ON THE PROPERTIES OF THE LOWEST COLLECTIVE STATES OF NUCLEI AROUND RADIUM*

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Energy of the lowest collective states and the electromagnetic transitions between them are studied theoretically for even-even nuclei in the radium region. Quadrupole- and octupole-transition probabilities B(E2) and B(E3) are considered. The energy as well as the probabilities are found to be sensitive functions of the shape of the collective potential energy of a nucleus.

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1. Introduction

There is continuous interest in the explanation of the abnormal properties of nuclei around radium. In particular, such unusual properties are observed in the spectra of the lowest states of these nuclei. In even-even nuclei, they consist in a very small energy of the lowest negative-parity states and in an anharmonic character of the spectra [1]. Both properties are usually interpreted as being connected with a strong anharmonicity of the collective potential energy of a nucleus, treated as a function of the octupole deformation ε_3. The anharmonicity consists in an appearance of a minimum of the energy at the octupole deformation different from zero, $\varepsilon_3^0 \neq 0$. Such minimum has been obtained in theoretical calculations [2, 3]. The position of the minimum (i.e. the value of the octupole deformation) and its depth (which is a measure of the stability of the deformation) is, however, much dependent on specifics of the models used in the calculations (cf. e.g. [4, 5]). It seems reasonable then to calculate a quantity which much depends on the potential and which can be measured. Possible measurement of it could thus help in deciding how large and how stable the octupole deformation of these nuclei is. The reduced octupole-transition probability B(E3), besides the energy of the collective states, is such a quantity.

The scope of the present paper is to calculate the B(E3) and B(E2) transition probabilities between the lowest collective states, for even-even nuclei in the radium region, and

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discuss the sensitivity of these quantities to the exact shape of the collective potential energy. The energy of the states is also calculated. The paper is an extension of the study [6], where B(E3) has only been calculated for the first negative-parity state. Part of the results discussed here have been presented in Ref. [7].

2. Method of calculations

We consider the lowest intrinsic collective excitations of a nucleus as being of vibrational nature. Two lowest-multipolarity degrees of freedom are taken: quadrupole and octupole deformations. They are described by the Nilsson parameters ε_2 and ε_3, respectively. The hamiltonian, when taken in its classical form, is

$$H = \tfrac{1}{2}(B_{\varepsilon_2\varepsilon_2}\dot{\varepsilon}_2^2 + B_{\varepsilon_3\varepsilon_3}\dot{\varepsilon}_3^2) + V(\varepsilon_2, \varepsilon_3), \tag{1}$$

where $B_{\varepsilon_i\varepsilon_i}$ is the mass parameter describing the inertia of a nucleus with respect to the deforamtion ε_i ($i = 2, 3$) and V is the potential energy. The potential V is calculated microscopically. The mass parameters are taken constant (independent of deformation), for simplicity. The values $B_{\varepsilon_2\varepsilon_2} = B_{\varepsilon_3\varepsilon_3} = 300\,\hbar^2\,\mathrm{MeV}^{-1}$ and $B_{\varepsilon_2\varepsilon_3} = 0$ are used. They are based on the cranking results of Ref. [8]. Only the intrinsic excitations are considered. Thus, no rotation is included.

The quantum hamiltonian (being the quantized version of the classical hamiltonian (1)) is numerically diagonalized in the basis of two one-dimensional oscillators, corresponding to ε_2 and ε_3 degrees of freedom. The eigenvalues give the collective energies, while the wave functions are used for the calculations of B(E2) and B(E3).

The form of the hamiltonian (1) tells us that the excitations are the quadrupole and octupole vibrations of a nucleus, coupled to each other by the potential energy $V(\varepsilon_2, \varepsilon_3)$.

3. Results and discussion

Before presenting the results for the energy spectra and the transition probabilities B(E2) and B(E3), obtained with the potential V based on a microscopic model of a nucleus, let us remind the situation for a pure one-dimensional oscillator. This is shown in Fig. 1. The energy levels are equidistant and the transition probability $B(E\lambda; i \to i-1)$ satisfies a simple rule. It increases linearly with the number of the level i: $B(\mathrm{E}\lambda; i \to i-1) = i \cdot B(\mathrm{E}\lambda; 1 \to 0)$. Thus, if our two-dimensional potential $V(\varepsilon_2, \varepsilon_3)$ was simple superposition of two oscillators in ε_2 and ε_3, our spectrum E_i and transitions $B(\mathrm{E}\lambda; i \to k)$ would also be simple superpositions of the oscillators spectra and transitions.

Fig. 2 shows the potential energy V obtained for ^{224}Ra in a microscopic approach. It is calculated in three different ways. The potentials (D) and (Y) are obtained by the macroscopic-microscopic method with two different smooth parts of the energy. In the case (D), the droplet model with the parameters of Ref. [9] is used for the smooth part, while in the case (Y) the Yukawa-plus-exponential model with the parameters of Ref. [3] is taken for it. In both cases, the microscopic part (shell correction) is calculated by the Strutinski method, based on the Nilsson potential with the "$A = 225$" parameters [10].

