$\gamma N \to \pi N$ PHOTOPRODUCTION: A MODEL WITH CROSSING SYMMETRY

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Various dispersion relations techniques have been revisited in order to check the recent conjecture of Jurewicz that the u-channel exchange of the Δ (1232) governs the dynamics of the $E^{3/2}_{1+}$ amplitude in the $\gamma N \to \pi N$ reaction. The role of crossing symmetry in this process is clarified and both the $E^{3/2}_{1+}$ and $M^{3/2}_{1+}$ amplitudes are calculated in a crossing-symmetric model.

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1. Introduction

Dispersion relations techniques have been used with success for nearly thirty years in describing photoproduction of pions on nucleons in the low energy region up to the first πN resonance $\Delta(1232)$. With the advent of high precision data it becomes possible to extract the individual monopole amplitudes directly from experiment and these monopoles can be confronted with theoretical predictions. Owing to the inelasticity appearing in some amplitudes at photon lab energy of about 400 MeV, it has become conventional to restrict the comparison of dispersion relations calculations with experiment to photon energies below 450 MeV. In particular, the two dominant monopoles, viz. the magnetic dipole $M^{3/2}_{1+}$ and the electric quadrupole $E^{3/2}_{1+}$ have been evaluated by many workers and compared with experiment. While the magnetic dipole amplitude is well understood, most calculations failed to reproduce the electric quadrupole amplitude even qualitatively in the resonance region. Since the disagreement was most severe at the high energy end, the consensus was that the data apparently called for some extra input information to be used for extending the existing models.

Opposite views have been expressed by Jurewicz who presented recently [1] a parameter free calculation in which the electric quadrupole amplitude was found to be in perfect quantitative agreement with experiment. It has been suggested in [1] that the $\Delta(1232)$ exchange in the u-channel of $\gamma N \to \pi N$ reaction might be an important contributor to the

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dynamics of the $E^{3/2}_{1+}$ amplitude. To verify this conjecture, however, a model with explicit crossing is needed. It should be noted here that in most papers [1, 5, 6] crossing symmetry is maintained only at the initial stage of the calculation but is destroyed later on in result of partial wave projection.

The purpose of this paper is to resolve the controversy about $E^{3/2}_{1+}$ amplitude and to clarify the role of crossing symmetry. We are going to show that the constraint used in [1] to dispose of the polynomial ambiguity follows naturally as the stability requirement imposed on the solution. In contrast to Jurewicz [1] who uses fixed angle dispersion relations, we chose to stick to the fixed t approach devised a long time ago by Blankenbecler and Gartenhaus [3]. The latter model is manifestly crossing-symmetric and allows for an exact treatment of nuclear recoil. On the other hand, the solution of the underlying integral equations given in [3] is mathematically not quite correct. In the following we improve upon this point and come up with the complete solution. The resulting $E^{3/2}_{1+}$ and $M^{3/2}_{1+}$ amplitudes turn out to be in good agreement with a typical dispersion theoretic prediction [2], i.e. we find marked difference with the results presented in [1]. Our finding is that with the crossing symmetry maintained throughout the calculation the u-channel Δ exchange contributes negligibly to either $M^{3/2}_{1+}$ or $E^{3/2}_{1+}$ amplitudes. Consequently, the good agreemen with the data achieved in [1] is not to be attributed to the u-channel Δ exchange and must have a different explanation. We discuss this point in some detail in the Appendix.

2. *Theory*

The analyticity postulate for the four invariant amplitudes of $\gamma N \to \pi N$ reaction implies the dispersion relations

$$D_i(x, v) = B_i^{33}(x, v) + \frac{1}{\pi}\int_{x_0}^{x_c} \operatorname{Im} D_i(x', v)\left\{\frac{1}{x'-x} + \frac{\eta_i}{x'+x+2v}\right\} dx', \tag{1}$$

where, following the notation of Ref. [3]

$$D_i(x, v) = A_i(x, v) - C_i(x, v).$$

$A_i(x, v)$ are the standard amplitudes of photoproduction with $i = 1, \ldots, 4$ and suppressed isospin indices. $C_i(x, v)$ are the comparison functions, chosen according to Ref. [3] to be the small phase shifts contribution to the Born amplitudes $B_i(x, v)$ i.e.

