ANALYTIC BREMSSTRAHLUNG INTEGRATION FOR THE PROCESS $e^+e^- \to \mu^+\mu^-\gamma$ IN QED

BY O. M. FEDORENKO

Department of Physics and Mathematics, State University of Petrosavodsk

AND T. RIEMANN

Joint Institute for Nuclear Research, Dubna*

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The photon bremsstrahlung correction to the differential cross-section and to the charge asymmetry A_{FB} of e^+e^--annihilation into two fermions has been integrated analytically over the complete photon phase space and is discussed in the context of the standard electroweak theory.

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1. Introduction

One of the most important reactions observed in e^+e^--storage rings is the muon pair creation, or, generally, the creation of two fermions:

$$e^+e^- \to f^+f^-. \tag{1}$$

This process provides a unique possibility to study the standard electroweak theory [1] over a wide range of energies. Taking into account radiative corrections of order α^3, it is unavoidable to study in parallel the process

$$e^+e^- \to f^+f^-\gamma, \tag{2}$$

where the fermion pair created is accompanied by a bremsstrahlung photon. The analysis of (1) relies heavily on the differential cross-section $d\sigma/d\cos\theta$ with respect to the scattering angle θ. For unpolarized beams one usually determines the total cross-section

$$\sigma_{\mathrm{tot}} = \int\limits_{-1}^{+1} d\cos\theta \frac{d\sigma}{d\cos\theta}, \tag{3}$$

* Address: Joint Institute for Nuclear Research, Dubna, 101 000 Moscow, Head Post Office, P.O. Box 79, USSR.

and the forward-backward asymmetry,

$$A_{FB} = \frac{1}{\sigma_{tot}}\left[\int_0^1 d\cos\theta \frac{d\sigma}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\sigma}{d\cos\theta}\right]. \tag{4}$$

The total cross-section σ_{tot} is sensible to the C-even contributions, and the charge asymmetry A_{FB} measures the C-odd terms.

In this paper we study the bremsstrahlung contributions (2) to the observables (3, 4) within the standard electroweak theory. We obtain analytic expressions for $d\sigma/d\cos\theta$, A_{FB} and σ_{tot}, where the bremsstrahlung integration has been done over the complete photon phase space, beginning here with the QED corrections which are most easily obtained.

There are two extreme approaches to hard bremsstrahlung problems. One is the consequent numeric integration of the squared matrix element by Monte-Carlo (MC) methods as has been highly developed by a large theoretical collaboration [2]. There is no doubt that MC-integrated cross-sections are of great value for applications due to their flexibility concerning experimental cuts. The value of analytic results (the other extreme) is two-fold. Of course, it is desirable to get analytic results on simple processes even if they are not simple to obtain. Further, one may use the analytically integrated hard bremsstrahlung and subtract by MC-integration the not needed phase space regions to get a com-

