FROM STRANGE MATTER TO STRANGE STARS*

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A model of hypothetical strange matter, self-bound quark matter with strangeness per baryon $\cong -1$, which might be the true ground state of cold catalyzed matter, is considered. Potential possibilities of production of strange matter in violent processes occurring in the universe are reviewed. Properties of strange stars — astronomical objects built of strange matter — are described and compared to those of normal neutron stars.

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1. Introduction

We are all used to the fact that stable forms of matter have zero strangeness. Hyperons and hypernuclei are unstable. From the point of view of the fundamental quark structure of matter this amounts to saying that stable matter does not contain strange quarks.

Let us consider the case of matter at $T = 0$ K. The baryonic component of stable matter takes form of atomic nuclei — self-bound aggregates of finite number of nucleons, for which the surface (finiteness) effects play an important role. However, at the matter density as high as that in the interiors of heavy atomic nuclei (mass density $\varrho_0 \cong 2.5 \quad 10^{14}\,\mathrm{g\,cm^{-3}}$, baryon density $n_0 \cong 0.15\,\mathrm{fm^{-3}}$) stable matter can exist only in the form of homogeneous baryon-lepton plasma. At sufficiently high density, above normal nuclear density n_0, matter is expected to undergo some kind of transition from a state where quarks are localized (confined) inside baryons to a deconfined state of homogeneous quark plasma. In contrast to the case of confined baryon phase, quark matter with the strangeness per unit baryon number $S \cong -1$ is energetically preferred over the non-strange ($S = 0$) one. Consider the simplest case of the Fermi gas model of massless quarks at a given baryon density n. It is easy to show that the energy per unit baryon number (energy per baryon), $\mathscr{E}$, in the flavor symmetric, electrically neutral mixture of the u, d, s

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quarks is about 10% lower than that of a neutral mixture of the u, d quarks:

$$\text{fixed } n: \frac{\mathscr{E}(\text{uds}: S = -1)}{\mathscr{E}(\text{ud}: S = 0)} = (\tfrac{2}{3})^{1/4} = 0.904. \tag{1}$$

This is due to the exclusion principle effect. Inclusion of the strange quark mass and of the quark-quark interaction seems to not change this conclusion: strange quark matter is the ground state of the cold quark plasma. Let us notice that at $n = n_0$ the $S = -1$ baryon matter, composed of Λ^0 hyperons, would have an energy per baryon more than 20% higher than the $S = 0$ one, composed of nucleons.

In a recent paper Witten [1] has pointed out that $S \cong -1$ quark matter may be an *absolute* ground state* of matter at zero temperature and pressure (strange matter). The most stable nonstrange configuration is that of the ^{56}Fe crystal

$$\text{at } P = 0: \quad \mathscr{E}(^{56}\text{Fe}) = 930.4 \text{ MeV}. \tag{2}$$

Hence, the condition that the $S \cong -1$ quark matter be an absolute ground state of stable, self-bound (i.e., existing at $P = 0$) matter at $T = 0$ is

$$\text{at } P = 0: \quad \mathscr{E}(\text{uds}) < 930.4 \text{ MeV}. \tag{3}$$

Actually, a *necessary* condition for existence of *strange matter* is much weaker:

$$\text{at } P = 0: \quad \mathscr{E}(\text{uds}) < m_N c^2 \cong 939 \text{ MeV}. \tag{3'}$$

Such an intriguing possibility is not excluded by what we know from laboratory nuclear physics. The experiments tell us only that at $T = 0$, $P = 0$ nuclear matter is more stable than the non-strange quark matter:

$$\text{at } P = 0: \quad \mathscr{E}\,(\text{nuclear matter}) < \mathscr{E}\,(\text{ud}). \tag{4}$$

In the case when Eq. (3) is actually satisfied, ordinary nuclei do not convert to the strange state because of the difficulty in making transition to the strange configuration by a very high order weak interaction.

Let us consider a following example. One way of producing a *stable* drop of strange matter out of an aggregate of A neutrons in neutron matter of density n at a fixed pressure P, would be a deconfinement transition with simultaneous A-th order weak interaction changing A d-quarks into s ones. The A-th order weak interaction should change the strangeness of the droplet by $-A$ on the timescale characteristic of lifetime of the fluctuation leading to deconfinement (i.e. strong interaction timescale). To make things worst, very small droplets of strange matter are unstable because of the surface effects, so that A should be rather large. In the case of atomic nuclei transition is even more difficult.

The possible existence of strange matter seems thus to have no relevance for laboratory, or more generally, "terrestrial" physics. However, the height of the energy barrier separating the normal and strange configuration of matter decreases with increasing pressure. Strange matter could be spontaneously created in the extreme conditions existing in the early universe or in the central core of collapsing massive stars during supernova explosion. It might also exist inside massive neutron stars. Before discussing some of possible astronomical implications of existence of strange matter in the universe, we should consider more realistic models of quark matter.

2. *A model of strange matter*

The quark plasma will be described using the phenomenological MIT bag model (for a review see the les Houches lectures of Baym [2]). We determine the thermodynamic equilibrium of a mixture of the massless u, d quarks and electrons, and s quarks of finite mass m_s. We allow for the transformations mediated by the weak interactions between quarks and leptons. The quark gluon interaction will be included to lowest order in $\alpha_c = g^2/4\pi$. Actually, the renormalized coupling constant varies logarithmically with quark chemical potential. It is easy to see, however, that a consequent neglecting of higher than first order terms can be reconciled with the first law of thermodynamics only for a *density independent* α_c. Therefore, we assume a constant value of α_c. As we shall see, the maximum density in stable static configurations of strange matter is only $4\div5$ times higher than that at $P = 0$. Therefore, our assumption of a constant α_c is quite reasonable, as far as applications to stable, static configurations are concerned.

All quantities considered will be color and spin independent. However, even with this simplification we should consider a four component plasma. Suitable thermodynamic variables for such a multicomponent system are the chemical potentials of the constituents, μ_i (i = u, d, s, e). All thermodynamic quantities can be then determined from thermodynamic potential (per unit volume), Ω. To lowest order in α_c the contributions of u, d quarks are [2]

$$\Omega_f(\mu_f) = \frac{-\mu_f^4}{4\pi^2(\hbar c)^3}\left(1-\frac{2\alpha_c}{\pi}\right), \quad f = \mathrm{u, d}. \tag{5}$$

