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EQUATIONS FOR A SUBSPACE OF UNSTABLE PARTICLES

By K. Urbanowski

Department of Physics, Pedagogical University of Zielona Góra*

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The equations connecting the Hamiltonian of a physical system under consideration with the projection operator onto a subspace of unstable particles are given. Solutions of these equations are found for the Lee model.

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1. *Introduction*

All hitherto existing papers analysing the decay laws of unstable states (i.e. analysing a function which describes a probability that a system initially at $t = 0$ being in the unstable state stays in the same state up to time t) can be collected into a few thematic groups. For example, there is an analysis of a shape of the decay function following from the physical properties of the system — the question here is what are deviations of this function from the exponential form for very large and very small times [1–4]. In another group of papers an influence of the measurement methods on a shape of the decay function was analysed [3, 5–11]. Possibilities of extension of a standard formulation of Quantum Mechanics in the Hilbert space, in order that this extended theory included nonhermitean operators were also analysed. This led to a complex energy and states decaying with the increase of time, e.g. [12, 13]. Next, some authors assumed that unstable states were described by vectors which belonged to a proper subspace $\mathscr{H}_u$ of the state space $\mathscr{H}$ and the time evolution in this subspace was given by a semigroup of contractions. Then they tried to construct the whole of the state space and to find the total unitary group $U(t)$ describing the time evolution in $\mathscr{H}$, on the ground of Sz.-Nagy Theorem referring to the extension of the semigroup of contractions to the unitary group [14]. So, it was possible to find the generator of this group — a Hamiltonian [15–17].

In all these papers there was no discussion of how to find unstable states belonging to $\mathscr{H}_u$ having $\mathscr{H}$ and the Hamiltonian H. One of the earliest papers [18], connected with an analysis of the decay phenomenon in Quantum Mechanics contains a necessary and

* Address: Wyższa Szkoła Pedagogiczna, Zakład Fizyki, pl. Słowiański 6, 65-069 Zielona Góra, Poland.

sufficient condition for a given vector to describe unstable state. Unfortunately, this condition is not very helpful in searching for unstable particles in the system. Namely, let us follow Fock and Krylov [18] and consider the vector

$$|\alpha\rangle = \int_{\sigma_c(\bar{H})} c_\alpha(E)\,|E\rangle dE, \tag{1.1}$$

where $\sigma_c(H)$ is the continuous part of the spectrum of H, $|E\rangle$ is an eigenvector for H to $E \in \sigma_c(H)$ and

$$\langle E|E'\rangle = \delta(E-E'). \tag{1.2}$$

And now, if $|\alpha\rangle$ is an unstable state and $|\alpha; t\rangle = e^{-itH}|\alpha\rangle$ is the wave function of the system at time $t > 0$ then $|\langle\alpha|\alpha; t\rangle|^2$ is the probability that the system being at $t = 0$ in the unstable state $|\alpha\rangle$ is in the same state at time t and it is obvious that $|\langle\alpha|\alpha; t\rangle|^2$ must tend to zero as $t \to \infty$. Krylov and Fock derived the formula:

$$\langle\alpha|\alpha, t\rangle = \int_{\sigma_c(\bar{H})} e^{-iEt} dW_\alpha(E), \tag{1.3}$$

where

$$dW_\alpha(E) \equiv |c_\alpha(E)|^2 dE, \tag{1.4}$$

and proved that the necessary and sufficient condition for $|\langle\alpha|\alpha; t\rangle|^2 \to 0$ as $t \to \infty$ is the continuity of the integral energy distribution function $W_\alpha(E)$.

In this paper we shall give formulae different from the Krylov and Fock one, which enable us to construct the subspace of unstable particles which will be denoted by $\mathscr{H}_{up}$. It seems, this may be of some importance in investigations of multiparticle composite unstable objects. We start also from the function of type $|\langle\alpha|\alpha; t\rangle|^2$, but obtain a new necessary condition in order that vectors from some subspace of the Hilbert space of the system describe the unstable states — Section 2 gives explicit relations between given Hamiltonian H of the system and the projector P onto subspace $\mathscr{H}_{up}$. These relations are tested in Section 3 by applying them to the simplest version of the Lee model.