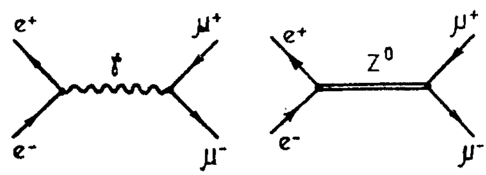

Fig. 1. Born diagrams for $e^+e^- \to f^+f^-$ in the electroweak standard theory

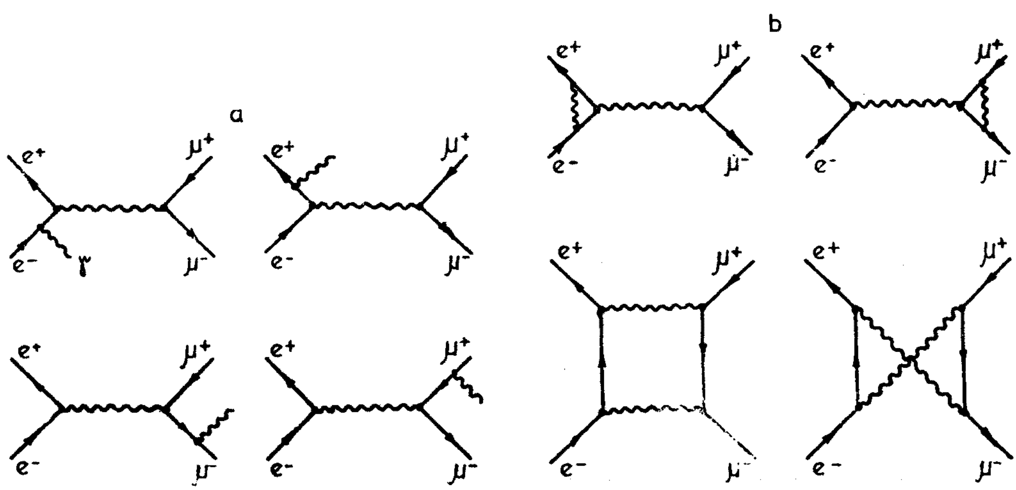

Fig. 2. Gauge invariant and infrared finite set of QED bremsstrahlung (a) and one loop diagrams (b)

pletely independent theoretical prediction for cross-sections with realistic cuts. Analytical integrations have been done by several groups. The first result on $d\sigma/d\cos\theta$ is in [3]. Expressions applicable over the whole relativistic energy range including the region of the Z-boson pole were given in [4] in which several distributions have been presented but not the $\cos\theta$-spectrum, and in [5] where the problem raised here has been studied but compact analytic expressions were not given.

In the present article, we derive analytic results on the pure QED bremsstrahlung, Fig. 2a, in connection with the Born cross-section of the standard electroweak theory, Fig. 1. Adding the QED virtual corrections of Fig. 2b we get a gauge invariant infrared finite set of diagrams. Inclusion of the fermionic vacuum polarization would complete the QED radiative corrections. The region of applicability of our approximation is defined by the relative magnitude of the two Born diagrams as functions of s. It is well-known that at PETRA-energies the electroweak radiative corrections and the genuine Z-boson exchange Born cross-section are quite small with the exception of QED corrections. So, for $s \lesssim 1600\ \mathrm{GeV}^2$ the dominant contributions are included. As already stated above, we also assume that m_e^2, $m_f^2 \ll s$.

The article is organized as follows. In Section 2 we introduce the notation and analytic results, Section 3 contains numerical results, and in Appendices A–C some formal intermediates are presented which are of interest also for the derivation of expressions for γZ--interference and pure weak integrated photon bremsstrahlung.

2. *Analytic results*

The cross-section for (1) together with (2) is in the adopted approximation:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left\{Q^2\left[\left(1+\frac{\alpha}{\pi}F_{\mathrm{vp}}\right)+\frac{\alpha}{\pi}(F_0+QF_1+Q^2F_2)\right]\right.$$
$$+2\,\mathrm{Re}\,\chi|Q|\,[v_e v(1+\cos^2\theta)+2a_e a\cos\theta]$$
$$\left.+|\chi|^2\,[(v_e^2+a_e^2)(v^2+a^2)(1+\cos^2\theta)+8v_e a_e va\cos\theta]\right\}. \tag{5}$$

Here, $s = 4E^2$, and θ is the emission angle of the created fermion f^+ with respect to the e^+-beam axis in the cms. Q, v, a are the charge, vector and axial. vector couplings, respectively ($Q_\mu = -1$):

$$v = 1-4s_W^2|Q|, \quad a = 1. \tag{6}$$

The relative weight of photon and Z-boson exchange is

$$\chi = \frac{1}{16s_W^2c_W^2}\,\frac{s}{s-M^2} = \frac{1}{8\pi\alpha}\,\frac{G_\mu}{\sqrt{2}}\,[1-\Delta r+O(\alpha^2)]\,\frac{M_Z^2}{s-M^2}, \tag{7a}$$

