SUB-QUARK STRUCTURE AND THE MAGNETIC MOMENTS OF THE U, D QUARKS IN THE RISHON MODEL*

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The magnetic moments of the U and D quarks are calculated using various spin flavor wave functions assuming that the subquarks are rishons. The interpretation that color is a spin flavor permutation symmetry leads to a correct prediction for μ_U/μ_D. The possibility that the T, V rishons form a flavor doublet is also exploited and the resulting values for μ_U/μ_D using various spin flavor combinations are calculated.

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1. Introduction

One of the great achievements of the quark model is that it predicted the ratio of the proton to neutron magnetic moments to within 2% of the experimental value of -1.47 [1]. Such a result is independent of the space orbital wave functions and depends on the SU(6) octet wave functions of the baryons. With all the interest in composite models it naturally becomes interesting to inquire if this result can be improved upon using a preon or sub-quark model [2–5]. In fact, it becomes a testing ground for composite model builders who are given the liberty to discriminate against models which give results for the quark magnetic moments at variance with experiment. Unless the moments of sub-quarks arise from purely anomalous effects of the very complicated structure of the internal hyper-colour dynamics we are permitted at least on provisional grounds to calculate moments based on symmetry patterns and compare them with those already used in previous calculations. This is the interest of this note and it is suggested below that the Rishon model can be salvaged if the correct spin-flavor function interpretation is given.

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2. *Model dependent magnetic moments*

Consider the following function for the Rishon model [6],

$$\psi_{U_1} = \frac{T(1)T(2)}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]V(3)\alpha(3), \tag{1}$$

where T refers to a 1/3 charge rishon and V refers to a 0 charge rishon, α, β represent spin up, spin down states. We interpret the above function to represent a red colored quark of a triplet combination TTV, TVT, VTT [7]. In this scheme the permutation symmetry determines the color of the quark. If we write

$$\mu_i = \mu_0\left(\frac{e_i}{e}\right)S_{z_i}$$

for the magnetic moment of the i^{th} rischon where μ_0 = *constant*, e_i = charge of the i^{th} rishon, e = electronic charge, S_{z_i} = z component of spin of the i^{th} rishon we have

$$\mu = \sum_{i=1}^{3} \mu_0\left(\frac{e_i}{e}\right)S_{z_i}$$

for the magnetic moment operator of the quark. Now

$$\mu_{U_1} = \langle\psi_{U_1}|\mu|\psi_{U_1}\rangle = 0$$

for the magnetic moment of the red quark. For U_2:

$$\psi_{U_2} = \frac{T(1)V(2)}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]T(3)\alpha(3),$$

$$\langle\psi_{U_2}|\mu|\psi_{U_2}\rangle = \frac{\mu_0}{6},$$

for U_3:

$$\psi_{U_3} = \frac{V(1)T(2)}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]T(3)\alpha(3),$$

$$\langle\psi_{U_3}|\mu|\psi_{U_3}\rangle = \frac{\mu_0}{6}.$$

The color average would be

$$\bar{\mu}_U = \frac{\mu_{U_1}+\mu_{U_2}+\mu_{U_3}}{3} = \frac{\mu_0}{9}.$$

Now consider the $\overline{\mathrm{D}}$ quark as *TVV, VTV, VVT.*

$$\psi_{\overline{\mathrm{D}}_1} = \frac{T(1)V(2)}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]V(3)\alpha(3),$$

$$\langle\psi_{\overline{\mathrm{D}}_1}|\mu|\psi_{\overline{\mathrm{D}}_1}\rangle = \langle\psi_{\overline{\mathrm{D}}_2}|\mu|\psi_{\overline{\mathrm{D}}_2}\rangle = 0,$$

$$\langle\psi_{\overline{\mathrm{D}}_3}|\mu|\psi_{\overline{\mathrm{D}}3}\rangle = \frac{\mu_0}{6},\quad \mu_{\overline{\mathrm{D}}} = \frac{0+0+\dfrac{\mu_0}{6}}{3} = \frac{\mu_0}{18} \tag{2}$$

By C invariance $\mu_{\mathrm{D}} = -\frac{\mu_0}{18}$. Thus $\mu_{\mathrm{U}}/\mu_{\mathrm{D}} = -2$, which agrees with the standard SU [6] result. We have here assumed that spin flavor function can be combined with hyper-color. space function to yield an anti-symmetric result.