2. *Equations for a subspace of unstable particles*

Let us assume that the physical system under consideration is described by a selfadjoint operator H — the Hamiltonian, acting in the Hilbert space $\mathscr{H}$ of states $|\alpha\rangle$, $|\varphi\rangle \in \mathscr{H}$ of the system. Let us assume further that the unitary group of operators $U(t) = e^{-itH}$ describes the total evolution of the system.

We are not interested in the properties and the behaviour of the whole system but only of its certain property described by the states in some closed subspace (let us denote it as $\mathscr{H}_{\parallel}$) of $\mathscr{H}$ [19, 8]. This subspace can be described by means of a projector P:

$$\mathscr{H}_{\parallel} \stackrel{\text{df}}{=} P\mathscr{H}. \tag{2.1}$$

The property we are interested in, is connected with the question if the system being at the initial moment $t = 0$ in a state $|\alpha\rangle \in \mathscr{H}_{\parallel}$ is at the latter instant t in $\mathscr{H}_{\parallel}$ or not. So, the probability that a state $|\alpha\rangle \in \mathscr{H}_{\parallel}$ created at $t = 0$ is in $\mathscr{H}_{\parallel}$ at $t > 0$ will be investigated, i.e. the function

$$\mathscr{P}(t, \alpha) \stackrel{\mathrm{df}}{=} \sum_{\nu\in\mathfrak{A}} |\langle\alpha_\nu|e^{-itH}|\alpha\rangle|^2, \tag{2.2}$$

where: $|\alpha\rangle$, $|\alpha_\nu\rangle \in \mathscr{H}_{\parallel}$ and $\{|\alpha_\nu\rangle\}_{\nu\in\mathfrak{A}}$ is an orthonormal and complete set in $\mathscr{H}_{\parallel}$:

$$\sum_{\nu\in\mathfrak{A}} |\alpha_\nu\rangle\langle\alpha_\nu| = \mathbf{1}_{\mathscr{H}_{\parallel}} \equiv P. \tag{2.3}$$

In quantum theory, a general notion "particles" and also "unstable particles" is usually joined with the eigenstates for some Hamiltonian H_0 (so-called free Hamiltonian) to the discrete eigenvalues E_ν:

$$H_0|\alpha_\nu\rangle = E_\nu|\alpha_\nu\rangle. \tag{2.4}$$

Thus the decay of unstable particles $|\alpha_\nu\rangle$ is due to the interaction H_1, and the behaviour of the system is described by a Hamiltonian $H = H_0 + H_1$. It means that a basis of the subspace of unstable particles $\mathscr{H}_{\mathrm{up}}$, or its closed part $\mathscr{H}_{\parallel} \subset \mathscr{H}_{\mathrm{up}}$, can be composed of the normalized eigenvectors $|\alpha_\nu\rangle$ of H_0.

The natural conclusion following from the above is that an operator P projecting onto an arbitrary closed subset $\mathscr{H}_{\parallel}$ of the subspace $\mathscr{H}_{\mathrm{up}}$ of the state space $\mathscr{H}$ commutes with H_0:

$$[P, H_0] = 0. \tag{2.5}$$

The quantity $\mathscr{P}(t, \alpha)$ is expected to converge to zero as $t \to \infty$ for every $|\alpha\rangle \in P\mathscr{H}$, if it were to describe a decay [18, 16] and if $P\mathscr{H}$ were to describe unstable particles, and it is called the decay law of the unstable state $|\alpha\rangle$ [17, 20, 21]. The smallest closed subspace of $\mathscr{H}$ which consists of all such $P\mathscr{H}$ will be denoted by $\mathscr{H}_{\mathrm{up}}$. It is assumed that $\mathscr{H} \ominus \mathscr{H}_{\mathrm{up}} \supset \mathscr{H}_{\mathrm{d}}$, i.e. that $\mathscr{H}$ contains unstable particles, decay products ($\mathscr{H}_{\mathrm{d}}$ is usually called the subspace of decay products) and other (stable) states [20].