The electron contribution is given by a standard formula

$$\Omega_e(\mu_e) = -\frac{\mu_e^4}{12\pi^2(\hbar c)^3}. \tag{6}$$

The formula for $\Omega_s(\mu_s)$ for the massive s quark is much more complicated (see, e.g., Farhi and Jaffe [3], but notice that the plus sign before the last term in the second line of their formula for Ω_s should be replaced by minus)

$$\Omega_s(\mu_s) = -\frac{1}{4\pi^2(\hbar c)^3}\Big\{\mu_s(\mu_s^2-m_s^2c^4)^{1/2}(\mu_s^2-\tfrac{5}{2}m_s^2c^4)$$

$$+\tfrac{3}{2}m_s^4c^8\ln\left(\frac{\mu_s+(\mu_s^2-m_s^2c^4)^{1/2}}{m_sc^2}\right)$$

$$-\frac{2\alpha_c}{\pi}\left[3\left(\mu_s(\mu_s^2-m_s^2c^4)^{1/2}-m_s^2c^4\ln\left(\frac{\mu_s+(\mu_s^2-m_s^2c^4)^{1/2}}{m_sc^2}\right)\right)^2\right.$$

$$-2(\mu_s^2-m_s^2c^4)^2-3m_s^4c^8\ln^2\left(\frac{m_sc^2}{\mu_s}\right)+6\ln\left(\frac{\tilde{\varrho}}{\mu_s}\right)\left(\mu_sm_s^2c^4(\mu_s^2-m_s^2c^4)^{1/2}\right.$$

$$\left.\left.-m_s^4c^8\ln\left(\frac{\mu_s+(\mu_s^2-m_s^2c^4)^{1/2}}{m_sc^2}\right)\right)\right]\Bigg\}. \tag{7}$$

The parameter $\tilde{\varrho}$ appearing in the above formula is the renormalization point for the strange quark mass. The standard choice is $\tilde{\varrho} = m_sc^2$. However, in order to consider also a case of low m_s (e.g., $m_sc^2 \leqslant 100$ MeV) it is suitable to choose $\tilde{\varrho}$ to be given by a characteristic, fixed energy scale equal to one third of the nucleon energy, $\tilde{\varrho} = 313$ MeV (see Farhi and Jaffe [3]). This choice of $\tilde{\varrho}$ will be adopted here.

The number densities of the constituents of strange matter can be expressed in terms of μ_i from the thermodynamic relation

$$n_i(\mu_i) = -\frac{\partial\Omega_i}{\partial\mu_i}\ (i = \mathrm{u, d, s, e}). \tag{8}$$

The quark plasma should be in equilibrium with respect to reactions mediated by weak interactions

$$\mathrm{d} \to \mathrm{u}+\mathrm{e}^-+\bar{\nu}_e,\quad \mathrm{u}+\mathrm{e}^- \to \mathrm{d}+\nu_e,\quad \mathrm{s} \to \mathrm{u}+\mathrm{e}^-+\bar{\nu}_e,\quad \mathrm{u}+\mathrm{d} \rightleftarrows \mathrm{u}+\mathrm{s}\ \text{etc.} \tag{9}$$

Assuming that neutrinos escape freely from quark matter we see that the condition of equilibrium with respect to weak interactions leads to two independent relations:

$$\mu_d = \mu_u+\mu_e, \tag{10}$$

$$\mu_d = \mu_s. \tag{11}$$

Strange matter should be electrically neutral. This leads to a third condition

$$\tfrac{2}{3}\,n_u(\mu_u) = \tfrac{1}{3}\,[n_d(\mu_d)+n_s(\mu_s)]+n_e(\mu_e). \tag{12}$$

Conditions (10)–(12) leave us with only *one* independent variable, which we denote by y. The value of pressure corresponding to a given value of y should be calculated taking into account the vacuum pressure of the bag model:

$$P(y) = -\sum_i \Omega_i(y)-B. \tag{13}$$

The values of the energy density, ϱc^2, and baryon density, n, of strange matter can then be calculated from

$$\varrho(y)c^2 = \sum_i (\Omega_i+\mu_in_i)+B, \tag{14}$$

$$n(y) = \tfrac{1}{3}\,[n_u+n_d+n_s]. \tag{15}$$

In order to obtain the equation of state of strange matter, we eliminate y variable from Eq. (13), getting $y = y(P)$, and express ϱ and n as functions of pressure using Eqs (14, 15).

The baryon chemical potential of strange matter, μ, is defined as minimal energy connected with the decrease of the baryon number of strange matter at fixed P by 1. It is given by the formula

$$\mu(P) = \frac{\varrho(P)c^2 + P}{n(P)}. \tag{16}$$

The energy per baryon

$$\mathscr{E} = \varrho c^2/n. \tag{17}$$

Within the framework of this very simple bag model we may calculate the properties of the quark plasma, assuming some values of three phenomenological parameters B, α_c and m_s. The dependence of the bulk properties of quark matter on the values of B, α_c and m_s has been studied in detail by Farhi and Jaffe [3]. In order to give an example of a definite model of strange matter, we shall consider one model from the three-parameter family studied in [3]. For a standard value of $B = 60$ MeV fm^{-3}, the condition for the existence of strange matter, Eq. (2), is satisfied for, e.g., $m_s c^2 = 200$ MeV *and* $\alpha_c < 0.2$. Let us consider a model for $\alpha_c = 0.17$ [4]. For such a model strange matter might exist, with

$$\Delta\mathscr{E} = \mathscr{E}(P = 0: \mathrm{uds}) - M(^{56}\mathrm{Fe})c^2/56 = -1.5 \text{ MeV}. \tag{18}$$

At zero pressure strange matter would have the density

$$n_s = 0.290 \text{ fm}^{-3} (\cong 2n_0), \qquad \varrho_s = 4.82 \cdot 10^{14} \text{ g cm}^{-3}. \tag{19}$$

In general, the values of $\Delta\mathscr{E}$, n_s and ϱ_s depend on the particular choice of the phenomenological parameters B, m_s and α_c. However, the composition of strange matter, and even more generally — of strange quark matter at any pressure — turns out to be quite model independent, provided one has made a reasonable choice $m_s c^2 < 300$ MeV and $\alpha_c < 0.5$. Strange quark phase is a (nearly) flavor symmetric mixture of the u, d, s quarks. The strangeness per baryon is very close to -1. The electron density is always very small, $n_e/n < 10^{-3}$.

3. *Possibilities of production of strange matter in the universe*

Before entering discussion of possible cosmic scenarios of creation of strange matter, let us consider the problem of stability of cold matter at a fixed pressure. We have considered three phases of cold matter: confined nucleon phase, non-strange quark matter and strange quark matter. They will be denoted by N, ud and S, respectively. At a given value of pressure, *stable* phase of matter is that with the least value of the chemical potential μ, which is defined as the minimum energy needed for changing the baryon number of the system by 1. Results of the calculation of $\mu(P)$ for the three phases of cold matter are schematically plotted in Fig. 1. At $P = 0$ nucleon matter is metastable with respect to strange matter, but is stable with respect to the non-strange quark phase. Because of the difficulty