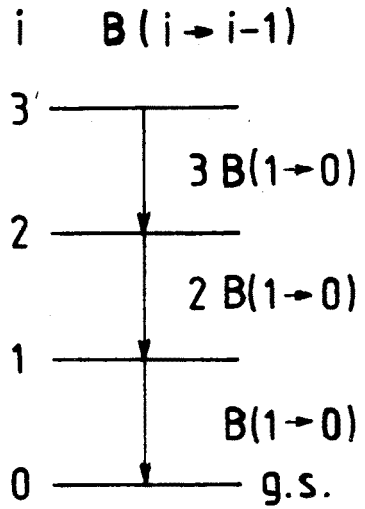

Fig. 1. Spectrum and transition probabilities for a one-dimensional oscillator

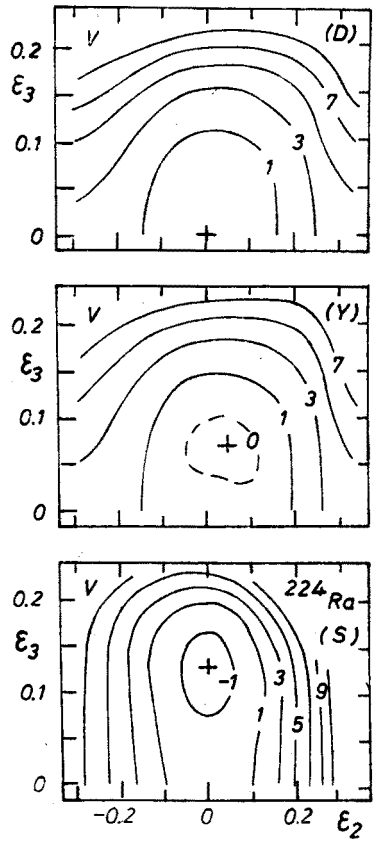

Fig. 2. Contour maps of the potential energy $V(\varepsilon_2, \varepsilon_3)$ obtained microscopically for ^{224}Ra in three variants: (D), (Y) and (S) described in text. The numbers at the contour lines give the values of the energy in MeV. For each variant, the cross shows the position of the minimum of the energy

The potential (S) is obtained as a total single-particle energy of a nucleus smoothed by the pairing interaction [11]. This potential is expected to be good for not too large changes in the deformation of a nucleus. It is also obtained with the Nilsson single-particle potential with the "$A = 225$" parameters.

One can see that the three potential energies V differ significantly. While in the case (D), the minimum of V is obtained at the octupole deformation $\varepsilon_3^0 = 0$, the minimum (of about 0.3 MeV depth with respect to the energy at $\varepsilon_3^0 = 0$) is observed at $\varepsilon_3^0 \approx 0.07$ in the case (Y). A deep (of about 1.6 MeV) minimum appears at $\varepsilon_3^0 \approx 0.13$ in the (S) case.

The appearance of the minimum at $\varepsilon_3^0 \neq 0$ is a manifestation of an anharmonicity of the potential V in the octupole degree of freedom ε_3. It is because the potential is symmetric in ε_3, $V(-\varepsilon_3) = V(\varepsilon_3)$, and the minimum at $\varepsilon_3^0 = 0$ is a necessary condition for the harmonicity of the potential. Due to this, the potential (S) is strongly anharmonic in the octupole degree of freedom ε_3. The large differences in V, calculated in different ways, stress the importance of a study of the sensitivity of the calculated quantities to the changes in the potential, as pointed out in the Introduction.

Figs 3–5 show the energy spectra and the B(E2) and B(E3) transition probabilities obtained with the (D), (Y) and (S) potentials, respectively. Seven lowest levels and the largest probabilities $B(E\lambda)$ of the transitions between them, given in the Weisskopf units (W.u.), are shown. Solid lines correspond to the levels of positive parity and dashed lines to those of negative parity. The E2 transitions are denoted by solid arrows and the E3 transitions by dashed ones. The state with the largest E2 transition to the ground state (g.s.) is considered as the first quadrupole excitation (1q), that with the largest E2 to the 1q state as the second quadrupole state (2q) and so on. The same concerns the octupole excitations, with E2 being replaced by E3. The states, which are mixtures of the above states, are denoted by $iq+jo+\ldots$

One can see in Fig. 3 that the spectra of both quadrupole and octupole excitations, obtained in the case (D), are close to harmonic (equidistant). Also the ratios of the transition

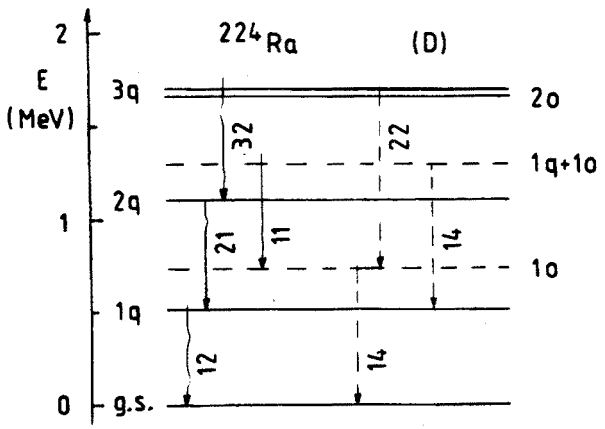

Fig. 3. Energy spectrum of the lowest seven states and the largest B(E2) and B(E3) transition probabilities, given in Weisskopf units, obtained with the variant (D) of the potential of Fig. 2

probabilities are close to those for the harmonic oscillator. The mixed state, 1q+1o, is rather direct superposition of the first quadrupole, 1q, and the first octupole, 1o, states. All the lowest mixed states are simple superpositions of the quadrupole and octupole, almost harmonic, oscillations, which are only weakly coupled to each other. One might expect such picture from the form, itself, of the potential energy (D), given in Fig. 2.