The formula (2.2) can also be written as follows (for $|\alpha\rangle \in P\mathscr{H} \subseteq \mathscr{H}_{\mathrm{up}}$)

$$\mathscr{P}(t, \alpha) = \langle\alpha|PP(t)P|\alpha\rangle, \tag{2.6}$$

where:

$$P(t) = e^{itH}Pe^{-itH}. \tag{2.7}$$

Now we can formulate the necessary and sufficient condition for vectors from a subspace $P\mathscr{H}$ to describe unstable particles. Namely, we can say that [22]

> vectors from a subspace $P\mathscr{H}$ describe unstable particles (i.e. $P\mathscr{H} \subseteq \mathscr{H}_{\mathrm{up}}$) if and only if for every $|\varphi\rangle \in \mathscr{H}$

$$\lim_{t\to\infty} \langle\varphi|PP(t)P|\varphi\rangle = 0. \tag{2.8}$$

Using the polar identity one can express the above criterion in terms of operators:

$$P\mathscr{H} \subseteq \mathscr{H}_{un} \Leftrightarrow \underset{t\to\infty}{\text{w-lim}}\, PP(t)P = 0, \tag{2.9}$$

where: w-lim is the weak limit in the set of all linear and bounded operators in $\mathscr{H}$.

Formally, it follows from (2.9) that we have for a projector P onto $\mathscr{H}_{\|} \subseteq \mathscr{H}_{up}$: either

$$\underset{t\to\infty}{\text{w-lim}}\, P(t) = 0, \tag{2.10}$$

or

$$\underset{t\to\infty}{\text{w-lim}}\, P(t) = \Pi = 0, \tag{2.11}$$

where $\Pi \neq \Pi^2$ is such linear, bounded and selfadjoint operator that

$$P\Pi = \Pi P = P\Pi P = 0, \tag{2.12}$$

(in other words

$$\Pi = (1-P)A + A^{+}(1-P) + (1-P)B(1-P), \tag{2.13}$$

and A, $B = B^{+}$ are any linear, bounded operators).

One of the obvious methods of studying the limits of type (2.9)–(2.11) is by taking the Laplace transforms and by applying Tauberian theorems. If $f(t) \to \gamma$ as $t \to \infty$ then (see e.g. [23])

$$\gamma = \lim_{\varepsilon\to 0+} \varepsilon\mathscr{L}\{f(t)\}(\varepsilon), \tag{2.14}$$

where:

$$\mathscr{L}\{f(t)\}(z) \overset{\text{df}}{=} \int_0^\infty f(t)e^{-zt}dt, \tag{2.15}$$

$$\operatorname{Re} z > 0.$$

Thus, one can conclude (taking into account (2.10), (2.11)) that the necessary condition in order that

$$\underset{t\to\infty}{\text{w-lim}}\, PP(t)P = 0 \tag{2.16}$$

is, either

$$\underset{\varepsilon\to 0+}{\text{w-lim}}\, \varepsilon\mathscr{L}\{P(t)\}(\varepsilon) = 0, \tag{2.17a}$$

or

$$\underset{\varepsilon\to 0+}{\text{w-lim}}\, \varepsilon\mathscr{L}\{P(t)\}(\varepsilon) = \Pi \neq 0, \tag{2.17b}$$

where: Π — see: (2.12), (2.13).