$$\chi \approx \frac{1}{8\pi\alpha}\,\frac{G_\mu}{\sqrt{2}}\,\frac{M_Z^2}{s-M^2}, \tag{7b}$$

where $\Delta r = X/4\pi$ is taken from [6, 7]. The definition of χ deserves some comment. We use the on mass shell renormalization framework of Sirlin [7, 8], where

$$s_W^2 = 1-c_W^2 = \sin^2\theta_W = 1-M_W^2/M_Z^2. \tag{8}$$

As has been discussed extensively in the literature [9], the definition (7b) is to be preferred because it includes some large virtual radiative corrections and will be used here. The complex parameter M^2

$$M^2 = M_Z^2 - iM_Z\Gamma_Z, \tag{9}$$

contains the physical mass and width of the Z-boson. In the framework chosen, M_Z is determined experimentally while Γ_Z is predicted by the theory [10]. To be definite, we use in the following $M_Z = 93$ GeV, the t-quark mass $m_t = 40$ GeV, and Higgs boson mass $M_H = 100$ GeV. This, together with the fine structure constant α and the muon decay Fermi constant G_μ allow sone to calculate $M_W = 82.0$ GeV, $\sin^2\theta_W = 0.222$, $\Gamma_Z = 2.17\,\mathrm{GeV} + \Gamma_Z(t)$, $\Gamma_Z(t)$ being the partial width for the t-quark channel. The integrated bremsstrahlung and virtual QED correction are contained in the F_i:

$$F_0 = F_v(m_e)(1+\cos^2\theta)+F_{br}^{i}, \tag{10}$$

$$F_1 = F_{box}+F_{br}^{int}, \tag{11}$$

$$F_2 = F_v(m_f)(1+\cos^2\theta)+F_{br}^{f}. \tag{12}$$

The virtual corrections are well-known (see App. A). The initial (F_{br}^{i}), interference (F_{br}^{int}), and final (F_{br}^{f}) bremsstrahlung contributions are derived in Appendices B and C. The compact final result is:

$$F_{0,2} = f_{0,2}(\cos\theta)+f_{0,2}(-\cos\theta), \tag{13}$$

$$F_1 = f_1(\cos\theta)-f_1(-\cos\theta), \tag{14}$$

$$\begin{aligned} f_0 = {} & \tfrac{10}{9}+\tfrac{4}{3}L_e-\tfrac{2}{3}L_f-\tfrac{2}{3}L_-+\tfrac{8}{3}c_-L_- \\ & +c_-^2\left[\tfrac{10}{3}-\tfrac{13}{3}L_e+\pi^2+2(1-L_e)(L_++L_-)-L_1^2\right] \\ & +\frac{1}{c_-}\left[-\tfrac{19}{9}+\tfrac{5}{3}L_e+\tfrac{2}{3}L_f+\tfrac{4}{3}L_-+2L_+\right] \\ & +\frac{1}{c_-^2}\left[(L_e-2)L_++\tfrac{1}{2}L_+^2-\Phi(c_-)\right], \end{aligned} \tag{15}$$

$$f_1 = -\frac{3}{c_-^2}L_+ - \frac{3}{c_-}+9L_++12c_-^2L_i+c_-\left[6+\frac{\pi^2}{3}+L_-+L_+-2L_-L_+\right], \tag{16}$$

$$f_2 = 1-\tfrac{3}{2}c_+^2. \tag{17}$$

The following abbreviations are used:

$$L_e = \ln \frac{s}{m_e^2}, \quad L_f = \ln \frac{s}{m_f^2},$$

$$c_+ = \tfrac{1}{2}(1+\cos\theta) = \cos^2\frac{\theta}{2}, \quad c_- = \tfrac{1}{2}(1-\cos\theta) = \sin^2\frac{\theta}{2},$$

$$L_+ = \ln\cos^2\frac{\theta}{2}, \quad L_- = \ln\sin^2\frac{\theta}{2}, \quad L_t = \ln\tan^2\frac{\theta}{2},$$

