# COULOMB EFFECTS IN DEUTERON STRIPPING REACTIONS AS A THREE-BODY PROBLEM

By A. Osman*

International Centre for Theoretical Physics, Trieste, Italy

*(Received January 31, 1986; revised version received April 21, 1986; final version received October 14, 1986)*

Deuteron stripping nuclear reactions are reconsidered as a three-body problem. The Coulomb effects between the proton and the target nucleus are investigated. The mathematical formalism introduces three-body integral equations which can be exactly calculated for such simple models. These coupled integral equations suitably include the Coulomb effects due to repulsive or attractive Coulomb potentials. Numerical calculations of the differential cross-sections of the reactions $^{28}$Si(d, p)$^{29}$Si and $^{40}$Ca(d, p)$^{41}$Ca are carried out, showing the importance of the Coulomb effects. The angular distributions of these reactions are theoretically calculated and fitted to the experimental data. From this fitting, reasonable spectroscopic factors are obtained. Inclusion of Coulomb force in three-body model are found to improve the results by about 6.826‰, as an average value corresponding to the different reactions considered.

PACS numbers: 25.45.-z

## 1. Introduction

Recently, nuclear reactions have been investigated [1–3] theoretically following different formalisms based on three-body problem approaches [1, 4, 5]. These studies result in the form of three-body integral equations. In developing these three-body integral equations, the short-range nuclear interactions are used, while the long-range Coulomb potentials are ignored. In cases of bound three particles, the approximation of considering only the short-range nuclear interactions is valid and reasonable results are obtained [6]. However, in the cases of nuclear reactions two charged particles separate asymptotically. In these latter cases (the nuclear reaction processes), the Coulomb potentials act. Thus, in nuclear reactions, the long-range Coulomb potentials must be included as well as the short-range nuclear interactions.

Different approaches [7–10] have been presented in formulating the three-body integral equations in the presence of the Coulomb forces. One of these approaches is that suggested by Schulman [7], executed by approximating Coulomb Green's functions in momentum

* Permanent address: Physics Department, Faculty of Science, Cairo University, Cairo, Egypt.

space. Introducing this approximation for the Coulomb forces in the three-body system, Osman [7] introduced modified Faddeev equations which are applied for different three--body problems [11]. The second approach, introduced by Noble [8] and Bencze [8], uses the Coulomb potentials only in free Green's functions in the three-body integral equations by which they obtained well-defined equations. In order to make these equations suitable for calculations, some approximations suggested by Adya [8] and Sawicki [8] must be used. The third approach was that introduced by Veselova [9] who investigated the Faddeev equations for screened Coulomb potentials making use of the Gorshkov's renormalization technique [9]. Veselova developed three-body integral equations for reactions of three particles with a kernel well-behaving in the limit of two-body transition operators. For this, Veselova restricted her approach to three-particle energy of negative values which are below the breakup threshold, thus excluding breakup reactions. The fourth approach was introduced by Alt, Sandhas and Ziegelman [10]. They described the three particles scattering problem by including the long-range Coulomb force together with any arbitrary short-range potentials in the AGS quasiparticle formulation [12] of the three-body problem. Alt et al. [10] in their approach have given Coulomb-modified integral equations which are exact for any general short-range interactions and suitable for negative as well as positive three-particle energies. The formalism of Alt et al. [10] has been considered by van Haeringen [10] for the simple model of proton-deuteron scattering.

To include the Coulomb forces in the three-body system, the two-body Coulomb $T$ matrix off-the-energy-shell have to be well defined in considering the Lippmann–Schwinger equation. For three-body systems, it is easier to introduce the studies of the $T$ matrix in momentum representation. Besides that, Merkuriev et al. [13] introduced formulae for the three-body integral equations in coordinate representation. For the three-particle systems, Alt et al. [10] derived effective two-particle integral equations for the $T$ matrix amplitudes valid for general separable and local potentials. In case of three charged particles, the Alt et al. [12] formalism is exact but very difficult to calculate [14]. Therefore, Coulomb-modified form factors with overlap integrals are needed in which the partial--wave projection of the Coulomb $T$ matrix is of great importance. Then, both the Coulomb $T$ matrix and its partial-wave projection are approximated by their first Born term.