Applying the Parseval relation between the Laplace transforms [23] to the $\mathscr{L}\{P(t)\}(z)$ we obtain instead of (2.17), either

$$\operatorname*{w-lim}_{\varepsilon\to 0+} \frac{\varepsilon}{\pi} \int_{-\infty}^{\infty} R(\lambda - i\varepsilon) P R(\lambda + i\varepsilon) d\lambda = 0, \tag{2.18a}$$

or

$$\operatorname*{w-lim}_{\varepsilon\to 0+} \frac{\varepsilon}{\pi} \int_{-\infty}^{\infty} R(\lambda - i\varepsilon) P R(\lambda + i\varepsilon) d\lambda = \Pi, \tag{2.18b}$$

where $R(z) \overset{\mathrm{df}}{=} (z-H)^{-1}$ is the resolvent of H, and the weak limit is taken for vectors from the domain of H which is supposed to be dense in $\mathscr{H}$. Using the property (2.14), identity

$$z\mathscr{L}\{P(t)\}(z) \equiv P - i\mathscr{L}\{[P(t), H]\}(z), \tag{2.19}$$

and the Parseval equality, two new equations for P can be easily derived: either

$$P - \frac{i}{2\pi} \operatorname*{w-lim}_{\varepsilon\to 0+} \int_{-\infty}^{\infty} R(\lambda - i\varepsilon)[P, H] R(\lambda + i\varepsilon) d\lambda = 0, \tag{2.20a}$$

or

$$P - \frac{i}{2\pi} \operatorname*{w-lim}_{\varepsilon\to 0+} \int_{-\infty}^{\infty} R(\lambda - i\varepsilon)[P, H] R(\lambda + i\varepsilon) d\lambda = \Pi. \tag{2.20b}$$

Of course, these equations are completely equivalent to (2.18) but not more singular than those.

From all solutions of equations (2.18) or (2.20) only those will be useful which additionally satisfy the following requirement

$$P = P^{+} = P^{2} \neq 0. \tag{2.21}$$

One should stress that equations (2.18a) and (2.18b) (or (2.20a) and (2.20b) will have different solutions, if they exist. Let us denote by P_a the solutions of the equation (2.18a) or (2.20a) and by P_b projectors obtained from (2.18b) or (2.20b). We have that

$$P_a \cdot P_b = 0. \tag{2.22}$$

All operators projecting onto subspaces contained in $\mathscr{H}_{\mathrm{up}}$, which describes unstable particles, must fulfill one of the above derived equations, thus generally

$$\mathscr{H}_{\mathrm{up}} \subset \overline{\bigcup_a P_a \mathscr{H}} \oplus \overline{\bigcup_b P_b \mathscr{H}}. \tag{2.23}$$

The set of projectors, for which the condition (2.9) takes place, is a subset of operators obtained for a given H as solutions of equations (2.18a), (2.18b) or (2.20a), (2.20b).

One can show that solutions of the equation (2.18a) project onto such subspace of $\mathscr{H}$, which describes unstable states of the system decaying irrespective of what states were occupied at the initial time $t = 0$ (i.e. irrespective whether states in $\mathscr{H} \ominus P_a\mathscr{H}$ were occupied or not).

The equation (2.18b) defines a subspace of $\mathscr{H}$ containing unstable states which would decay only if the system was in a state from $P_b\mathscr{H} \subseteq \mathscr{H}_{\text{up}}$ at $t = 0$ and the states described by vectors from $\mathscr{H} \ominus P_b\mathscr{H}$ were not occupied at this moment. This means, in terms of particles, that the system under consideration at the initial moment $t = 0$ cannot contain any other particles and sources besides the unstable one, so that the decay should be observed.

In principle, a search for projection onto single particle states in simple quantum field theory models will not be facilitated by these equations. Usually it is rather clear which particles should be unstable there. It seems that for more complicated states (e.g. multiparticle composite objects and the like) our equations should be more useful.

Let us stress that the fulfillment of the equation (2.18a) or (2.18b) (or (2.20a), (2.20b)) together with the requirements (2.5), (2.21) is the necessary condition for the subspace $P\mathscr{H}$ to describe unstable particles.

At the end of this Section we have to make two remarks. First, it seems that equations (2.20) have an advantage over (2.18) because they enable us to include the requirement (2.5) in a simple way and, of course, they are not more singular than (2.18). Second, there are Hamiltonians, for which equations (2.18a), (2.20a) have solutions (see the next Section) but the problem of the existence of solutions of equations (2.18b), (2.20b) is open at present.