$$\Phi(x) = -\int_0^1 \frac{dt}{t}\ln(1-xt). \tag{18}$$

Finiteness and integrability of (10)–(12) are ensured by the following modification near the end points $\cos\theta = \pm 1$ (see Appendix B)

$$\cos\theta \to \cos\left(\theta^2 + 4\frac{m_e^2}{s}\right)^{1/2}. \tag{19}$$

The initial radiative correction F_0 is dependent on s, m_e, m_f and θ while F_1 and F_2 are functions of the scattering angle only. Integration over $\cos\theta$ allows one to obtain analytic expressions for σ_{tot} and A_{FB}

$$\sigma_{tot} = \tfrac{4}{3}\pi\frac{\alpha^2}{s}\left\{Q^2\left[1+\frac{\alpha}{\pi}(F_0^{tot}+Q^2F_2^{tot})+\frac{\alpha}{\pi}F_{vp}\right]\right.$$

$$\left. +2\,\mathrm{Re}\,\chi|Q|v_e v + |\chi|^2(v_e^2+a_e^2)(v^2+a^2)\right\}, \tag{20}$$

$$A_{FB} = \frac{1}{\sigma_{tot}}\tfrac{4}{3}\pi\frac{\alpha^2}{s}\left[Q^3\frac{\alpha}{\pi}F_1^{tot} + \tfrac{3}{2}\,\mathrm{Re}\,\chi|Q|a_e a + 3|\chi|^2 v_e v a_e a\right], \tag{21}$$

$$F_{0,2}^{tot} = \tfrac{3}{8}\int_{-1}^{+1} d\cos\theta F_{0,2}, \tag{22}$$

$$F_1^{tot} = \tfrac{3}{8}\left(\int_0^1 d\cos\theta F_1 - \int_{-1}^{0} d\cos\theta F_1\right), \tag{23}$$

$$F_0^{tot} = \tfrac{2}{3} - \tfrac{7}{6}L_e + L_e L_f - L_f + \frac{\pi^2}{3}, \tag{24}$$

$$F_1^{tot} = \tfrac{3}{2} - \frac{\pi^2}{8} + \tfrac{3}{4}\ln^2 2 - \tfrac{15}{2}\ln 2 = -4.5720, \tag{25}$$

$$F_2^{tot} = \tfrac{3}{4}. \tag{26}$$

The $F_{0,2}^{tot}$ were first derived in [2, 3]. Concerning (15)–(17), we took advantage from a contact with the authors of [7] for an understanding of the partial disagreement with their Eq. (14).

3. *Numerical results*

Using the parameters as given above we obtain for s in GeV^2

$$\chi = -4.49 \times 10^{-5} s/(1 - s/8649). \tag{27}$$

The width of the Z-boson may be neglected in the energy range of interest here. As long as $s \lesssim 1600\ GeV^2$, the Born interference contribution does not exceed 15% of the photon exchange Born cross-section and the pure weak contribution is quite small (although in the numerics we will include it).

The pure QED radiative corrections (with exception of the vacuum polarization) is contained in the F_i, Eqs. (10–12). The dependence of the F_i on the scattering angle is shown in Fig. 3. The F_1 and F_2 are smooth over the complete kinematical region, whereas F_0 (initial state radiation) has sharp peaks of order s/m_e^2 times logarithms at the end points $|\cos\theta| = 1 - 2\,m_e^2/s$. These peaks are due to the well-known fermion mass singularities. Fig. 4 shows the differential cross-section. The QED Born cross-section is symmetric and, in the normalization chosen, independent of the energy. The Born cross-section of the GWS-theory is asymmetric in $\cos\theta$ (shown here for $s = 1600\ GeV^2$). The inclusion of the total Bremsstrahlung discussed here leads to a considerable modification, especially at the end points.