In the present work, we are interested in studying the direct nuclear reactions with nucleon transfer as a three-body problem. We consider that only two particles of the three--particle system are charged. In this way, the Coulomb force acts in one subsystem only. One of the most interesting tools in studying the static properties of nuclei, are the direct--stripping nuclear reactions. We consider in the present work the deuteron stripping nuclear reactions. In this type of nuclear reactions, we are considering a simple model of three interacting particles (the proton, the neutron and the target nucleus). This three-body system is simply including the Coulomb force in only one subsystem, since the Coulomb force acts only between the proton and the target nucleus. Following the Alt et al. [10] formalism, the three-body integral equations are developed including the Coulomb force. The Coulomb $T$ matrix is deduced. The nuclear reactions $^{28}$Si(d, p)$^{29}$Si and $^{40}$Ca(d, p)$^{41}$Ca are considered at incident deuteron energies 18.0 MeV and 7.0 MeV, respectively. The angular distributions of these stripping reactions, as well as for deuteron elastic scattering,

are calculated. The theoretical and numerical calculations are compared with the experimental measurements. From the fitting, the spectroscopic factors for the deuteron stripping reactions are extracted.

In Section 2, the three-body integral equations as well as the Coulomb $T$ matrix are introduced. In Section 3, we present the numerical calculations and results. Discussion and conclusions are given in Section 4.

## 2. *Three-body integral equations and Coulomb T matrix*

In the present work, we are interested in studying the rearrangement collision process between three particles as a three-body problem. Two particles are taken as charged particles, while the third particle will be taken as neutral particle. This present study enables us to study the deuteron stripping nuclear reactions. Let us now consider the particles 1 and 2 as the two charged particles and denote them generally with $i$ and $j$, while the particle 3 referred to as $k$ will be the neutral particle. Thus, the Coulomb force exists only in the subsystem 3 (denoting the Coulomb interaction between particles 1 and 2). We follow here the formalism of Alt, Grassberger and Sandhas [12] of the quasiparticle approach. Thus, the three-body equations for elastic and rearrangement scattering are given by the amplitudes

$$T_{ij} = v_{ij} + \sum_k v_{ik} G_{0;k} T_{kj}, \tag{1}$$

where $G_{0;k}$ is effective free Green's function given by

$$\langle \boldsymbol{q}_i | G_{0;i}(Z) | \boldsymbol{q}_i' \rangle = \delta(\boldsymbol{q}_i - \boldsymbol{q}_i') \frac{Z - q_i^2}{2M_i - E_i}. \tag{2}$$

If the masses of the three particles are $m_1$, $m_2$ and $m_3$ and their momenta are $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and $\boldsymbol{k}_3$, then the notations $\mu_i$ and $\boldsymbol{p}_i$ are used as the reduced mass and relative momentum of particles $j$ and $k$, respectively. On the other hand, $M_i$ and $\boldsymbol{q}_i$ are the reduced mass and corresponding relative momentum between particle $i$ and subsystem of particles $(j+k)$. Thus,

$$\mu_i = \frac{m_j m_k}{m_j + m_k}, \tag{3}$$

$$\boldsymbol{p}_i = \frac{m_k \boldsymbol{k}_j - m_k \boldsymbol{k}_k}{m_j + m_k}, \tag{4}$$

$$M_i = \frac{m_i (m_j + m_k)}{m_i + m_j + m_k}, \tag{5}$$

$$\boldsymbol{q}_i = \frac{(m_j + m_k)\boldsymbol{k}_i - m_i(\boldsymbol{k}_j + \boldsymbol{k}_k)}{m_i + m_j + m_k}. \tag{6}$$

$E_i$ is the two-particle binding energy of the pair $(j+k)$.

