible. The hadron nucleus interactions give us an additional parameter unavailable in the fundamental particle experiments.

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