$$C_i(x, v) = B_i(x, v) - B_i^{33}(x, v). \tag{2}$$

$B_i(x, v)$ are taken to be the minimal set of gauge invariant Born amplitudes as defined in Refs [1, 3, 5]. Our variables are $x = (W^2 - M^2)/2M$, where W is the energy in the center of mass system, M is the nucleon mass, $m_\pi = \hbar = c = 1$, $v = -q \cdot k/2M$, $x_0 = 1 + 1/2M$, x_c is a cut off parameter, $\eta_i = \pm 1$ according to the convention adopted in [3].

In order to obtain a unique solution of Eq. (1) several assumptions will be needed. (*i*) The (3.3) phase shift dominance: the phase of $D_i(x, v)$ is assumed to be equal to $\delta_{33}(x)$ for v in the neighbourhood of $v_0 = -(1 + M/(M+1))/4M$.

(*ii*) The contribution from $x > x_c$ energy range is neglected and the minimal set of gauge invariant Born amplitudes is used.
(*iii*) To extract the contributions of Δ(1232) poles in the complex x plane the elastic phase shift δ_{33} of $D_i(x, \nu_0)$ will be extrapolated to π at x_c as was done in Ref. [1].

These assumptions deserve a few words of comment. As a matter of fact, in this model x_c has to be regarded as a parameter and variation of x_c produces important changes in the magnitude of the solutions what is a simple consequence of the fact that $B_i^{33}(x, \nu)$ does not vanish sufficiently rapidly above x_c. This indicates that $B_i^{33}(x, \nu)$ should be somehow modified at higher energies by other effects as will become evident later. A simple modification is suggested by quark models [11, 12], where the πNN strong vertex is shown to contain a formfactor rapidly decreasing with the pion momentum. In the actual calculations x_c was fixed at 770 MeV.

The particular choice of ν_0 in the assumption (*i*) is motivated by the fact that this is the only value of ν for which no unphysical values of the production angle appear on both cuts in the x plane. It is assumed that the phase is slowly varying as a function of ν around ν_0 i.e. $\delta_{33}(x, \nu) \approx \delta_{33}(x, \nu_0) = \delta_{33}(x)$. It should be noted, however, that in general the phase of $D_i(x, \nu)$ does depend on ν what may be easily seen by taking sufficiently large values of ν to make the right (R) and the left (L) hand cuts in the x plane overlap. Then, the phase of $D_i(x, \nu)$ must be zero on $L \cap R$ if one wants to be consistent with Eq. (1).

Under the assumptions (*i*), (*ii*), (*iii*) the complete solution of Eq. (1) may be obtained from the solution of the associated Riemann boundary condition problem [4]. To show that let us define the following function in the complex x plane

$$\Phi_i(x) = \frac{1}{2\pi i}\int_R D_i(t, \nu_0)h^*(t)\left[\frac{1}{t-x} + \frac{\eta_i}{t+x+2\nu_0}\right]dt, \tag{3}$$

where $h^*(t) = e^{-i\delta_{33}(t)}\sin\delta_{33}(t)$, $R = [x_0, x_c]$, $L = [-x_c-2\nu_0, -x_0-2\nu_0]$ and let us introduce several functions defined on $L \cup R$

$$\tilde{\delta}_{33}(x) = \delta_{33}(x),$$

$$\tilde{\delta}_{33}(-x-2\nu_0) = -\delta_{33}(x),$$

$$\tilde{B}_i(x) = B_i^{33}(x, \nu_0),$$

$$\tilde{B}_i(-x-2\nu_0) = \eta_i B_i^{33}(x, \nu_0),$$

$$G(x) = \exp(2i\tilde{\delta}_{33}(x)),$$

$$g_i(x) = \tilde{B}_i(x)e^{i\tilde{\delta}_{33}(x)}\sin\tilde{\delta}_{33}(x) \equiv \tilde{B}_i(x)\tilde{h}(x). \tag{4}$$

In the above definitions of the $\tilde{\delta}_{33}(x)$ and $\tilde{B}_i(x)$ functions x belongs to the $[x_0, x_c]$ interval. These functions provide natural continuations of the $\delta_{33}(x)$ and $B_i^{33}(x, \nu_0)$ defined on R only.