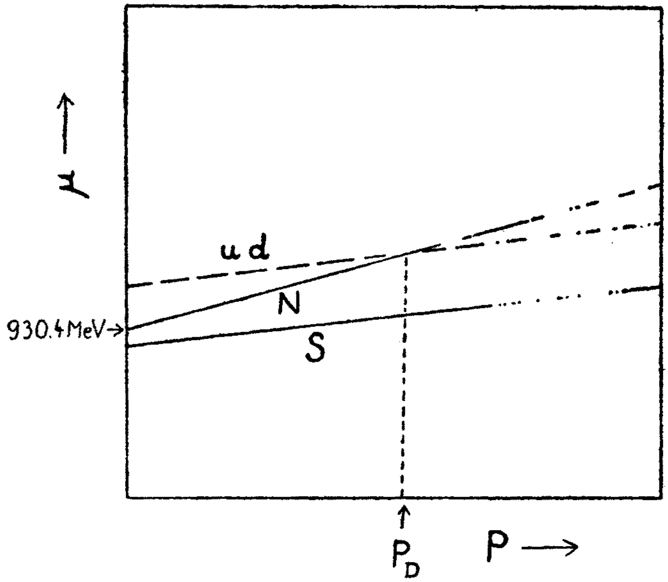

Fig. 1. Chemical potential of cold catalyzed baryonic matter (N), non-strange quark matter (dashed line, ud) and strange matter (S) versus pressure

in making the transition from the N to the S phase, the N phase can be however considered in practice as stable at $P = 0$. The situation changes if we go to high values of P. At some $P = P_D$ one has $\mu_{ud}(P_D) = \mu_N(P_D)$ and deconfinement transition, which does not involve strangeness changing weak interaction, can occur through a first order phase transition.

The ud phase is unstable with respect to weak interactions, which transform it into the S one. In the case strange matter might exist, the appearance of even a small "nucleus" of the uds phase leads to transformation of the whole system built previously of nucleons into a piece of strange matter. Because $\mu_S(P) < \mu_N(P)$ at *any* P there is no possibility of stable coexistence of the S phase and homogeneous N plasma. A very particular case of *metastable* coexistence of the N and S phase will be considered in Section 4.

Our discussion of possible scenarios of the cosmic production of strange matter will be based on Fig. 1.

a) Gravitational collapse of massive stars

Present day theory of stellar evolution tells us that stars with masses greater than 6–8 $M_\odot$ ($M_\odot$ is the mass of the Sun, equal to $1.989 \cdot 10^{33}$ g) end their life in a gigantic implosion of their inner core, which eventually leads to the supernova explosion of whole star. Several years ago it was believed that the maximum density of matter in the collapsing core of a 15 $M_\odot$ star is $2 \div 3\varrho_0$. Recent investigations of the properties of dense matter in imploding stellar cores lead to a much softer equation of state in the $2\varrho_0 < \varrho < 4\varrho_0$ region than the previous one (see, e.g. [5]). Consequently, maximum densities reached in numerical simulations of implosion are larger, up to $5 \div 8\varrho_0$. This compressed matter is heated to $T \sim 10^{11}$ K. In case the density reached is greater than the threshold density for the N $\rightarrow$ ud phase transition, $n_{D(ud)}$, local deconfinement occurs, leading eventually to the appearance of the expanding S matter. Even if the threshold density is not reached, thermal fluctuations may lead to the appearance of a nucleus of the S phase, which then spontaneously grows. The leftovers of some of the supernova explosions could then be strange stars: neutron star-like objects built of strange matter.

b) Massive neutron stars

On the basis of the present evolutionary scenarios, a newly born neutron star is expected to have a mass $M \cong 1.1 \div 1.6\ M_{\odot}$. It is initially very hot, with interior temperature $> 10^{10}$ K, and rapidly rotating. During its evolution neutron star may increase its central density due to such processes as slowing down of rotation, cooling down, or accretion of matter onto its surface. The last process may increase the mass of the star, possibly up to about $2\ M_{\odot}$. If central density reaches $n_{D(ud)}$ then a nucleus of the ud phase may spontaneously appear at the star center and transform into the S phase through the weak interactions. Some massive neutron stars might then transform into strange ones.

c) Neutron star collisions

Close binary system consisting of two neutron stars shrinks due to emission of gravitational waves. Binary pulsar PSR 1913+16 belongs to such a system. The pulsar will collide with its invisible companion after about $3 \cdot 10^8$ years. Huge densities and temperatures reached during such a catastrophic event could lead to the appearance of strange matter. Neutron star collisions seem to be most probable in the central region of the globular clusters. These spherical objects, consisting of up to 10^5 stars, bound by mutual gravitational attraction, are as old as galaxies. Because of their age and lack of the interstellar gas and dust which could be used in the star formation, they contain a rather high percentage of compact stellar remnants (white dwarfs and neutron stars). The density of neutron stars near the center of globular cluster is expected to be so high, that it could be considered as a suitable place for production of strange matter.

d) Confinement transition in the early universe

The shock waves accompanying the confinement transition could lead to the production of some amount of strange matter. This scenario has been advanced in the paper of Witten [1]. However, such a cosmological strange matter, produced at $k_B T \sim 100 \div 200$ MeV evaporates completely by the nucleon emission as the universe cools to $k_B T \sim 10$ MeV [6].

4. *Strange star models*

Strange matter can form stable, self-bound configurations. The properties of microscopic droplets of strange matter (strangelets), characterized by a very low Z/A ratio, were studied by Farhi and Jaffe [3]. Here, we shall consider the properties of macroscopic objects built of strange matter, with maximum size of about 10 km: strange stars.

The models of spherically symmetric configurations of cold strange matter, corresponding to non-rotating strange stars, can be obtained through the numerical integration of the general relativistic equations of hydrostatic equilibrium (see, e.g., Shapiro and Teukolsky [8]). The use of the general relativistic equations is obligatory because of significant relativistic effects for $M > 1\, M_\odot$. The relativistic equations of hydrostatic equilibrium for a spherical configuration of strange matter read:

$$\frac{dP}{dr} = -\frac{Gm\varrho}{r^2}\left(1+\frac{4\pi r^3 P}{mc^2}\right)\frac{1+\dfrac{P}{\varrho c^2}}{1-\dfrac{2Gm}{rc^2}},$$

$$\frac{dm}{dr} = 4\pi r^2 \varrho. \tag{20}$$

For a given equation of state, $P = P(\varrho)$, the equilibrium configuration is completely determined by the value of the central density ϱ_c (or pressure, P_c). The radius of strange star, R, is determined from the condition $P(R) = 0$. The total gravitational mass of the star is $M = m(R)$; it is related to the total star energy (which includes also the rest energy) by $M = E/c^2$. One can also calculate the baryon number of the star, A and stellar moment of inertia, for slow ($\omega^2 \ll GM/R^3$) and rigid rotation, I. In this way one gets a one-parameter family of *all* equilibrium configurations of strange stars: $M(\varrho_c)$, $R(\varrho_c)$, $A(\varrho_c)$, $I(\varrho_c)$, etc. Results presented in Figs. 2–6 have been obtained for the strange matter model described in Section 2 ($B = 60$ MeV fm^{-3}, $m_s c^2 = 200$ MeV, $\alpha_c = 0.17$). In Fig. 2 this family is represented by a curve in the (M, R) plane. Moving along the M–R curve from the ($M = 0$, $R \doteq 0$) point one passes through the subfamily of "golf ball" strange stars with $M \cong \frac{4}{3}\pi R^3 \varrho_s$. The binding of "golf ball" stars is provided not by gravitation, as in the case