In the present work we use separable nuclear potentials and non-separable Coulomb potentials which act only in one subsystem. Thus, the effective potential $v_{ij}$ has a form which can be represented as

$$\langle q_i|v_{ij}(Z)\,|q_i'\rangle = \left[D_i\left(Z-\frac{q_i^2}{2M_i}\right)\right]^{1/2}\langle q_i|\langle\psi_i|\,(1-\delta_{ij})$$

$$\{G_0(Z)+[\delta_{i3}+\delta_{j3}+(1-\delta_{i3})\,(1-\delta_{j3})]G_0(Z)T_C(Z)G_0(Z)\}$$

$$+\delta_{ij}(1-\delta_{i3})G_0(Z)T_C(Z)G_0(Z)\,|\psi_j\rangle\,|q_j\rangle\left[D_j\left(Z-\frac{q_j^2}{2M_j}\right)\right]^{1/2}. \tag{7}$$

In equation (7), $G_0(Z)$ is three-particle free Green's function

$$G_0(Z) = (Z-H_0)^{-1}. \tag{8}$$

$D_i(Z)$ is defined for the subsystem $(j+k)$ and $(k+i)$ as

$$D_i(Z_i) = 1/\langle\psi_i|G_0(E_i)G_0(Z_i)\,|\psi_i\rangle \tag{9}$$

for $i = 1$ and 2. For the subsystem $(i+j)$

$$D_3(Z_3) = 1/\langle\psi_3|G_C(E_3)G_C(Z_3)\,|\psi_3\rangle, \tag{10}$$

where $G_C(Z)$ is full Green's function for a screened Coulomb potential, and is given as

$$G_C(Z) = (Z-H_0-V_C)^{-1}. \tag{11}$$

Here, $V_C$ is the screened Coulomb potential represented as

$$V_C(r) = Z_1Z_2e^2\,\frac{e^{(-r/r_0)}}{r} \tag{12}$$

and it depends on the screening radius $r_0$. Also $|\psi_i\rangle$ are the form factors. $T_C$ is the screened Coulomb amplitude and satisfies the Lippmann-Schwinger equation via equations (11) and (12).

The effective potential $v$ can be decomposed in two parts as

$$v_{ij} = v_{ij}^l+v_{ij}^s, \tag{13}$$

where $v_{ij}^l$ is the long-range part describing the pure screened Coulomb scattering of the charged particle $i$ from the centre of mass of particles $j$ and $k$. $v_{ij}^s$ is the shorter-range contribution. Then,

$$v_{ij}^l(q_i, q_j'; Z) = \delta_{ij}(1-\delta_{i3})V_C(q_i-q_i') \tag{14}$$

and

$$v_{ij}^s(q_i, q_j'; Z) = \left[D_i\left(Z-\frac{q_i^2}{2M_i}\right)D_j\left(Z-\frac{q_j^2}{2M_j}\right)\right]^{1/2}\langle q_i|\,\langle\psi_i|\,\{(1-\delta_{ij})$$

$$\times[G_0(Z)+G_0(Z)T_C(Z)G_0(Z)]+\delta_{ij}(1-\delta_{i3})G_0(Z)$$

$$\times[T_C(Z)-V_C]G_0(Z)\}\,|\psi_j\rangle\,|q_j'\rangle+\delta_{ij}(1-\delta_{i3})V_C$$

$$\times(q_i-q_i')\,\{[B_i(q_i, q_i'; Z)-1]+[F_i(q_i, q_i'; Z)-B_i(q_i, q_i'; Z)]\}. \tag{15}$$

The function $B_i$ and the factor $F_i$ are given and represented as

$$B_i(q_i, q_i'; Z) = \int d^3k \frac{|\psi_i(k)|^2\left[D_i\left(Z-\frac{q_i^2}{2M_i}\right)D_i\left(Z-\frac{q_i'^2}{2M_i}\right)\right]^{1/2}}{\left(\frac{k^2}{2\mu_i}-\frac{Z-q_i^2}{2M_i}\right)\left(\frac{k^2}{2\mu_i}-\frac{Z-q_i'^2}{2M_i}\right)} \tag{16}$$

and

$$F_i(q_i, q_i'; Z) = \left[D_i\left(Z-\frac{q_i^2}{2M_i}\right)\right]^{1/2}\int d^3k$$

$$\times\frac{\overset{*}{\psi}_i(k)\psi_i\left[k+\frac{m_k}{m_j+m_k}(q_i-q_i')\right]}{\left\{\left[k+\frac{m_k}{m_j+m_k}(q_i-q_i')\right]^2\Big/2\mu_i-\frac{Z-q_i^2}{2M_i}\right\}\left(\frac{k^2}{2\mu_i}-\frac{Z-q_i'^2}{2M_i}\right)}\left[D_i\left(Z-\frac{q_i'^2}{2M_i}\right)\right]^{1/2}. \tag{17}$$