3. *An example: the Lee model*

In this Section equations (2.20) will be tested by applying them to the well known Lee model in its simplest version [24, 25]. This model describes two spinor particles V and N interacting through spinless boson θ-particle according to the Hamiltonian

$$H = H_0 + H_1, \tag{3.1}$$

where

$$H_0 = m_V \int d^3\vec{p}\, V^+(\vec{p})V(\vec{p}) + m_N \int d^3\vec{p}\, N^+(\vec{p})N(\vec{p}) + \int d^3\vec{k}\, \omega(\vec{k})a^+(\vec{k})a(\vec{k}), \tag{3.2}$$

and

$$H_1 = \frac{g_0}{(2\pi)^{3/2}} \int d^3\vec{k}\, \frac{f[\omega(\vec{k})]}{\sqrt{2\omega(\vec{k})}} \int d^3\vec{p}\,[V^+(\vec{p})N(\vec{p}-\vec{k})a(\vec{k}) + \text{h.c.}]. \tag{3.3}$$

Spinor particles V and N are static while the θ-particles, associated with the a^+, a operators are relativistic ones with energies

$$\omega(\vec{k}) = \sqrt{\vec{k}^2 + \mu^2}. \tag{3.4}$$

The real cut-off function $f[\omega(\vec{k})]$ and the coupling constant g_0 together with bare masses m_V, m_N and μ are parameters of the model. The creation and annihilation operators V^+, V,

N^+, N and a^+, a fulfill obvious commutation relations for bosons and fermions [25, 26]. The Hilbert space of states is constructed in the standard way [25, 26]. The "charges" $Q_1 = n_V + n_N$ and $Q_2 = n_N + n_\theta$ (where n_V is the V-particle number operator, n_θ is the θ-particle number operator, and so on) are conserved in the model. Hence, the Hilbert space of states may be decomposed into the direct sum of sectors $\mathscr{H}(q_1, q_2)$ having definite values q_1, q_2 of Q_1 and Q_2. The first nontrivial sector is $\mathscr{H}(1, 1)$ (where $q_1 = q_2 = 1$) to which we shall confine our attention in what follows. A typical element from this sector is

$$|\psi_V, \psi_{N\theta}\rangle \equiv |\psi_V, 0\rangle + |0, \psi_{N\theta}\rangle = \int d^3\vec{p}\,\psi_V(\vec{p})\,|V_{\vec{p}}\rangle$$
$$+ \int d^3\vec{p}\,d^3\vec{q}\,\psi_{N\theta}(\vec{p}, \vec{q})\,|N_{\vec{p}-\vec{q}}, \theta_{\vec{q}}\rangle, \tag{3.5}$$

where $\psi_V(\vec{p})$ and $\psi_{N\theta}(\vec{p}, \vec{q})$ are both square integrable with respect to $\vec{p}$ and $\vec{p}$, $\vec{q}$ respectively, and vectors

$$|V_{\vec{p}}\rangle \stackrel{\text{df}}{=} V^+(\vec{p})\,|0\rangle,$$
$$|N_{\vec{p}-\vec{q}}, \theta_{\vec{q}}\rangle \stackrel{\text{df}}{=} N^+(\vec{p}-\vec{q})a^+(\vec{q})\,|0\rangle, \tag{3.6}$$

($|0\rangle$ — is the normalized eigenvector for H_0 and H to the zero eigenvalue — a vacuum — defined as usually [25]) are eigenvectors for the H_0 to the m_V and $(m_N + \omega(\vec{q}))$ eigenvalues respectively, and they form and orthogonal and complete basis there.

The condition (2.5) implies for the considered sector that generally

$$P = \int d^3\vec{p}\,p_V(\vec{p})\,|V_{\vec{p}}\rangle\langle V_{\vec{p}}|$$
$$+ \int d^3\vec{p}\,d^3\vec{q}\,p_{N\theta}(\vec{p}, \vec{q})\,|N_{\vec{p}-\vec{q}}, \theta_{\vec{q}}\rangle\langle\theta_{\vec{q}}, N_{\vec{p}-\vec{q}}|. \tag{3.7}$$

Thus the problem of finding the subspace of unstable states in the $\mathscr{H}(1, 1)$-sector resolves itself into a calculation of functions $p_V(\vec{p})$ and $p_{N\theta}(\vec{p}, \vec{q})$. One can do this by means of equations (2.20).