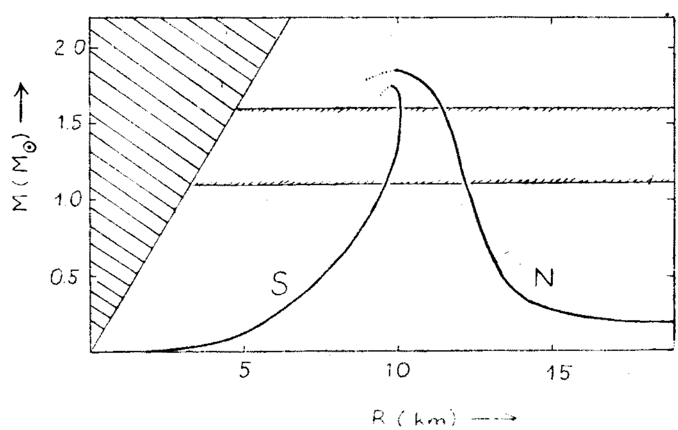

Fig. 2. Gravitational mass, M, plotted versus stellar radius, R, for strange stars (S) and normal neutron stars (N). Configurations to the left of the maximum on each curve are unstable with respect to radial perturbations (dotted line) and thus cannot exist in the universe. Hatched area corresponds to configurations with $R < 2GM/c^2$, for which no equilibrium exists in general relativity. The masses within two horizontal lines correspond to probable range for stable remnants of stellar implosions

of more massive stars, but rather by the QCD interaction. As the gravity starts to dominate, the curve bends upwards. The maximum mass, M_{max}, is reached at $\varrho_c = \varrho_{max}$; for $\varrho_c > \varrho_{max}$ strange stars are *unstable* with respect to small radial perturbations (the necessary condition for stability, $dM/d\varrho_c > 0$ (see, e.g., [8]), is violated) and thus cannot exist in the universe. For our model of strange matter $M_{max} = 1.75\, M_\odot$ and $\varrho_{max} = 2.40 \cdot 10^{15}$g cm^{-3} ($n_{max} = 1.20$ fm^{-3}). In order to compare the properties of strange stars with those of the ordinary neutron stars we show in Fig. 2 the M–R curve obtained for the model I of Bethe

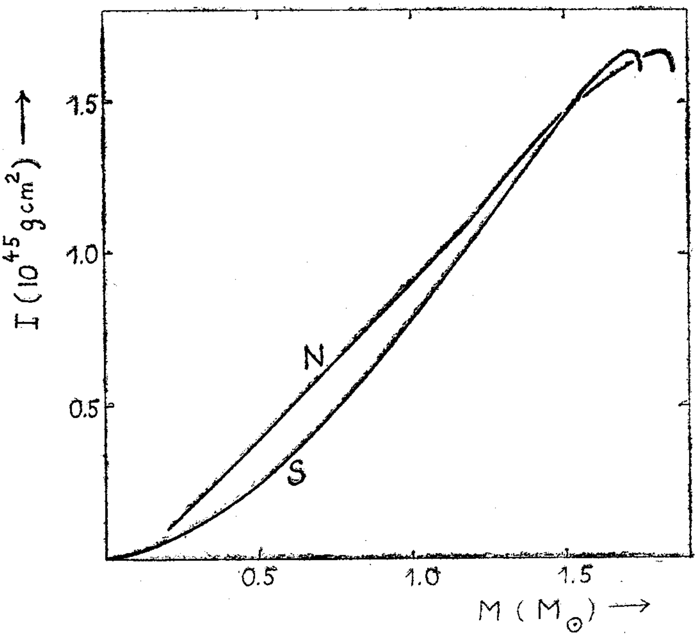

Fig. 3. Moment of inerti a, I, versus stellar mass, M, for strange stars and normal neutron stars

and Johnson equation of state of cold dense baryonic matter [9]. For $M > M_\odot$ both curves are quite similar. The parameters of the extremal configurations with maximum mass are also not very different. Drastic differences concern low mass region, $M < 0.5\, M_\odot$. They are due to the fact that ordinary neutron stars possess a *crust* with surface density $\cong 8$ g cm^{-3}, while strange stars have a superdense surface of the density $\varrho_s \cong 5 \cdot 10^{14}$g cm^{-3}. Small strange stars are well bound by the QCD forces. The low mass neutron stars, consisting mostly of the solid crust, are very loosely bound by gravitation; this leads to the existence of the *minimum* mass for neutron stars, $M_{min} \cong 0.09\, M_\odot$ [8]. There is no (macroscopic) lower bound for the masses of strange stars.

In Fig. 3 we compare the I versus M curves for strange and normal stars. Again, for $M > M_\odot$ the differences are rather small.

The surface redshift, z_s, of dense star is of particular interest, because it is, in principle, an observable quantity. This parameter corresponds to the redshift that would experience a radially propagating photon while travelling from the surface of the star to infinity.

It is related to M and R by a simple formula

$$z_s = (1-2GM/Rc^2)^{-1/2}-1. \tag{21}$$

Surface redshifts of strange stars are systematically lower than those corresponding to normal neutron stars of the same mass (Fig. 4).

According to current theories of the final stages of stellar evolution the mass of the dense stellar remnant left after the supernova explosion of a massive star (or after implosion of an evolving white dwarf which has crossed the Chandrasekhar limit) is expected to be about 1.4 $M_\odot$. This result seems to be corraborated by most of the recent estimates of masses of neutron stars in interacting binary systems [10] as well as by the value of the mass of the binary radio pulsar [11]. The 1.4 $M_\odot$ neutron star may thus be considered as a "standard" one. In Table I we compare the parameters of the strange star and normal neutron star, both of $M = 1.4\ M_\odot$. The radius of strange star is about 20% smaller than that of the normal one. Consequently, the surface redshift for strange star is about 20% larger. Generally, the parameters of the 1.4 $M_\odot$ stars are quite similar. In contrast to normal neutron stars, which have a huge density gradient near the surface, strange stars have a rather flat density profile (Fig. 6). The electron fraction increases when one moves from the star center to its surface.