Then, the effective transition operator $T_{ij}$ can be decomposed as

$$T_{ij} = T_{ij}^{1}+T_{ij}^{s}. \tag{18}$$

In this decomposition, the operator $T^1$ is defined by the Lippmann-Schwinger equation

$$T_{ij}^{1} = v_{ij}^{1}+\sum_k v_{ik}^{1}G_{0;k}T_{kj}^{1}. \tag{19}$$

The second quantity on the right-hand side of equation (18) is expressed as

$$T_{ij}^{s} = \sum_{kl}(1+T^1G_0)_{ik}v_{kl}(1+G_0T)_{lj}. \tag{20}$$

If the operator $T^1$ is given in terms of another amplitude $T_C^1(Z)$ as

$$T_{ij}^{1}(Z) = \delta_{ij}(1-\delta_{i3})T_C^1(Z), \tag{21}$$

then, equation (21) stands for the definition of the amplitude $T_C^1(Z)$. Then, from equations (19) and (21), making use of equation (14), we have in momentum space an expression:

$$T_C^1(q_i, q_i'; Z) = V_C(q_i-q_i')+\int d^3q_i''\frac{V_C(q_i-q_i'')T_C^1(q_i, q_i''; Z)}{Z-E_i-\frac{q_i''^2}{2M_i}}. \tag{22}$$

For physical three-particle energies in which the momentum $q_i$ is equal to its on-shell value, equation (22) is the momentum representation of the Lipmann-Schwinger equation of two-particle energy $Z_i = Z - E_i$. Thus, $T_C^1(q_i, q_i'; Z)$ is the screened Coulomb amplitude for two particles, one of them of a mass $m_i$ and the other of a mass $(m_j + m_k)$. Also, the second amplitude $T_{ij}^s$ represented by equation (20), is the Coulomb-modified strong amplitude. This last amplitude can be calculated using a partial wave expansion. The expressions of the two amplitudes, the screened Coulomb amplitude and the Coulomb-modified strong amplitude, enable us to calculate the Coulomb corrections in the three-body problem. These amplitudes can be applied to study the elastic and rearrangement scattering processes in nuclear reactions for one neutral and two charged particles as a three-body problem.

### *3. Numerical calculations and results*

In the preceding section, three-body integral equations and the $T$ matrix for three--body system with one neutral particle and two charged particles are given by including the Coulomb force in one system. These equations can be applied for the case of deuteron stripping nuclear reactions. In these equations the short-range nuclear forces are considered as rank-one separable interactions. For deuteron stripping reactions, we have a deuteron bound state between the neutron and the proton in the initial channel, while in the final channel we have a residual nucleus, $R$, which is a bound state of the captured neutron and the target nucleus, $T$. For the short-range nuclear interactions we use here simple rank-one two-body separable interactions of the Yamaguchi [15] form as

$$v_d(q^2) = \lambda_d/(q^2 + \beta_d^2) \tag{23}$$

and

$$v_R(q^2) = \lambda_R/(q^2 + \beta_R^2), \tag{24}$$

where $\lambda_d$ and $\lambda_R$ are the renormalized coupling constants. The different parameters in the Yamaguchi two-body nuclear interaction expressed by equation (23), for the deuteron bound-state, are determined by varying the parameters $\lambda_d$ and $\beta_d$ independently, to fit the deuteron binding energy $\varepsilon_d$ taken as 2.225 MeV. The relation between these parameters is given by

$$m\lambda_d = 8\pi\beta_d[\sqrt{m}\,\varepsilon_d + \beta_d]^2. \tag{25}$$

The values of the different parameters of the neutron-proton potential are determined by minimization referring to the low-energy parameters, scattering length and effective range, and to the experimental phase shifts. The numerical values of the other two parameters $\lambda_R$ and $\beta_R$ which appeared in equation (24), can be obtained by expressions similar to that of equation (25), for the different reactions considered. The Coulomb force exists only between the proton and the target nucleus and thus it acts in one subsystem only and is represented by a screened Coulomb potential expressed by equation (12). Using the method of minimization and by varying the parameters $\lambda_d$ and $\beta_d$ independently to fit