It is well-known that the total cross-section σ_{tot}^0 of the GWS-theory in Born approximation is very near to σ_{tot} of QED for mixing angles around 0.25 since then the vector couplings of leptons become small: $v_e = v_\mu \simeq 0.01$ for $\sin^2\theta = 0.222$ as chosen here. This may be seen in Fig. 5. The QED bremsstrahlung correction to σ_{tot} is considerable due to F_0 and becomes much smaller if a cut on $\cos\theta$ is applied.

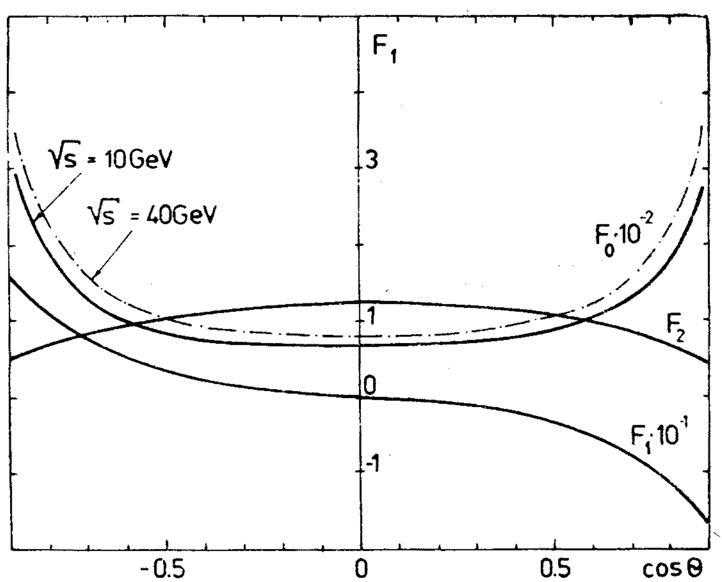

Fig. 3. The QED α^3 corrections of Fig. 2 as defined in (10)–(12) as function of $\cos\theta$ with parameter s

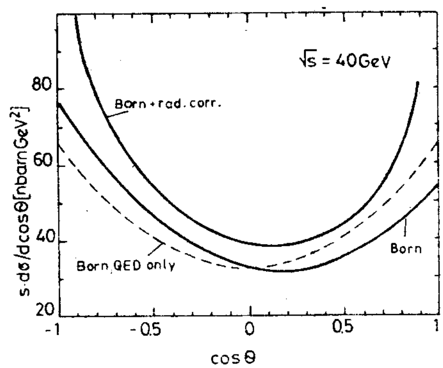

Fig. 4. The differential cross-section $d\sigma/d\cos\theta$

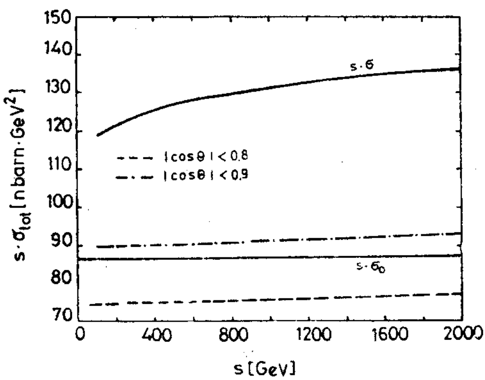

Fig. 5. The total cross-section σ_{tot} as function of s (σ_0 — Born cross-section; the slashed and slashed dotted lines are radiatively corrected with cuts as explained in the text)