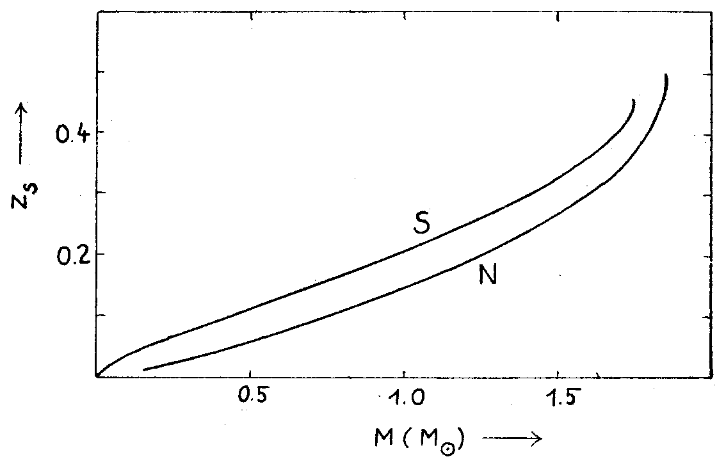

Fig. 4. Surface redshift, Z_s, versus stellar mass, M, for strange stars and normal neutron stars

TABLE I

Stellar parameters of the $M = 1.4\ M_\odot$ configurations for the model I of Bethe and Johnson (N) and strange matter model of Section 2 (S)

	A (10^{57})	I (10^{45} g cm^2)	R (km)	z_s
N	1.841	1.361	11.91	0.238
S	1.937	1.323	10.13	0.300

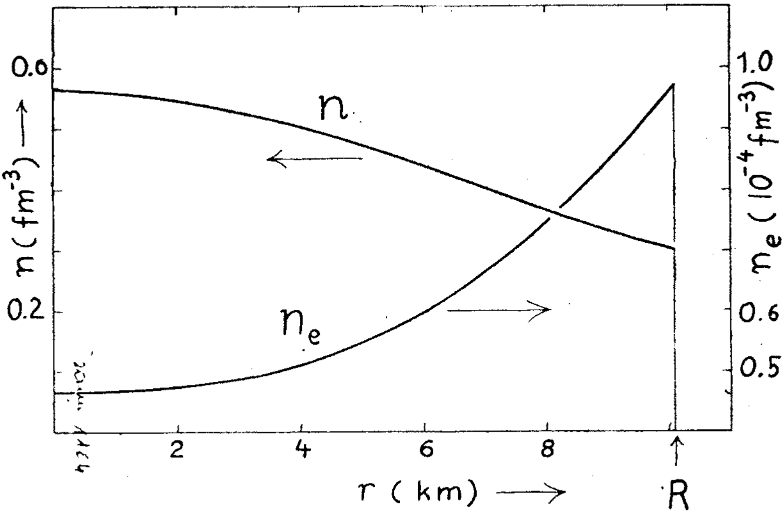

Fig. 5. The baryon density profile, $n(r)$ (in fm^{-3}), and the electron density profile, $n_e(r)$, for the 1.4 $M_\odot$ strange star

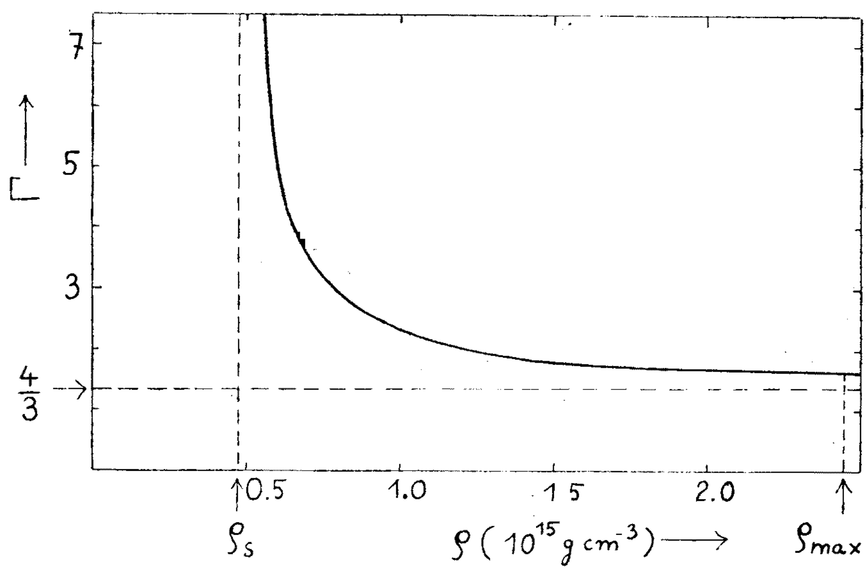

Fig. 6. Relativistic adiabatic index of strange matter, Γ, versus density, ϱ (in the units of 10^{15} g cm^{-3})

The fact that massive strange stars composed of quasi-free, ultrarelativistic quarks are *stable* in the same mass range as those constructed for a rather stiff Bethe Johnson equation of state of baryon matter (model I, Ref. [9]) may seem surprising. For many years, the standard argument for non-existence of quark stars has been that equilibrium configurations of quasi-free ultrarelativistic quarks (adiabatic index = 4/3) do not satisfy the stability condition for relativistic stars. The relativistic adiabatic index of cold matter is defined as

$$\Gamma = \frac{\varrho + P/c^2}{P} \frac{dP}{d\varrho}. \tag{22}$$

To first order in the ratio r_g/R ($r_g = 2GM/c^2$ is the gravitational (Schwarzschild) radius for mass M), which measures the importance of the general relativistic effects, the stability of a stellar model, built of cold matter with Γ = const, with respect to small radial perturbations, requires that (see e.g., [8])

$$\Gamma > \tfrac{4}{3} + \kappa \frac{r_g}{R}. \tag{23}$$

The coefficient κ, which depends on the structure of the star, is of order of unity. From Fig. 1 one finds that r_g/R is as large as 0.3 even for a 1 $M_\odot$ strange star.

Clearly, for $\alpha_c = 0.17$ the first order corrections, included in our model of strange matter, cannot stabilize strange stars. The stability of strange stars results from the stabilizing effect of the confinement forces, represented by the bag constant B. Strange star is a huge bag of the QCD vacuum, immersed in ordinary vacuum. At sufficiently high pressure ($P \gg B$) one has indeed $\Gamma \cong 4/3$. However, for $P \sim B$ the adiabatic index is greater than 2 and increases rapidly with decreasing P (Fig. 6).

The above discussion shows how important for the stability of strange star are the long range quark confining forces represented in the bag model by the bag constant B. Of course, the confinement effects are important only when the deconfinement transition occurs at $P_D \gtrsim B$. For strange matter $P_D = 0$ (if one neglects the metastability of the N phase). For N → ud phase transition one has indeed $P_D \gg B$. If one restricts to the deconfinement transitions in dense cold matter occurring "safely" far from the normal nuclear density, then the adiabatic index is always close to 4/3 and quark stars, composed mostly of quark matter, could not exist.