the deuteron binding energy and the other observables, making use of equation (25), the numerical values of the two-body potentials can be obtained. The numerical values of the parameters in the neutron–proton potentials are: the range $\beta_d^{-1} = 0.752$ fm; equivalent strength $\lambda_d = 113.9$ MeV. Similarly, for the neutron-target nucleus two-particle interaction the numerical values of the parameters are: the range $\beta_R^{-1} = 1.028$ fm; equivalent strength $\lambda_R = 73.303$ MeV and $\lambda_R = 74.261$ MeV for the two reactions considered using the $^{28}$Si and $^{40}$Ca target nuclei, respectively.

The formalism presented in Section 2 is very suitable for computation and manageable

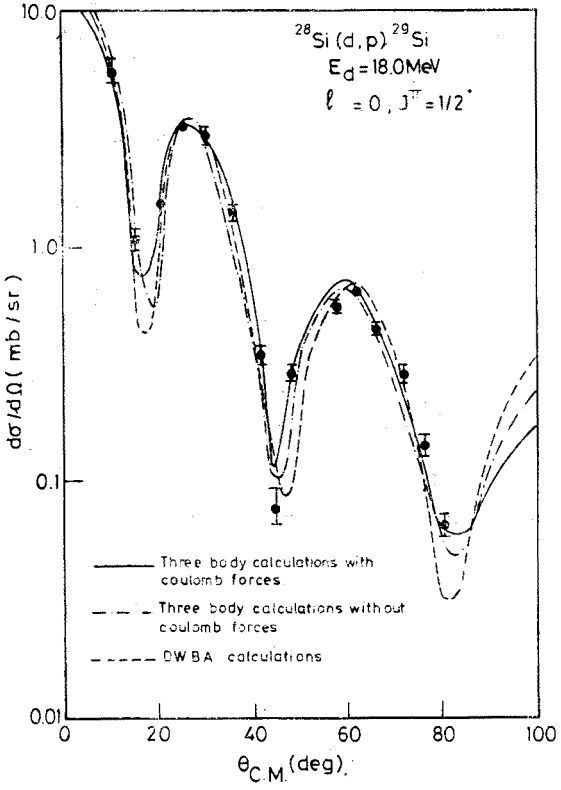


Fig. 1

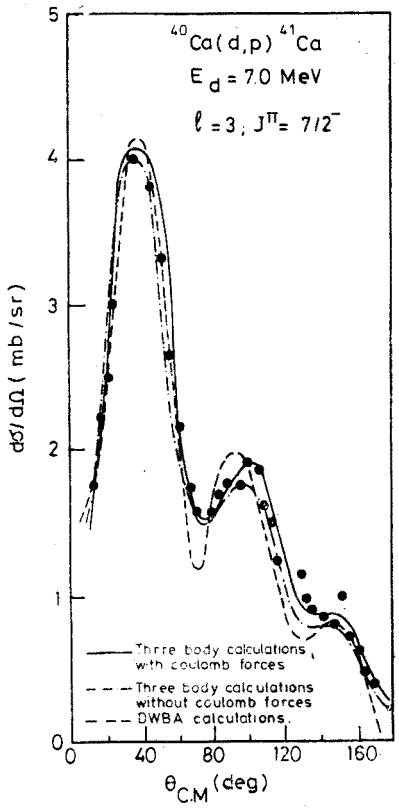


Fig. 2

Fig. 1. The differential cross-sections of the nuclear stripping reaction $^{28}$Si(d, p)$^{29}$Si of incident deuteron energy 18.0 MeV, leaving the residual nucleus $^{29}$Si in its ground state. The solid curve is our present three--body calculation with Coulomb forces included. The dashed-dotted curve is the three-body calculation without Coulomb forces. The dashed curve is the DWBA calculation. The experimental data are taken from Ref. [17]

Fig. 2. The differential cross-sections of the nuclear stripping reaction $^{40}$Ca(d, p)$^{41}$Ca of incident deuteron energy 7.0 MeV, leaving the residual nucleus $^{41}$Ca in its ground state. The solid curve is our present three--body calculation with Coulomb forces included. The dashed-dotted curve is the three-body calculation without Coulomb forces. The dashed curve is the DWBA calculation. The experimental data are taken from Ref. [18]

on the computer. The method of Kopal [16] is used in performing the numerical calculations. The integrals are solved and are replaced by mesh points.