Eq. (1) is equivalent to the following Riemann boundary condition on $L \cup R$

$$\Phi_i^+(x) = G(x)\Phi_i^-(x) + g_i(x), \tag{5}$$

where

$$\Phi_i^\pm(x) \equiv \Phi_i(x \pm i\varepsilon).$$

The crossing symmetry condition may be expressed in terms of $\Phi_i^\pm$ as

$$\Phi_i^+(x) = \eta_i \Phi_i^-(-x-2\nu_0). \tag{6}$$

Although assumption (*iii*) makes the index of the problem formulated in Eq. (5) equal to 2, the crossing symmetry condition (6) reduces the polynomial ambiguity of the solution to a single constant C. The full solution of (5) may be written as

$$\Phi_i(x) = X(x) \cdot \frac{1}{2\pi i}\left[\int_R \frac{\tilde{B}_i(t)\tilde{h}(t)dt}{X^+(t)(t-x)} + \int_L \frac{\tilde{B}_i(t)\tilde{h}(t)dt}{X^+(t)\,(t-x)} + C\right], \tag{7}$$

$$X^+(t) = X(t+i\varepsilon),$$

where $X(x)$ denotes the solution of the homogeneous problem

$$\Phi^+(x) = G(x)\Phi^-(x). \tag{8}$$

It should be stressed that in view of the crossing symmetry condition (6), the homogeneous solutions $X(x)$ are different for the two values of $\eta_i = +1$ and $\eta_i = -1$ (a point not recognized in Ref. [3]). According to the assumption (*iii*) we have

$$X(x, \eta_i = +1) = \frac{1}{(x_c-x)\,(x_c+x+2\nu_0)} \exp\left\{\frac{1}{\pi}\int_R \delta_{33}(t)\left[\frac{1}{t-x} + \frac{1}{t+x+2\nu_0}\right]dt\right\} \tag{9}$$

while for $\eta_i = -1$ (odd under crossing), one obtains

$$X(x, \eta_i = -1) = (x+\nu_0)\cdot X(x, \eta_i = +1). \tag{10}$$

The asymptotic behaviour of our $X(x, \eta_i)$ may be easily deduced from Eq. (3). One sees immediately that for $x \to \infty$ these functions should behave like $1/x$ and $1/x^2$ for $\eta_i = -1$ and $\eta_i = +1$, respectively. Furthermore, $X(x, \eta_i = -1)$ should have a zero at $x = -\nu_0$ what leads to expressions (9) and (10). The important point is that one solves the boundary condition problem (5) or (8) with the additional requirement that the solution should be expressible in the form (3). This definition incorporates properly the crossing symmetry condition. The flaw in Ref. [3] consisted in that only the function even under crossing was used and the homogeneous term in (7) was absent.

The final form of the solutions satisfying assumption (*iii*) may be obtained from

$$D_i(x, \nu_0) = B_i^{33}(x, \nu_0) + 2i\Phi_i^+(x). \tag{11}$$

The constant C is fixed by the requirement that

$$D_i(x_c, \nu_0) = 0, \quad i = 1, \ldots, 4. \tag{12}$$

In order to obtain the $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ amplitudes the solutions (11) were projected onto partial waves and evaluated numerically. The results of these calculations are shown in Figs 1, 2. The obtained energy dependence is similar to that found by Adler [5] and earlier by Finkler [6]. These authors, in fact, fix the polynomial ambiguity in their equations in the same way as it was done in [1] and in the present paper. It is evident that this method