In view of the crucial role played by the bag constant B, it is interesting to note that the properties of strange stars scale with the value of B in a rather simple way. For the configurations with the maximum mass, one has, within a fraction of a percent (for fixed values of m_s and α_c, [4])

$$M_{max} = 1.75\, M_\odot (B/B_0)^{1/2}, \quad R = 9.81 \text{ km} (B/B_0)^{1/2},$$
$$I = 1.58 \cdot 10^{45} \text{ g cm}^2 (B/B_0)^{3/2},$$

where $B_0 = 60$ MeV fm^{-3}. Actually, the value of B relevant for strange stars is bounded from above. Consider a simplest model of strange matter with massless, noninteracting quarks. The equation of state reads then

$$P(\varrho) = \tfrac{1}{3}(\varrho c^2 - 4B). \tag{24}$$

The self-bound state appears at $\varrho_s = 4B/c^2$, so that $n_s = 0.287$ fm^{-3} $(B/B_0)^{3/4}$. Notice how close is this value of n_s to that obtained for our more realistic model of strange matter with nonzero m_s and α_c! The condition for the self-bound state to be energetically preferred to that of ^{56}Fe crystal reads

$$4B/n_s = 837.2\,(B/B_0)^{1/4}\ \mathrm{MeV} < 930.4\ \mathrm{MeV}. \tag{25}$$

This condition is satisfied for $B < 1.525\,B_0$. For this simplest model of strange matter the maximum mass of strange stars cannot be larger than $2.6\,M_\odot$. The inclusion of finite m_s and α_c lowers somewhat this bound.

5. *Properties of strange stars*

The mass — radius relation for the $M \gtrsim 0.5\,M_\odot$ strange stars is drastically different from that for normal neutron stars. Small, low mass strange stars are stable and could, in principle, exist in the universe. However, their creation would require very violent cosmic scenarios. Examples of such scenarios are: disruption of a *massive* strange star by tidal forces in the vicinity of a massive black hole, collisions of neutron stars in dense old stellar systems like globular clusters and collapse of the binary neutron star system. Such a low mass strange stars could not be a product of stellar evolution, where the compact remnants of $1.1 \div 1.6\,M_\odot$ are expected. As far as the observational criteria are concerned, we may only say that a compact object with, say $M = 0.1\,M_\odot$ and $R < 10$ km, and which for sure is not a black hole (i.e., which, e.g., radiates from its surface) cannot be but a low mass strange star. How to detect such an object is of course an open problem.

If strange stars are born as an outcome of stellar evolution, then they are initially very hot. Strange stars born in some (?) supernova explosions are expected to have an initial internal temperature $T \sim 10^{11}$ K. Newly born normal neutron stars have similar internal temperatures. However, their subsequent cooling is dramatically different. For internal temperature above 10^9 K, the dominating mechanism of cooling is in both cases the neutrino emission. For $T \lesssim 10^{10}$ K both neutron star matter and strange matter are transparent for neutrinos because the neutrino mean free path in dense hot medium is then larger than the stellar radius. Hot strange matter has a neutrino emissivity dramatically larger than that of ordinary neutron star matter [12]. The neutrinos are produced in strange matter mainly in reactions:

$$\mathrm{d} \to \mathrm{u} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}, \quad \mathrm{u} + \mathrm{e}^- \to \mathrm{d} + \nu_\mathrm{e}. \tag{26}$$

For noninteracting, massless quarks the energy and momentum conservation requires that all the particle momenta must be collinear. One can show that in such a case the transition amplitude for Eq. (26) vanishes. However, the matrix element is *finite* if one takes into account quark-quark interaction: energy and momentum can then be conserved for a finite angle between the momenta of particles participating in reactions (26). The neutrino emissivity of strange matter (the energy radiated per 1s from 1 cm^3 of strange matter) is given by the formula [12] ($T_{10} = T/10^{10}$ K, $Y_e = n_e/n$)

$$\varepsilon_\nu^s \cong 4 \cdot 10^{30} \frac{\alpha_c}{0.1} \frac{n}{n_0} \left(\frac{Y_e}{10^{-4}}\right)^{1/3} \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}\, T_{10}^6. \tag{27}$$

For $\alpha_c = 0.17$, $n = 3n_0$ and $Y_e = 10^{-4}$ (typical values for our model of strange matter, considered in Section 3) we have $\varepsilon_\nu^s = 2 \cdot 10^{31}\, T_{10}^6$ erg/cm³ s.

In the case of normal neutron star matter with $Y_e < 0.08$ the simplest processes

$$n \rightarrow p+e^- +\bar{\nu}_e, \quad p+e^- \rightarrow n+\nu_e \tag{28}$$

are blocked because of impossibility of satisfying simultaneously the momentum and energy conservation. Consider the beta decay reaction. Let us denote energy and momentum of initial state by E_i, p_i. Because we are dealing with degenerate n, p, e fluids, the energies and momenta of proton and electron produced in the decay cannot be smaller than the corresponding Fermi energies and momenta: $E_p \geqslant E_{Fp}$, $p_p \geqslant p_{Fp}$, $E_e \geqslant E_{Fe}$, $p_e \geqslant p_{Fe}$. The energy in the final state is thus $E_f \geqslant E_{Fp}+E_{Fe}+E_\nu$. However, energy of decaying neutron can be *at most* E_{Fn}. The chemical equilibrium condition requires that $E_{Fn} = E_{Fp}+E_{Fe}$ and in order to satisfy the energy conservation one should thus have $E_n = E_{Fn}$, $E_p = E_{Fp}$, $E_e = E_{Fe}$ so that the neutrino energy is at most of the order of the thermal energy: $E_\nu \sim k_B T$. Neglecting neutrino momentum one has thus $p_{f,\max} = p_{Fe}+p_{Fp}$ which for $Y_e < 0.08$ is significantly *less* than the neutron Fermi momentum. For $Y_e < 0.08$ the momentum carried by a decaying neutron is too large to be distributed between the decay products.

The beta decay (and its inverse) can proceed only in the presence of an additional nucleon interacting with decaying neutron (or with absorbing proton), which can carry out the momentum excess

$$n+n \rightarrow p+e^- +\bar{\nu}_e, \quad p+n+e^- \rightarrow n+n+\nu_e. \tag{29}$$

Compared to reactions (27) and (28), reactions (29) involve an additional nucleon in the initial and final state. Because of the Pauli exclusion principle effect for degenerate neutrons, protons and electrons, this leads to additional T^2 factor in the emissivity resulting from reactions (29), which at $n = n_0$ reads [13]

$$\varepsilon_\nu^N \cong 10^{29} T_{10}^8 \,\frac{\text{erg}}{\text{cm}^3\,\text{s}}\,. \tag{30}$$

Neutrino emissivity of strange matter exceeds that of normal neutron star matter by a factor of ~ 100 at $T = 10^{10}$ K and by four orders of magnitude at $T = 10^9$ K. Actually, the neutrino emissivity of neutron star matter for $T < 10^{10}$ K is smaller. If one takes into account the superfluidity of nucleons, the neutrino emissivity is reduced by a factor $\exp\{-[\Delta_n(0)+\Delta_p(0)]/k_B T\}$ where $\Delta_n(0)$ and $\Delta_p(0)$ are the ground-state superfluid gap energies for the neutrons and protons, respectively [13].

