A HOMOGENEOUS PERFECT FLUID COSMOLOGICAL MODEL IN GENERAL RELATIVITY

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A solution of Einstein field equations has been obtained for a plane symmetric space-time filled with perfect fluid. Certain physical and geometrical properties of the model have been examined.

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1. *Introduction*

Plane symmetric space-times representing distributions of perfect fluid with isentropic flow were discussed in detail by Taub [1]. A plane symmetric non-static homogeneous cosmological model of perfect fluid distribution was derived by Singh [2] from the considerations of class one metric. A metric is of class one if it can be locally imbedded in a five dimensional flat space-time. The universe obtained by them is irrotational and expanding. The metric form studied by them can be expressed as

$$ds^2 = (1+\alpha t)^2 (dt^2 - dx^2 - dy^2) - (1+\beta t)^2 dz^2 \tag{1.1}$$

where α and β are positive constants. The metric (1.1) admits a four parameter group of motions. In this paper we consider the line element

$$ds^2 = A^2(dt^2 - dx^2 - dy^2) - B^2 dz^2 \tag{1.2}$$

where the metric potentials are function of t alone. The line element (1.2) has been studied by Patel and Vaidya [3] for dust and disordered radiation. In this paper we have obtained a new exact solution of Einstein field equations for the line element (1.2) of a space-time filled with perfect fluid which is assumed to satisfy the condition $\frac{\sigma}{\theta} = \text{constant}$ [4] where σ is shear and θ is expansion. The matter distribution in the obtained model automatically satisfies the equation of state, pressure = energy density.

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2. *Derivation of the line element*

The energy momentum tensor for the perfect fluid distribution is given by

$$T_{ij} = (\varrho+p)V_iV_j-pg_{ij} \tag{2.1}$$

together with

$$g^{ij}V_iV_j = 1 \tag{2.2}$$

ϱ being the density, p the pressure and V_i the flow vector. The field equations to be satisfied are

$$R_{ij}-\tfrac{1}{2}Rg_{ij} = -8\pi T_{ij}. \tag{2.3}$$

Equations (2.1) and (2.3) for the line element (1.2) lead to

$$V_1 = V_2 = V_3 = 0. \tag{2.4}$$

Hence the field equations (2.3) lead to

$$-8\pi pA^2 = \left[\left(\frac{A_4}{A}\right)_4+\frac{B_{44}}{B}\right], \tag{2.5}$$

$$-8\pi pA^2 = \left[\left(\frac{A_4}{A}\right)_4+\frac{A_{44}}{A}\right], \tag{2.6}$$

$$-8\pi\varrho A^2 = \left[-2\frac{A_4B_4}{AB}-\frac{A_4^2}{A^2}\right] \tag{2.7}$$

From equations (2.2) and (2.4) we have

$$V_4 = A. \tag{2.8}$$

From equations (2.5) to (2.8) we have four equations in five unknown A, B, ϱ, p and V_4. Therefore, for complete determination of this set we require an additional condition. We assume that $\frac{\sigma}{\theta}$ = constant [4] which ultimately leads to

$$A = B^n, \tag{2.9}$$

where n is a real constant. From Eq. (2.5) and (2.6) we have

$$\frac{A_{44}}{A} = \frac{B_{44}}{B}. \tag{2.10}$$

Solving Eqs. (2.9) and (2.10) and using the suitable transformation of coordinates we have the line element in the form

$$ds^2 = T^{2(1-m)}dT^2-dX^2-dY^2-T^{2m}dZ^2, \tag{2.11}$$

where $m = \frac{1}{n+1}$. The expressions for density (pressure) and non-vanishing component of flow vector for the model (2.11) are given by

$$8\pi\varrho = 8\pi p = \frac{(1-m^2)}{T(3-m)} \tag{2.12}$$

$$\dot{V}_4 = T^{(1-m)}. \tag{2.13}$$

The acceleration vector $\dot{V}_i$ and rotation ω are given by

$$\dot{V}_i = 0, \tag{2.14}$$

$$\omega = 0. \tag{2.15}$$

The scalar of expansion θ and shear σ are given by

$$\theta = \frac{(2-m)}{T^{(2-m)}} \tag{2.16}$$

$$\sigma = \frac{(1-2m)}{\sqrt{3}} \frac{1}{T^{(2-m)}}. \tag{2.17}$$

3. *Discussion*

The model is clearly expanding and shearing having geodesic lines of flow. The equation of state $\varrho = p$ is automatically satisfied. The reality condition $p > 0$ requires that $|m| \ll 1$.

The model starts with a big bang at $T = 0$ and evolves smoothly till $T = \infty$, where expansion and shear become zero. The model has a point singularity at $T = 0$.

We also note that the ratio $\frac{\sigma}{\theta} = \frac{(1-2m)}{\sqrt{3}(2-m)} \ll 1$ and non-vanishing components of conformal curvature tensor C_{hijk} tend to zero for large value of T. Therefore, we conclude that the model approaches FRW model for large values of time.

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