From (2.21) it follows that

$$p_V(\vec{p}) = p_V^2(\vec{p}),$$
$$p_{N\theta}(\vec{p}, \vec{q}) = p_{N\theta}^2(\vec{p}, \vec{q}), \tag{3.8}$$

i.e. that values taken by $p_V(\vec{p})$, $p_{N\theta}(\vec{p}, \vec{q})$ can be equal to 0 a.e. or 1 a.e.; (2.21) implies that $p_V(\vec{p})$, $p_{N\theta}(\vec{p}, \vec{q})$ can be some characteristic functions on momentum space, thus generally

$$0 \leqslant p_V(\vec{p}) \leqslant 1, \quad 0 \leqslant p_{N\theta}(\vec{p}, \vec{q}) \leqslant 1. \tag{3.9}$$

For such P there does not exist any solution of the equation (2.20b) while the equation (2.20a) is solvable and gives

$$\langle 0, \psi'_V|P|\psi_V, 0\rangle - \frac{i}{2\pi}\,\underset{\varepsilon\to 0+}{\text{w-lim}} \int_{-\infty}^{\infty} \langle 0, \psi'_V|R(\lambda - i\varepsilon)\,[P, H_1]$$
$$\times R(\lambda + i\varepsilon)\,|\psi_V, 0\rangle d\lambda = 0, \tag{3.10}$$

and analogously for matrix elements of $\langle 0, \psi'_V|(\cdot)|0, \psi_{N\theta}\rangle$ and $\langle \psi'_{N\theta}, 0|(\cdot)|0, \psi_{N\theta}\rangle$ type. We have

$$R(z)\,|\psi_V, \psi_{N\theta}\rangle \overset{\mathrm{df}}{=} |\varrho_V(\psi_V, \psi_{N\theta}; z), \varrho_{N\theta}(\psi_V, \psi_{N\theta}; z)\rangle$$

$$= \int d^3\vec{q}\,\varrho_V(\psi_V, \psi_{N\theta}; z; \vec{q})\,|V_{\vec{q}}\rangle + \int d^3\vec{p}\,d^3\vec{q}\,\varrho_{N\theta}(\psi_V, \psi_{N\theta}; z; \vec{p}, \vec{q})\,|N_{\vec{p}-\vec{q}}, \theta_{\vec{q}}\rangle, \tag{3.11}$$

where:

$$\varrho_V(\psi_V, \psi_{N\theta}; z; \vec{q}) \overset{\mathrm{df}}{=} \{z-m_V+\varphi(z-m_N)\}^{-1}$$

$$\times\left\{\psi_V(\vec{q})-g_0\int d^3\vec{r}\,\frac{f[\omega(\vec{r})]}{\sqrt{2\omega(\vec{r})}}\cdot\frac{\psi_{N\theta}(\vec{q}, \vec{r})}{\omega(\vec{r})-(z-m_N)}\right\}, \tag{3.12}$$

$$\varrho_{N\theta}(\psi_V, \psi_{N\theta}; z; \vec{p}, \vec{q}) \overset{\mathrm{df}}{=} (z-m_N-\omega(\vec{q}))^{-1}$$

$$\times\left\{g_0\varrho_V(\psi_V, \psi_{N\theta}; z; \vec{p})\,\frac{f[\omega(\vec{q})]}{\sqrt{2\omega(\vec{q})}}+\psi_{N\theta}(\vec{p}, \vec{q})\right\}, \tag{3.13}$$

and

$$\varphi(z) \overset{\mathrm{df}}{=} g_0^2\int\frac{f^2[\omega(\vec{k})]}{2\omega(\vec{k})}\cdot\frac{1}{\omega(\vec{k})-z}\,d^3\vec{k},$$

$$\varphi(x\pm i0) \overset{\mathrm{df}}{=} F(x)\pm\frac{i}{2}\,\Gamma(x). \tag{3.14}$$