Cooling of strange star by neutrino emission is very rapid. In order to discuss the qualitative features of thermal evolution of strange stars let us consider an approximate constant density model of our 1.4 $M_\odot$ star with $n = 3n_0$ and $Y_e = 1.4 \cdot 10^{-4}$. For such a model, internal temperature of a several minutes old strange star evolves according to a power law [14]

$$T = 1.1 \cdot 10^9 \left(\frac{t}{1\text{h}}\right)^{-1/4} \text{K}. \tag{31}$$

The temperature within the star is constant, independently of initial conditions. Strange quark star cools down to 10^9 K after $\sim$1 h. In order to follow the cooling process for $T < 10^9$ K one should include the effect of photon cooling. Photons diffuse out from the hot interior and are radiated from the star surface. This problem has been considered in [14]. For our simplified model of strange star the cooling process can be studied by numerical integration of the equation for the thermal evolution

$$\frac{\partial}{\partial t} U = H - \varepsilon_\nu, \tag{32}$$

where internal energy of a unit volume $U = \frac{1}{2} \tilde{c}_v T^2$. The last term describes the neutrino radiation, $\varepsilon_\nu = \tilde{\varepsilon}_\nu T^6$, H describes the heat transport by diffusion or convection, and $\tilde{c}_v$ and ε_ν are constants. Eq. (15) should be solved for $T(r, t)$, with an appropriate boundary condition at $r = R$, starting with an assumed initial temperature profile. Model solutions to Eq. (32) have been obtained in the purely diffussive regime (i.e. assuming no convection in strange matter). In this case

$$H = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \kappa \frac{\partial T}{\partial r} \right), \tag{33}$$

where κ is the thermal conductivity of strange matter, which can be written in the form $\kappa = \tilde{\kappa}/T$, $\tilde{\kappa}$ being in our case a constant. The calculation of thermal conductivity of strange matter has been recently done by Haensel and Jerzak [15]. Heat transport in strange matter is limited by a color-screened QCD interaction. At $n = 3n_0$ one gets for $\alpha_c = 0.17$, $\tilde{\kappa} = 10^{33}$ erg/cm s. If one assumes that the strange star surface radiates like a black body, then after 8 days of nonisothermal evolution the star becomes isothermal. Actually, this time is much shorter because the star turns out to be unstable to convection. If strange star radiates like a black body then it cools down to 10^5 K after $\sim$5000 yr [14] (because of the thermally insulating solid crust normal neutron star would need $\sim 10^7$ yr to cool down to the same surface temperature!). Actually, the cooling could be much slower than this. The plasma frequency of strange matter is of the order of tens of MeV and thus, at first glance, strange matter seems to be a very poor radiator of the thermal keV photons [7]. The problem of the photon cooling of strange stars for $T < 10^9$ K requires a further study.

If strange stars are born in some (?) supernova explosions then because of enormous electric conductivity of strange matter they should possess huge frozen-in magnetic field. In this respect strange stars could be used as models of pulsars. The calculation of electric conductivity, σ, of strange matter has been recently done by Haensel and Jerzak [15]. Charge transport in strange matter is dominated by quarks. The value of σ is determined by the color-screened QCD interaction. For $\alpha_c = 0.17$ and $n = 3n_0$ one gets $\sigma = 10^{29} T_{10}^{-2}$ s^{-1}. At room temperature electric conductivity of strange matter is thus seventeen orders of magnitude larger than that of copper! However, it is only several times larger than electric conductivity of normal neutron star matter of the same density and temperature. In the case of neutron star matter the charge carriers are electrons.

The magnetic field of strange stars will decay due to ohmic dissipation of currents in strange matter. For the dipole magnetic field the decay time is (see, e.g., [16])

$$\tau_D \cong \frac{4\sigma R^2}{\pi c^2}. \tag{34}$$

For $T < 10^9$ K and $R = 10$ km we get $\tau_D > 4 \cdot 10^{11}$ yr. The ohmic dissipation of magnetic field in a strange star is thus negligible.

The observational data for radio pulsars seem to indicate that their magnetic field decays on the timescale $\sim 10^6$ yr. One may thus be tempted to conclude, that pulsars cannot be strange stars. However, similar apparent contradiction is also characteristic of realistic models of normal neutron stars. Therefore, it seems unwise to use the magnetic field decay argument as an observational evidence against the existence of strange stars in the universe.

Simplest models of strange stars, considered in the present paper, have a quark surface. Consider a 1.4 $M_\odot$ strange star. Strange quark matter, confined to a huge bag of 10 km radius, contains a small admixture of electrons. Near the surface we have $n_e \cong 10^{-4}\ \mathrm{fm}^{-3}$ (Fig. 5). Electrons, which do not feel the existence of the MIT bag, should be confined to the star volume by electromagnetic forces. The confinement of electrons is supplied by electric forces of the net positive charge of quarks. The maximum kinetic energy of degenerate electrons in strange star near the stellar surface is in our case $\mu_e - m_e c^2 = 27$ MeV. This number gives us an estimate of the height, V, of the electric potential barrier that confines electrons to the star volume. We have $V_e \cong V$ inside the star. The shape of the potential barrier has been calculated by Alcock et al. [7]. They show that V_e extends about 10^{-10} cm outside the quark surface: this is the thickness of the surface of the strange star. The directed outwards electric field at the strange star surface is estimated as $\sim 10^{18}$ V/cm [7].

Energetics of accretion of plasma onto the strange star surface is in some cases somewhat different from that for normal stars. Neglecting the magnetic field, the kinetic energy of a freely falling proton at the moment it hits quark surface is estimated as

$$E_{\mathrm{kin}} = \frac{GMm_p}{R} - Ve. \tag{35}$$

For our 1.4 $M_\odot$ strange star $GMm_p/R = 200$ MeV and electrostatic barrier is easily overcome. When the proton touches the surface, its bag fuses with the stellar bag. This fusion is an exothermic process, with energy release

$$\varepsilon_f = m_p c^2 - 2\mu_u - \mu_d. \tag{36}$$

For our model we get $\varepsilon_f = 37$ MeV. The infalling plasma is electrically neutral and, in average, each reaction of fusion of a proton with stellar bag is accompanied by absorbing of an electron, with an energy release $\cong Ve$. The total heating rate at the strange star surface is thus

$$W = \left(\varepsilon_{\rm h} + \frac{GMm_{\rm p}}{R}\right)\frac{dA}{dt}, \quad \varepsilon_{\rm h} = \varepsilon_{\rm f} - \mu_{\rm e} + m_{\rm e}c^2, \tag{37}$$

where $\varepsilon_{\rm h} dA/dt$ is an additional term, characteristic of strange star.