Using the present theoretical expressions, numerical calculations are carried out for deuteron-induced nuclear reactions. The differential cross-sections for deuteron stripping as well as for deuteron scattering reactions are calculated. Theoretical calculations of the angular distributions for the deuteron stripping reactions $^{28}$Si(d, p)$^{29}$Si and $^{40}$Ca(d, p)$^{41}$Ca are performed at deuteron incident energies of 18.0 MeV and 7.0 MeV and are introduced in Figs 1 and 2, respectively. The present theoretical calculations using three-body model which include the Coulomb force are shown by solid curves. Numerical calculations using the same three-body model but with the Coulomb force excluded, have been carried out for the same reactions and are shown by the dashed–dotted curves. These curves for both calculations of the three-body model, with and without the Coulomb force, are compared with the experimental data represented in Figs 1 and 2 by the points and are taken from Refs [17] and [18], respectively. DWBA calculations following a perturbation approach

TABLE I

Extracted spectroscopic factors

| Reaction | Incident energy (MeV) | $l$ | $J^\pi$ | Present TBM with CF | Spectroscopic factors | | Previous work |
|---|---|---|---|---|---|---|---|
| | | | | | Present TBM without CF | DWBA calculations[a] | |
| $^{28}$Si(d, p)$^{29}$Si | 18.0 | 0 | $\frac{1}{2}^+$ | 0.9961 | 0.9376 | 0.9841 | 0.530 |
| $^{40}$Ca(d, p)$^{41}$Ca | 7.0 | 3 | $\frac{7}{2}^-$ | 0.9284 | 0.8643 | 0.8421 | 0.742 |

[a] See Ref. [19]

[19] are also done for these reactions for the purpose of comparison and are represented by the dashed curves in the same figures. From the comparison of the present theoretical calculations with the experimental data, the spectroscopic factors in each case are extracted for the different reactions considered. All the values of the extracted spectroscopic factors in the cases considered are shown in Table I. For the purpose of comparison also, the previously obtained spectroscopic factors are shown in Table I, taken from Refs [17] and [18], respectively.

Deuteron elastic scattering on the target nuclei $^{28}$Si and $^{40}$Ca is considered. The numerical calculations of the angular distributions of the deuteron elastic scattering are carried out using the present three-body model and are represented in Figs 3 and 4. These calculations are represented by solid curves for the case including the Coulomb force and by dashed-dotted curves for the cases without Coulomb force.

From the values of the spectroscopic factors listed in Table I, we see that the effect of including the Coulomb force in the three-body model of deuteron stripping reactions is that it improves the results by 6.241% to 7.411%.