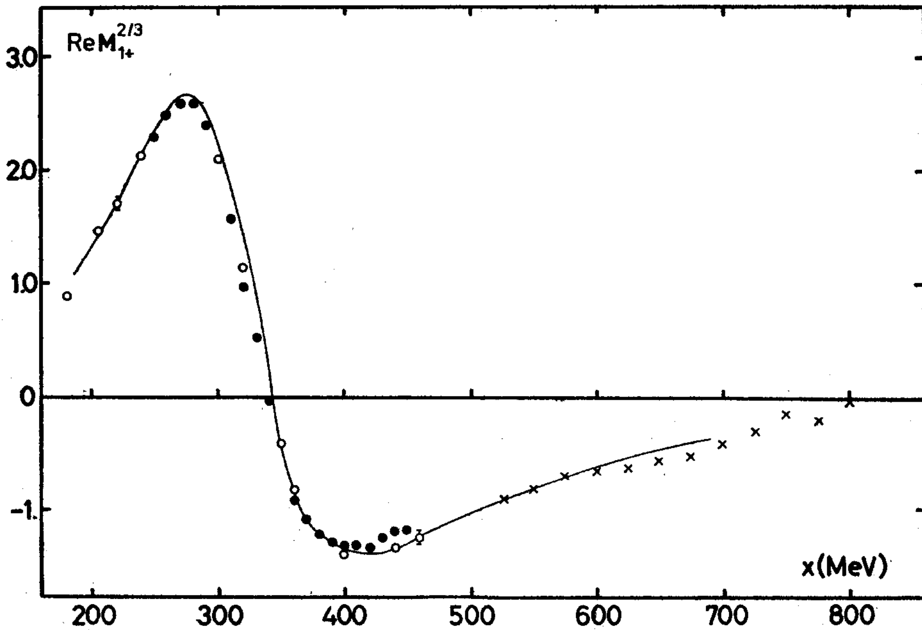

Fig. 1. Re $M_{1+}^{3/2}(x)$ in units $10^{-2}\ \hbar/(m_\pi c)$ vs photon energy in the laboratory frame. Data from Berends and Donnachie [8] (●), Pfeil and Schwela [9] (○), Berends and Donnachie [10] (×)

is the only one consistent with assumption (*ii*) above, for any other choice of the constant in Eq. (7) would produce a finite change in the solutions under an infinitesimal variation $x_c \to x_c - \varepsilon$, $\varepsilon > 0$ with $\delta_{33}(x)$ fixed. Therefore a nonstability of the solutions would appear what is in conflict with assumption (*ii*).

This procedure was also applied with success by Jurewicz [7] to the electroexcitation of the $\Delta(1232)$, emphasizing the crucial role of the homogeneous solution in building the resonant shape of the $M_{1+}^{3/2}$. It is the function $X(x)$ which is responsible for the resonant shape of the $M_{1+}^{3/2}$ in this paper. In the narrow resonance approximation $X(x)$ would give two poles on the real x axis corresponding to both s and u channel resonances. While the s channel pole has important effect on the solution, the crossed channel pole plays no important role because it is far from the physical region and because of the crossing symmetry condition constraining the residues.

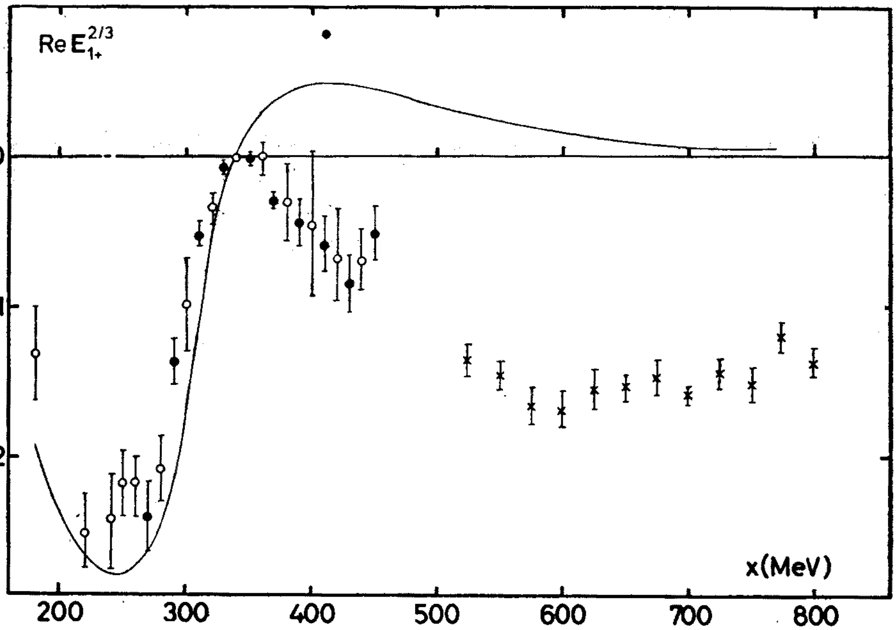

Fig. 2. Re $E_{1+}^{3/2}(x)$ in units 10^{-2} $\hbar/(m_\pi c)$ vs photon energy in the laboratory frame. Data from Refs [8, 9, 10] as in Fig. 1