In the case when the plasma is accreted via accretion disc, protons may be stopped by the electric forces at the strange star surface. Electric forces prevent accreted protons from touching the stellar bag and a normal matter envelope can build up [7]. However, the thickness of this envelope is strongly limited by the condition of a metastability of the inner edge of normal matter with respect to nucleon fusion with stellar bag. The resulting normal crust would be very thin, because the maximum density of the crust could not reach the neutron drip point ($4 \cdot 10^{11}$ g cm^{-3}). Normal solid crust could not be thicker than about 100 m. This is the thickness of the "outer" solid crust with $\varrho < 4 \cdot 10^{11}$g cm^{-3} in the normal 1.4 $M_\odot$ neutron star. The photon cooling of the strange star with a solid crust would be quite similar to that of the normal neutron star: the temperature gradient in normal neutron star builds up in the $\varrho \lesssim 10^{10}$ g cm^{-3} outer crust, which is similar to that which could enclose strange stars. Notice, that such a thin crust could not contribute significantly to the ohmic dissipation of the strange star magnetic field.

Simplest models of bare strange stars (without solid crust) do not provide any possibility of explaining the pulsar glitches. A 100 m thick solid crust seems to be too thin to reproduce observed glitches by abrupt modification of its structure.

Compact supernova remnant is expected to be born in a state of large amplitude vibrations. Internal instabilities in evolving dense stars can also lead to the excitation of their vibrations. Pulsational properties of strange stars turn out to be quite different from those of the normal neutron stars [17]. In both cases the nonradial vibrations are very effectively damped by the emission of gravitational radiation, typically on the timescale ~0.1 s for a 1 $M_\odot$ star [19]. We shall thus restrict to the case of radial pulsations. Consider the radial pulsation of a strange star. The order-of-magnitude estimate of pulsation period (see, e.g., [18]) is $\tau_{\rm p} \sim (G\varrho)^{-1/2} \sim 10^{-4}$ s. Consider a small element of strange matter inside strange star. In absence of vibrations matter is in equilibrium with respect to reactions

$$\mathrm{d} \to \mathrm{u} + \mathrm{e}^- + \bar{\nu}_{\rm e}, \quad \mathrm{u} + \mathrm{e}^- \to \mathrm{d} + \nu_{\rm e}, \tag{38a}$$

$$\mathrm{u} + \mathrm{s} \rightleftarrows \mathrm{u} + \mathrm{d}, \quad \mathrm{s} \to \mathrm{u} + \mathrm{e}^- + \bar{\nu}_{\rm e}, \quad \mathrm{u} + \mathrm{e}^- \to \mathrm{s} + \nu_{\rm e}. \tag{38b}$$

The equilibrium condition corresponds to the relations involving chemical potentials: $\mu_{\rm d} = \mu_{\rm u} + \mu_{\rm e}$, $\mu_{\rm s} = \mu_{\rm d}$. In presence of pulsations the element of strange matter undergoes periodic compressions and rarefactions. Reactions (38a) involve only ultrarelativistic particles and thus a change of the density with respect to equilibrium value does not alter the relation $\mu_{\rm d} = \mu_{\rm u} + \mu_{\rm e}$ (all $\mu_{\rm i}$ are proportional to $\varrho^{1/3}$). However, because of finite $m_{\rm s}$, the value of $\mu_{\rm s}$ increases with ϱ less rapidly than $\mu_{\rm u}$, $\mu_{\rm d}$ and $\mu_{\rm e}$: pulsation disturbs the equilibrium with respect to reactions involving s quark (Eq. (38)). In accordance with le Châtelier principle, following reactions occur:

$$\left.\begin{array}{l} u+e^- \rightarrow s+\nu_e, \\ u+d \rightarrow u+s, \end{array}\right\} \quad \delta P > 0$$

$$\left.\begin{array}{l} s \rightarrow u+e^- +\bar{\nu}_e, \\ u+s \rightarrow u+d. \end{array}\right\} \quad \delta P < 0 \qquad (39)$$

In the case of degenerate strange matter these reactions are so slow that the equilibrium value for strange quark fraction cannot be reached on the timescale $\sim 10^{-4}$ s [17]. The system is out of equilibrium and dissipation of mechanical energy occurs. The reactions $u+s \leftrightarrows u+d$ turn out to be a main source of dissipation. This gives a very effective mechanism of damping of strange star vibrations. For $m_s c^2 = 150$ MeV and $M = 1\ M_\odot$, pulsation amplitude $\delta R/R = 10^{-2}$ is damped to 10^{-3} in less than 0.05 s. Strange stars are pulsationally dead. Let us notice that in the case of a massive neutron star, similar damping due to slowness of the β decay and electron capture reactions (even in the presence of a $\frac{1}{2} R$ pion condensed core) would take years [17]. The presence of the hyperons in a neutron star core could lead to a much more effective damping. The energy dissipation is then due to the lag of equilibration resulting from slowness of reactions $n+n \rightarrow p+\Sigma^-$, $p+\Sigma^- \rightarrow n+n$. If Σ^- hyperons are present in a $\frac{1}{2} R$ core, then damping from $\delta R/R = 10^{-2}$ to $\delta R/R = 10^{-3}$ occurs in about 1 s [20]. However, recent calculations of the neutron star models for "standard" realistic equations of state indicate that the presence of a significant number of hyperons in a 1.4 $M_\odot$ (or less massive) neutron star is unlikely.

6. *Conclusions*

The possibility of existence of stable self-bound strange matter could have important consequences for neutron stars: some compact, dense stars could be strange stars. Discovery of a strange star in the universe would be a confirmation of the validity of our present theory of the structure of matter. In view of this, we should look for the signatures of strange stars. Univocal and detectable signature of strange star would be a key to its possible detection.

Low mass strange stars are much smaller than neutron stars. There is no lower limit for strange star mass.

Newly born strange stars are much more powerful emitters of neutrinos than neutron stars. However, this property is also characteristic of neutron star with a large quark core.

Pulsations of newly born strange star are damped in a fraction of second: after this time copious neutrino flux from them should not show pulsating features. Unfortunately, the same would be true for the neutrino emission from a neutron star with a large quark core.

Photon cooling of bare strange star could be significantly different from that of neutron stars. If quark surface is not an extremely poor emitter of photons, then absence of insulating crust could lead to a relatively fast photon cooling. In principle, a well established upper limit of surface temperature of neutron star-like object of known age, which is well below the estimates for an object with crust, could be a signature of a bare strange star. A neutron star-like object *with crust* which is 10^4y old cannot have a surface temperature lower than 10^6 K. Unfortunately, surface (black-body) emission flux decreases as a fourth power of temperature. The present day X-ray satellite detector cannot detect an object of 10 km radius at 100 pc = $3 \cdot 10^{15}$ km (typical distance to the nearest observed point X-ray sources) if its surface black body temperature is less than a few times 10^5 K.

The search for the detectable signatures of strange stars should be continued. After all, chances of producing strange matter in our laboratories are negligibly small compared to those for its creation during 10^{10} years in the immense cosmic laboratory of the universe.

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Note added in proof. Severel important papers concerning strange matter have been published after submission of the present paper for publication. Madsen et al. [21] reconsidered the problem of evaporation of strange matter in the early universe. Bethe et al. [22] presented some arguments against the existence of strange stars. Kaplan and Nelson [23] pointed out an intriguing possibility of producing strange matter via kaon condensation in dense nucleonic matter.

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