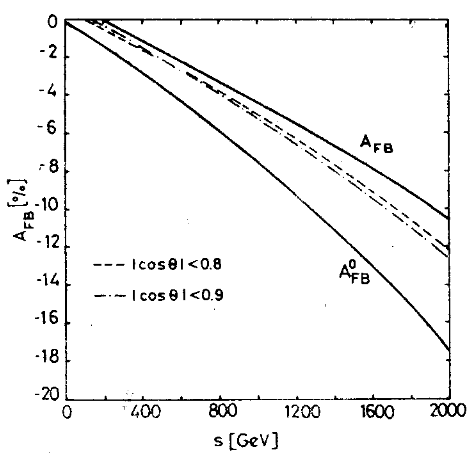

Fig. 6. The integrated forward-backward asymmetry A_{FB} as function of s

The integrated forward-backward asymmetry A_{FB} as function of s is shown in Fig. 6. In Born approximation, A^0_{FB} is negative and nearly linearly rising with s in the depicted energy range. The bremsstrahlung correction $Q_\mu F_1^{tot}$ is positive and constant. Correspondingly, its relative influence diminishes with rising s. In real experiments a cut is applied on the scattering angle, e.g. $|\cos\theta| < 0.8 \div 0.9$. The result of those cuts is shown, too. Like for σ_{tot}, the correction becomes much smaller. Interestingly, the corrected A_{FB} is closer to its Born value for $|\cos\theta| < 0.9$ than for $|\cos\theta| < 0.8$. This is due to a mismatch of the tendency of F_1 to add larger contributions to A_{FB} from scattering angles near $|\cos\theta| = 1$ against the peaking of F_0 at the same points influencing the denominator of A_{FB} in opposite direction.

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APPENDIX A

The QED vertex correction is in the approximation of small fermion masses and in the normalization adopted here:

$$F_v(m_f) = -2(L_f-1)\left(P_{IR}-\ln\frac{\eta}{m_f}\right)-2+\tfrac{2}{3}\pi^2+\tfrac{3}{2}L_f-\tfrac{1}{2}L_f^2, \tag{A1}$$

the $F_v(m_e)$ being defined analogously. The contribution of the two QED box diagrams is:

$$F_{Box} = f_{box}(\cos\theta)-f_{box}(-\cos\theta), \tag{A2}$$

$$f_{box} = 4(1+\cos^2\theta)L_-\left(P_{IR}-\ln\frac{\eta}{m_f}\right)+4C_-^2L_iL_f+2C_-L_+-C_-(L_-^2+L_+^2) \tag{A3}$$

The vacuum polarization is:

$$F_{vp} = \sum_{\text{leptons}} Q_l^2F_{vp}(m_l)+F_{vp}^{\text{hadrons}}, \tag{A4}$$

$$F_{vp}(m) = -\tfrac{10}{9}-\tfrac{8}{3}\frac{m^2}{s}+\frac{2\sqrt{n}}{3s}\left(1+\frac{2m^2}{s}\right)\ln\frac{s+\sqrt{n}}{s-\sqrt{n}}, \tag{A5}$$

$$F_{vp}(m) \approx -\tfrac{10}{9}+\tfrac{2}{3}L_l \quad (m^2 \ll s),$$

$$n = s^2-4m^2s. \tag{A6}$$

In the numerical results we do not include the F_{vp}. This causes a shift for σ_{tot} compared to a complete QED one loop calculation.

APPENDIX B

Here we sketch our derivation of a soft photon contribution to the bremsstrahlung cross-section:

$$e^-(k_1)+e^+(k_2) \to f^-(p_1)+f^+(p_2)+\gamma(p). \tag{B1}$$

We start with

$$M_\beta^{\rm IR} = M_0 \cdot \varphi_\beta^{\rm IR}, \tag{B2}$$

$$M_0 = \frac{i}{s}\, Q\bar{u}(k_2)\gamma^\mu u(k_1)\bar{u}(p_1)\gamma^\mu u(p_2), \tag{B3}$$

$$\varphi_\beta^{\rm IR} = \left(\frac{2k_{2\beta}}{\bar{Z}} - \frac{2k_{1\beta}}{Z}\right) + Q\left(\frac{2p_{1\beta}}{v} - \frac{2p_{2\beta}}{\bar{v}}\right). \tag{B4}$$

The Z, $\bar{Z}$ and v, $\bar{v}$ are the fermion propagators,

$$\overset{(-)}{Z} = -2k_{1(2)}p, \quad \overset{(-)}{v} = -2p_{1(2)}p. \tag{B5}$$