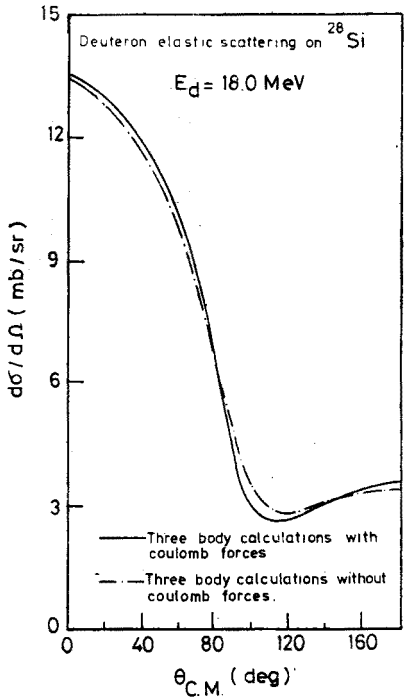


Fig. 3

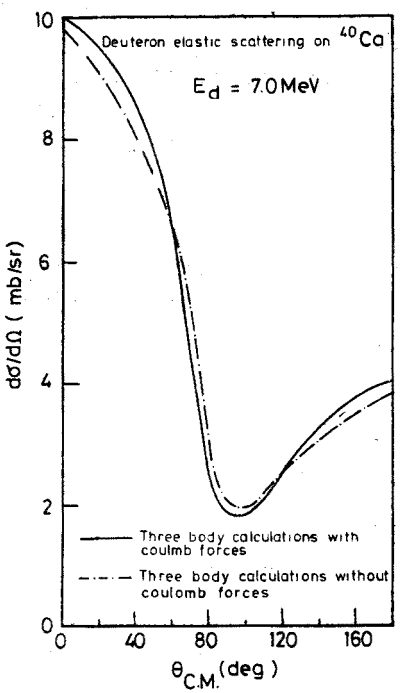


Fig. 4

Fig. 3. The angular distributions of deuteron elastic scattering on $^{28}$Si at deuteron incident energy of 18.0 MeV. The solid curve is our present three-body calculation including Coulomb forces. The dashed--dotted curve is the three-body calculation without Coulomb forces

Fig. 4. The angular distributions of deuteron elastic scattering on $^{40}$Ca at deuteron incident energy of 7.0 MeV. The solid curve is our present three-body calculation including Coulomb forces. The dashed--dotted curve is the three-body calculation without Coulomb forces

## *4. Discussion and conclusions*

In the present work, the three-body integral equations and the $T$ matrix of a three--body system composed of a neutral particle and two charged particles are given. In these equations, the Coulomb force is included in only one subsystem of the two charged particles. The present three-body equations including the Coulomb forces are applied to the deuteron stripping and deuteron elastic scattering nuclear reactions. The present theoretical and numerical calculations of the differential cross-sections are shown in Figs 1–4. These theoretical calculations are compared with the experimental measurements in Figs 1 and 2. From Figs 1 and 2, we see that the present three-body model including the Coulomb forces introduce good agreements with the experimental angular distributions. From the comparison of all the present theoretical calculations and the experimental data, spectroscopic factors are extracted and listed in Table I. Better and reasonable values of the spectroscopic factors are obtained for the case of three-body model including the Coulomb force. The

inclusion of the Coulomb force in only one subsystem improves the results by about 6.826%.

We can conclude that the present three-body model, in spite of its simplicity, accounts for all the characteristic features of direct nuclear reactions. We see that the Coulomb forces are very important and must be included in three-body calculations.

I am very grateful to Professor Abdus Salam and Professor Paolo Budinich, the International Atomic Energy Agency and UNESCO, for hospitality at the International Centre for Theoretical Physics, Trieste, where most of this work was done. Thanks are also due to the Centro di Calcolo dell'Università di Trieste for the use of their facilities.

## REFERENCES

[1] R. D. Amado, *Phys. Rev.* **132**, 485 (1963).

[2] W. Glöckle, W. D. Heiss, *Nucl. Phys.* **A132**, 468 (1969); W. Glöckle, *Nucl. Phys.* **A141**, 620 (1970).

[3] A. Osman, *Part. Nucl.* **3**, 28 (1972); *Atomkernenergie* **22**, 62 (1973); *Part. Nucl.* **7**, 29 (1974); *Int. J. Theor. Phys.* **23**, 15 (1984).

[4] L. D. Faddeev, *Zh. Eksp. Teor. Fiz.* **39**, 1459 (1960) (English translation: *Sov. Phys. JETP* **12**, 1014 (1961)); *Dokl. Akad. Nauk SSSR* **138**, 565 (1961) (English translation: *Sov. Phys. Dokl.* **6**, 384 (1961)); *Dokl. Akad. Nauk SSSR* **145**, 301 (1962) (English translation: *Sov. Phys. Dokl.* **7**, 600 (1963)); *Mathematical Aspect of the Three-Body Problems in the Quantum Scattering Theory*, translated from Russian by the Israel Programme for Scientific Translations, Jerusalem, D. Davey and Company, New York 1965.

[5] A. N. Mitra, *Nucl. Phys.* **32**, 529 (1962); A. N. Mitra, V. S. Bhasin, *Phys. Rev.* **131**, 1265 (1963); A. N. Mitra, *Phys. Rev.* **139**, B1472 (1965).