From (3.7) and (3.10) we obtain an equation for $p_V(\vec{p})$ and $p_{\theta N}(\vec{p}, \vec{q})$. For $m_V > m_N+\mu$ it has the following form

$$\int d^3\vec{p}\,p_V(\vec{p})\psi'^*_V(\vec{p})\psi_V(\vec{p})\left\{1-\frac{1}{2\pi}\int_\mu^\infty\frac{\Gamma(\lambda)}{h_0^2(\lambda)+\frac{1}{4}\Gamma^2(\lambda)}\,d\lambda\right\}$$

$$-\frac{i}{2\pi}\int d^3\vec{p}\,\psi'^*_V(\vec{p})\psi_V(\vec{p})\int_{-\infty}^{\infty}d\sigma\left\{\int d^3\vec{q}\,p_{N\theta}(\vec{p}, \vec{q})\right.$$

$$\times g_0^2\frac{f^2[\omega(\vec{q})]}{2\omega(\vec{q})}\left[\frac{1}{\omega(\vec{q})-\sigma-i0}-\frac{1}{\omega(\vec{q})-\sigma+i0}\right]$$

$$\left.\times\frac{1}{h_0^2(\sigma)+\frac{1}{4}\Gamma^2(\sigma)}\right\} = 0, \tag{3.15}$$

for all $\psi'_V(\vec{p}), \psi_V(\vec{p}) \in \mathscr{L}^2(d^3\vec{p})$,

$$h_0(\sigma) \overset{\mathrm{df}}{=} \sigma-m_V+m_N+F(\sigma). \tag{3.16}$$

The integral over λ (with $\mu < \lambda < \infty$) multiplied by $\frac{1}{2\pi}$ in the first term of the equation (3.15) is equal to 1 (see: [24]) and the integral over σ in the second term of this equation reduces itself for $p_{N\theta}(\vec{p}, \vec{q}) = \text{const.} \cdot p'_{N\theta}(\vec{p}) > 0$ to the previous one, and generally for $p_{N\theta}(\vec{p}, \vec{q}) > 0$ it differs from zero. This and (3.8) mean that nontrivial solutions of the equation (3.15) are

$$p_{N\theta}(\vec{p}, \vec{q}) = 0 \text{ a.e.}, \tag{3.17}$$

and

$$p_V(\vec{p}) = \chi_S(\vec{p}), \tag{3.18}$$

where: $\chi_S(\vec{p})$ is equal to 1 if $\vec{p} \in S \subset R^3$ and is equal to zero if $\vec{p} \in R^3 \backslash S$, and S is an arbitrary set of non-zero measure in the momentum space R^3, or

$$p_V(\vec{p}) = 1 \text{ a.e.}. \tag{3.19}$$

Equations of type (3.15) for matrix elements $\langle \psi'_{N\theta}, 0| (\cdot) |0, \psi_{N\theta}\rangle$ and $\langle 0, \psi'_V| (\cdot) |0, \psi_{N\theta}\rangle$ do not give any additional information about $p_V(\vec{p})$ and $p_{N\theta}(\vec{p}, \vec{q})$.

Finally, one can come to a conclusion that the operator

$$P = \int d^3\vec{p} |V_{\vec{p}}\rangle \langle V_{\vec{p}}| \tag{3.20}$$

is the maximal projector onto subspace of unstable particles in the $\mathscr{H}(1, 1)$-sector (compare: [26]).

Let $m_V < m_N + \mu$. Then the equation (3.15) and the similar one for other matrix elements have no solutions besides the trivial one:

$$p_V(\vec{p}) = p_{N\theta}(\vec{p}, \vec{q}) = 0 \text{ a.e.}, \tag{3.21}$$

and then

$$P = 0. \tag{3.22}$$

This means that there are no unstable particles in the $\mathscr{H}(1, 1)$-sector if $m_V < m_N + \mu$, as was to be expected.

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