Notice that the result for a positron

$$\psi_{e^+} = T(1)T(2)T(3)\frac{1}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]\alpha(3)$$

with

$$\mu_{e^+} = \sum_{i=1}^{3}\mu_0\left(\frac{e_i}{e}\right)S_{z_i}$$

yields a value of $\mu_0/6$. This is obviously at variance with experiment and suggests perhaps strong color, hyper-color dynamics to enhance the magnetic moment of e^+ up to its known value of $e\hbar/2mc$. Such modifications might serve as guidelines to the underlying preon structure. After all if rishons are viable candidates for sub-quarks, the electron would be more dominated by a chiral symmetry than the quarks.

Suppose instead of the above construction we view *T* and *V* as two flavors of an underlying flavor symmetry of the preon dynamics [8]. Whether the flavor symmetry emerges from the hyper-color dynamics is not important, since for at least the quarks, the electromagnetic properties should depend, for the most part, on the spin space flavor function. Suppose the spin flavor function for *T*, *V* rishons composing the U quark is

$$\psi_{\mathrm{U}} = \frac{1}{\sqrt{18}}\left(2\overset{\uparrow\downarrow\uparrow}{TVT}+2\overset{\uparrow\uparrow\downarrow}{TTV}+2\overset{\downarrow\uparrow\uparrow}{VTT}-\overset{\uparrow\downarrow\uparrow}{TTV}-\overset{\uparrow\uparrow\downarrow}{TVT}-\overset{\uparrow\downarrow\uparrow}{TVT}-\overset{\uparrow\downarrow\uparrow}{VTT}-\overset{\uparrow\uparrow\downarrow}{VTT}-\overset{\downarrow\uparrow\uparrow}{TTV}\right) \tag{3}$$

which is the SU_2[*flavor*]$\times SU_2$[*spin*] part of the SU_6 wave function used for quarks in a proton only here we substitute rishons and describe the quark by the same function. Then

$$\mu_{\mathrm{U}} = \langle\psi_{\mathrm{U}}|\mu|\psi_{\mathrm{U}}\rangle = \langle\psi_{\mathrm{U}}|\sum_{i=1}^{3}\mu_0\frac{e_i}{e}S_{z_i}|\psi_{\mathrm{U}}\rangle = \frac{2\mu_0}{9}. \tag{4}$$

Now if we choose

$$\psi_{\bar{D}} = \frac{1}{\sqrt{18}}\left(\overset{\uparrow\downarrow\uparrow}{-2VTV}\overset{\uparrow\uparrow\downarrow}{-2VVT}\overset{\downarrow\uparrow\uparrow}{-2TVV}+\overset{\uparrow\downarrow\uparrow}{TVV}+\overset{\uparrow\uparrow\downarrow}{VTV}+\overset{\downarrow\uparrow\uparrow}{VTT}+\overset{\uparrow\downarrow\uparrow}{VVT}+\overset{\uparrow\uparrow\downarrow}{TVV}+\overset{\downarrow\uparrow\uparrow}{VVT}\right)$$

$$\mu_{\bar{D}} = \langle\psi_{\bar{D}}|\sum_{i=1}^{3}\mu_0\left(\frac{e_i}{e}\right)S_{z_i}|\psi_{\bar{D}}\rangle = 0 \tag{5}$$

which is the SU_2[*flavor*]$\times SU_2$[*spin*] part of the SU_6 wave function used for the neutron only here we use rishons and describe the $\bar{D}$ quark by the same function.