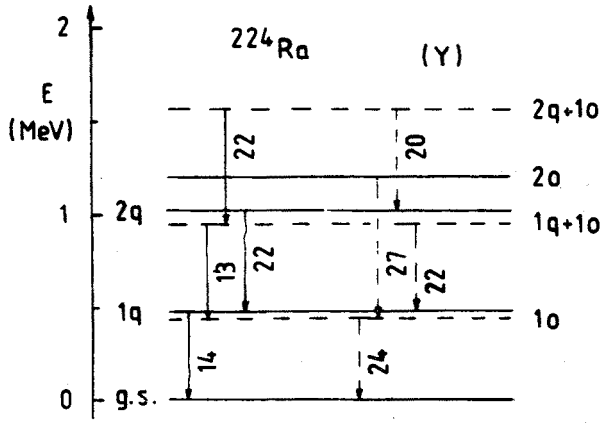

Fig. 4. Same as in Fig. 3, but for the variant (Y) of the potential

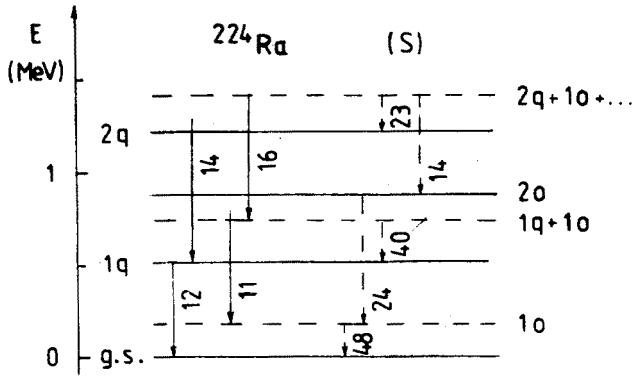

Fig. 5. Same as in Fig. 3, but for the variant (S) of the potential

In the case (Y), Fig. 4, the quadrupole excitations are still close to harmonic. The octupole excitations, however, are more distant from this limit. The ratio of the energy of the second excited state to that of the first excited state is: $E(2o)/E(1o) = 2.71$ and the ratio of the octupole transition probabilities: $B(E3; 2o \rightarrow 1o)/B(E3; 1o \rightarrow g.s.) = 1.13$. This is connected with the appearance of a flat minimum (of about 0.3 MeV depth) of the potential V at $\varepsilon_3^0 \neq 0$, which means an anharmonicity of the potential in the $\dot{\varepsilon}_3$ degree of freedom, as stressed above.

The largest anharmonicity in the octupole excitations appears in the case (S), Fig. 5.

This is due to the deep minimum (about 1.6 MeV) of the potential energy, appearing at $\varepsilon_3^0 \neq 0$. The largest B(E3) probabilities appear in this case for transitions between the following pairs of states: 1o → g.s., 3o → 2o, 5o → 4o, etc., characteristic for the model case of the double-oscillator potential [12]. The strongest is, naturally, the transition between the lowest pair, 1o → g.s., for which the wave functions of its members are most similar to each other (when neglecting the difference in the parity) and the corresponding energy levels are most close. Structure of higher states (e.g. the seventh state in the figure) appears to be complex, in this strongly anharmonic case.

4. *Conclusions*

The following conclusions may be drawn from our study:

(1) The energy spectrum and especially the transition probability between the levels are sensitive functions of the shape of the potential energy.

(2) The energy spectrum of the octupole excitations and the probability B(E3) of the octupole transitions between them are particularly sensitive to the position ε_3^0 and to the depth ΔV_3 of the minimum of the potential energy. This depth (being the energy gain due to the octupole degree of freedom) may be considered as a measure of the anharmonicity of V, treated as a function of ε_3.

(3) For large ε_3^0 and large ΔV_3, the spectrum is much anharmonic. Also the rules for B(E3) are much different, in that case, from those for the harmonic oscillator. The largest B(E3) appear for the following transitions: 1o → g.s., 3o → 2o, 5o → 4o, etc. The strongest one is that for 1o → g.s.

(4) The transition probability B(E3) from the lowest negative-parity state (1o) to the ground state (g.s.) seems to be the most direct measure of the value of the octupole deformation of a nucleus.

(5) All transition probabilities between the lowest states may be divided in to two groups: large (allowed) transitions and very small (forbidden) ones, indicating for a simple structure of the states. This structure becomes, however, more complex with increasing excitation energy of the states.

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