A common feature of the above mentioned works [1, 5, 6], in contrast to the Blankenbecler-Gartenhaus approach lies in their use of singular integral equations derived for the particular multipole amplitudes what makes the role of crossing symmetry obscure. Although the s channel Δ pole may be included explicitly, it does not seem possible to treat the u channel exchange symmetrically after the partial wave projection has been performed. The reason for the agreement between the results of this paper and those of Refs [5, 6] stems from the smallness of the crossed channel contributions. It should be stressed here that the operations of solving a singular integral equation and performing partial wave projection on it in general do not commute.

3. *Discussion*

In concluding it should be noted that in contrast to the $M_{1+}^{3/2}$ the behaviour of the $E_{1+}^{3/2}$ amplitude above the resonance region is still mysterious. Our solutions could be in principle corrected by successive iterations of the method in which the new comparison functions would be taken from the results of previous calculations [3]. These iterations, however, are not expected to bring significant corrections because they represent the small phase shifts influence on the dominant multipoles.

The couplings of the photon to the $\Delta(1232)$ were described in this work solely in terms of $\gamma\pi N$ dynamics as it was done frequently in the past. This procedure applied consistently has led us to a prediction of $E_{1+}^{3/2}$ amplitude which differs significantly from the experimental data. The foregoing discussion suggests that the assumption (*ii*) is likely to be incorrect and a question arises what effects are responsible for the observed behaviour of the $E_{1+}^{3/2}$. There are several possibilities. The Born terms B_i^{33} may be modified by quarks inside the nucleon coupled directly to photons and to pions. Additional pole terms may also be accounted for, and what seems very probable, the high energy contributions from the dispersion integrals may not be negligible. Finally, the data points in Fig. 2 indicate that inelastic effects may be rather important for $E_{1+}^{3/2}$ as this amplitude clearly does not vanish anywhere in the region of x_c.

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APPENDIX

As far as the outstanding results from [1] are concerned we wish to stress that they have been obtained on the basis of different equations then those used in [3, 5, 6] and also in the present work. The underlying fixed angle scheme used in [1] is not free from serious formal objections. To be more specific, the equations for the invariant amplitudes used in [1] are

$$H_{i0}(v, c) = B_{i0}(v, c)+ \frac{1}{\pi}\int\limits_0^\infty \frac{dv'}{v'-v}\sum_{j=0}^{3} \Omega_j(v, v')\,\mathrm{Im}\, H_{ij}(v', c), \tag{13}$$

where v, c mean in the c.m. system of the s-channel of $\gamma N \to \pi N$ the final three-momentum squared and the cosine of the production angle, j labels the four Riemann sheets of the (v, c) space resulting from the transformation from (s, t, u) to (v, c) variables. B_{i0} are the invariant Born amplitudes and $\Omega_j(v, v')$ are known functions [1] with the asymptotic behaviour $\Omega_j(v, v') \sim v$ for $v \to \infty$ and any finite v'. Let us note, however, that once a cut-off has been introduced, as follows from (13), $H_{i0}(v, c)$ will show a rather unusual asymptotic behaviour, viz. $H_{i0}(v, c) \to$ const for $v \to \infty$. Such a slow fall-off together with the assumption that the high energy tails of the dispersion integrals are negligible is in conflict with the derivation of Eq. (13) itself, unless the above mentioned constant is equal to zero, but this is, of course, not guaranteed by the dispersion equations alone. In consequence, the resulting equations have excessively large kernels producing distortions in $E_{1+}^{3/2}$ which rather fortuitously go in the right direction.

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