If, however, we choose

$$\psi_{\bar{D}} = \tfrac{1}{2}(TV-VT)(V)(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow),$$

we have

$$\mu_{\bar{D}} = \langle\psi_{\bar{D}}|\sum_{i=1}^{3}\mu_0\left(\frac{e_i}{e}\right)S_{z_i}|\psi_{\bar{D}}\rangle = \frac{\mu_0}{12}.$$

Thus

$$\mu_D = \frac{-\mu_0}{12} \qquad \mu_U = \frac{2\mu_0}{9}, \quad \frac{\mu_U}{\mu_D} = \frac{-24}{9}, \tag{6}$$

and

$$\frac{\mu_P}{\mu_N} = \frac{4\mu_U-\mu_D}{-\mu_U+4\mu_D} = \frac{4\dfrac{\mu_U}{\mu_D}-1}{\dfrac{-\mu_U}{\mu_D}+4} = \frac{-(\frac{24}{9})4-1}{\frac{24}{9}+4} = -1.75. \tag{7}$$

Where μ_P/μ_N is calculated from the SU_6 quark model in terms of μ_U, μ_D. Using the $\bar{D}$ function

$$\psi_{\bar{D}} = \frac{1}{\sqrt{2}}(TV-VT)(V)\left[\sqrt{\tfrac{2}{3}}(\downarrow\uparrow\uparrow)-\frac{1}{\sqrt{6}}(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow)\right], \tag{8}$$

we have

$$\mu_{\bar{D}} = \langle\psi_{\bar{D}}|\sum_{i=1}^{3}\mu_0\left(\frac{e_i}{e}\right)S_{z_i}|\psi_{\bar{D}}\rangle = \frac{\mu_0}{36} \tag{9}$$

and for

$$\mu_D = \frac{-\mu_0}{36}, \quad \mu_U = \frac{2\mu_0}{9}, \quad \frac{\mu_U}{\mu_D} = -8, \quad \frac{\mu_P}{\mu_N} = \frac{-33}{12} = -2.75.$$

Using the function

$$\psi_U = \frac{(TV-VT)T}{2}(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow) \tag{10}$$

for U we find

$$\mu_U = \langle\psi_U|\sum_{i=1}^{3}\mu_0\left(\frac{e_i}{e}\right)S_{z_i}|\psi_U\rangle = \frac{\mu_0}{12}.$$

Using

$$\psi_U = \frac{(TV-VT)T}{\sqrt{2}}\left[\sqrt{\tfrac{2}{3}}(\downarrow\uparrow\uparrow)-\frac{1}{\sqrt{6}}(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow)\right] \tag{11}$$

for the U function we have

$$\mu_U = \langle\psi_U|\sum_{i=1}^{3}\mu_0\left(\frac{e_i}{e}\right)S_{z_i}|\psi_U\rangle = \frac{\mu_0}{6}.$$

For this function and $\mu_D = -\mu_0/12$, we have

$$\frac{\mu_U}{\mu_D} = \frac{\frac{\mu_0}{6}}{\frac{-\mu_0}{12}} = -2, \tag{12}$$

and

$$\frac{\mu_P}{\mu_N} = \frac{\frac{4\mu_U}{\mu_D}-1}{\frac{-\mu_U}{\mu_D}+4} = -1.5. \tag{13}$$

Recalling

$$\left(\frac{\mu_P}{\mu_N}\right)_{EXP} = -1.47, \text{ see Ref. [1].}$$

Table I summarizes the various possible combinations of the U, D spin flavor functions along with the magnetic moments of the U, D quarks they predict and the resultant value of μ_P/μ_N.