[6] For a recent review, see *Few-Body Systems and Nuclear Forces* I and II, Eds. H. Zingl, M. Haftel and H. Zankel, Lecture Notes in Physics, Vols. **82** and **87**, Springer-Verlag, Heidelberg 1978,

[7] L. Shulman, *Phys. Rev.* **156**, 1129 (1967); A. Osman, *Phys. Rev.* **C4**, 302 (1971): A. Osman, *Int. J. Theor. Phys.* **22**, 341 (1983).

[8] J. V. Noble, *Phys. Rev.* **161**, 945 (1967); S. Adya, *Phys. Rev.* **166**, 991 (1968); S. Adya, *Phys. Rev.* **177**, 1406 (1969); K. A.-A. Hamza, S. Edwards, *Phys. Rev.* **181**, 1494 (1970); Gy. Bencze, *Nucl. Phys.* **A196**, 135 (1972); N. Sawicki, *Nucl. Phys.* **A215**, 447 (1973).

[9] A. M. Veselova, *Teor. Mat. Fiz.* **3**, 326 (1970) (English translation: *Theor. Math. Phys.* **3**, 542 (1970)); V. G. Gorshkov, *Zh. Eksp. Teor. Fiz.* **40**, 1481 (1961) (English translation *Sov. Phys. JETP* **13**, 1037 (1961)); L. D. Faddeev, in *Three-Body Problem in Nuclear and Particle Physics*, Eds. J. S. C. McKee and P. M. Rolph, North Holland Publishing Company, Amsterdam 1970.

[10] E. O. Alt, W. Sandhas, H. Ziegelman, *Phys. Rev.* **C17**, 1981 (1978); H. van Haeringen, *Nucl. Phys.* **A327**, 77 (1979).

[11] A. Osman, *Nuovo Cimento* **42A**, 397 (1977); **46A**, 477 (1978); **48A**, 121 (1978); A. Osman, *Phys. Rev.* **C17**, 341 (1978); **C19**, 1127 (1979).

[12] E. O. Alt, P. Grassberger, W. Sandhas, *Nucl. Phys.* **B2**, 167 (1967); E. O. Alt, W. Sandhas, H. Zankel, H. Ziegelman, *Phys. Rev. Lett.* **37**, 1537 (1976); E. O. Alt, in *Few Body Dynamics*, Eds A. N. Mitra et al., North Holland Publishing Company, Amsterdam 1976.

[13] S. P. Merkuriev, *Yad. Fiz.* **19**, 447 (1974) (English translation: *Sov. J. Nucl. Phys.* **19**, 222 (1974)); C. Gignoux, A. Laverne, S. P. Merkuriev, *Phys. Rev. Lett.* **33**, 1350 (1974); S. P. Merkuriev, *Yad. Fiz.* **24**, 289 (1976) (English translation: *Sov. J. Nucl. Phys.* **24**, 150 (1976)); S. P. Merkuriev, C. Gignoux, A. Laverne, *Ann. Phys.* (USA) **99**, 30 (1976); S. P. Merkuriev, *Teor. Mat. Fiz.* **32**, 187 (1977) (English translation: *Theor. Math. Phys.* **32**, 680 (1977)).

[14] A. M. Veselova, *Teor. Mat. Fiz.* **13**, 368 (1972) (English translation: *Theor. Math. Phys.* **13**, 1200 (1973)).

[15] Y. Yamaguchi, *Phys. Rev.* **95**, 1628 (1954); Y. Yamaguchi, Y. Yamaguchi, *Phys. Rev.* **95**, 1635 (1954).
[16] Z. Kopal, *Numerical Analysis*, John Wiley & Sons Inc., New York 1955.
[17] M. C. Mermaz, C. A. Whitten, Jr., J. W. Champlin, A. J. Howard, D. A. Brommely, *Phys. Rev.* **C4**, 1778 (1971).
[18] L. L. Lee, Jr., J. P. Schiffer, B. Zeidman, G. R. Satchler, R. M. Drisko, B. H. Bassel, *Phys. Rev.* **136**, B971 (1964).
[19] A. Osman, M. Zaki, *Phys. Rev.* **C13**, 1 (1976).