TABLE I

Rishon spin flavor function	μ_U[a]	μ_D[b]	μ_U/μ_D[c]	μ_P/μ_N[d]
$\psi_U = \frac{1}{\sqrt{18}}\left(2\overset{\uparrow\downarrow\uparrow}{TVT}+2\overset{\uparrow\uparrow\downarrow}{TTV}+2\overset{\downarrow\uparrow\uparrow}{VVT}-\overset{\uparrow\downarrow\uparrow}{TTV}-\overset{\uparrow\uparrow\downarrow}{TVT}-\overset{\downarrow\uparrow\uparrow}{TVT}-\overset{\uparrow\downarrow\uparrow}{VVT}-\overset{\uparrow\uparrow\downarrow}{VTT}-\overset{\downarrow\uparrow\uparrow}{TTV}\right)$ $\psi_{\bar{D}} = \frac{1}{\sqrt{18}}\left(2\overset{\uparrow\downarrow\uparrow}{VTV}-2\overset{\uparrow\uparrow\downarrow}{VVT}-2\overset{\downarrow\uparrow\uparrow}{TVV}+\overset{\uparrow\downarrow\uparrow}{TVV}+\overset{\uparrow\uparrow\downarrow}{VTV}+\overset{\downarrow\uparrow\uparrow}{VTT}+\overset{\uparrow\downarrow\uparrow}{VVT}+\overset{\uparrow\uparrow\downarrow}{TVV}+\overset{\downarrow\uparrow\uparrow}{VVT}\right)$	$\frac{2\mu_0}{9}$	0	∞	-4
$\psi_U = \frac{1}{\sqrt{18}}\left(2\overset{\uparrow\downarrow\uparrow}{TVT}+2\overset{\uparrow\uparrow\downarrow}{TTV}+2\overset{\downarrow\uparrow\uparrow}{VTT}-\overset{\uparrow\downarrow\uparrow}{TTV}-\overset{\uparrow\uparrow\downarrow}{TVT}-\overset{\uparrow\downarrow\uparrow}{TVT}-\overset{\uparrow\downarrow\uparrow}{VTT}-\overset{\uparrow\uparrow\downarrow}{VTT}-\overset{\downarrow\uparrow\downarrow}{TTV}\right)$ $\psi_{\bar{D}} = \frac{1}{2}(TV-VT)V(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow)$	$\frac{2\mu_0}{9}$	$\frac{-\mu_0}{12}$	$\frac{-24}{9}$	-1.75
$\psi_U = \frac{(TV-VT)T}{2}(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow)$ $\psi_D = \frac{1}{\sqrt{2}}(TV-VT)(V)\left[\sqrt{\frac{2}{3}}(\downarrow\uparrow\uparrow)-\frac{1}{\sqrt{6}}(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow)\right]$	$\frac{\mu_0}{12}$	$\frac{-\mu_0}{36}$	-3	-1.85
$\psi_U = \frac{(TV-VT)T}{\sqrt{2}}\left[\sqrt{\frac{2}{3}}(\downarrow\uparrow\uparrow)-\frac{1}{\sqrt{6}}(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow)\right]$ $\psi_{\bar{D}} = \frac{(TV-VT)V}{2}(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow)$	$\frac{\mu_0}{6}$	$\frac{-\mu_0}{12}$	-2	-1.5
$\psi_U = \frac{1}{\sqrt{18}}\left(2\overset{\uparrow\downarrow\uparrow}{TVT}+2\overset{\uparrow\uparrow\downarrow}{TTV}+2\overset{\downarrow\uparrow\uparrow}{VTT}-\overset{\uparrow\downarrow\uparrow}{TTV}-\overset{\uparrow\uparrow\downarrow}{TVT}-\overset{\downarrow\uparrow\uparrow}{TVT}-\overset{\uparrow\downarrow\uparrow}{VTT}-\overset{\uparrow\uparrow\downarrow}{VTT}-\overset{\downarrow\uparrow\uparrow}{TTV}\right)$ $\psi_{\bar{D}} = \frac{1}{\sqrt{2}}(TV-VT)V\left[\sqrt{\frac{2}{3}}(\downarrow\uparrow\uparrow)-\frac{1}{\sqrt{6}}(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow)\right]$	$\frac{2\mu_0}{9}$	$\frac{-\mu_0}{36}$	-8	-2.75

[a] Magnetic moment of U quark. [b] Magnetic moment of D quark. [c] μ_U/μ_D = ratio of μ_U to μ_D.
[d] μ_P/μ_N = ratio of proton moment to neutron moment.

3. *Conclusion*

We have seen that the interpretation of the three possible spin-paired-flavor rishon functions in terms of color and the resultant average over these for the magnetic moment of the U, D quarks has led to the correct value of $\mu_U/\mu_D = -2$. Thus color might be a spin permutation symmetry. The assumption of a flavor symmetry for *T*, *V* rishons has led us to various possibilities and can lead to the value $\mu_P/\mu_N = -1.5$ if the correct spin-flavor rishon functions are chosen. The calculation of the positron magnetic moment leads to unreasonable results and intimates the existence of strong hyper-color modifications if the Rishon model proves to be correct. Gluck, Ref. [6], has given a brief discussion of the magnetic moments of the quarks using the Rishon model but did not discuss the other color combination. It is encouraging to conclude that if the correct flavor-spin functions are chosen and the color average computed, the Rishon model is in harmony with the quark structure of matter.

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