Formally, neglecting the photon momentum everywhere but in the denominators of $\varphi_{\rm IR}$, we introduce

$$\tfrac{1}{64}\sum_{\rm spins} |M_\beta^{\rm IR}|^2 \to T^A(x,t)B^{\rm IR}(t, Z, \bar{Z}, v, \bar{v}), \tag{B6}$$

$$T^A(x,t) = Q^2\,\frac{t^2+(s-t)^2}{s^2},$$

$$B^{\rm IR} = \left(-\frac{m_e^2}{Z^2} - \frac{m_e^2}{\bar{Z}^2} + \frac{s}{Z\bar{Z}}\right) + Q\left(\frac{t}{\bar{Z}\bar{v}} + \frac{t}{Zv} - \frac{s-t}{\bar{Z}v} - \frac{s-t}{Z\bar{v}}\right)$$
$$+Q^2\left(-\frac{m_f^2}{v^2} - \frac{m_f^2}{\bar{v}^2} + \frac{s}{v\bar{v}}\right). \tag{B7}$$

Here

$$t = -2k_2p_2 = \frac{x}{2} - \frac{1}{2s}\sqrt{s^2-4m_e^2 s}\,\sqrt{x^2-4m_f^2 s}\,\cos\theta, \tag{B8}$$

$$t \simeq xC_-, \tag{B9}$$

$$x = -2p_2(k_1+k_2) = 2\sqrt{s}\,p_2^0, \tag{B10}$$

where p_2^0 is the energy of f^+ in the cms: $x \in (0, s)$. For scattering angles with $\cos\theta$ very close to ± 1, the approximation (B9) is too crude and has to be replaced by

$$t \simeq \frac{x}{2}\left[1-\left(1-2\frac{m_e^2}{s}\right)\cos\theta\right] \simeq \frac{x}{2}\left[1-\cos\left(\theta^2+\frac{m_e^2}{E^2}\right)^{1/2}\right]. \tag{B11}$$

The integration over the photon phase space has been performed with the method developed in [11], i.e. using dimensional regularization and the R_γ-system, the rest system of (f–γ) $\vec{p}_1^{\,R}+\vec{p}^{\,R} = 0$. We write the soft bremsstrahlung contribution to (10)–(12) as follows:

$$\frac{d\sigma^s}{d\cos\theta} = \frac{d\sigma_{\mathrm{Born}}}{d\cos\theta}\,\frac{\alpha}{\pi}\,\delta_{\mathrm{soft}}, \tag{B12}$$

$$\delta_{\mathrm{soft}} = \int_0^{\bar{\omega}} d\omega I^n(\omega), \tag{B13}$$

$$I^n(\omega) = \frac{2}{(2\sqrt{\pi})^{n-4}\Gamma\left(\frac{n}{2}-1\right)}\,\frac{1}{\omega}\left(\frac{\omega}{2m_f}\right)^{n-4}\int_0^1 d\alpha \int_{-1}^{+1} d\xi (1-\xi^2)^{(n-4)/2} B(\alpha,\xi), \tag{B14}$$

$$\begin{aligned} B(\alpha,\xi) = {} & \left[-\frac{m_e^2}{k_{10}^2(1-\beta_1\xi)^2} - \frac{m_e^2}{k_{20}^2(1-\beta_2\xi)^2} \right. \\ & \left. + \frac{s}{k_{30}^2(1-\beta_3\xi)^2}\right] + Q\left[\frac{t}{p_{t_0}^2(1-\beta_t\xi)^2} \right. \\ & \left. + \frac{t}{m_f k_{10}(1-\beta_1\xi)} - \frac{s-t}{p_{st_0}^2(1-\beta_{st}\xi)^2} - \frac{s-t}{m_f k_{20}(1-\beta_2\xi)}\right] \\ & + Q^2\left[-\frac{m_f^2}{p_{20}^2(1-\beta_{p_2}\xi)^2} + \frac{s}{m_f p_{20}(1-\beta_{p_2}\xi)} - 1\right], \end{aligned} \tag{B15}$$

$$\xi = \cos\theta_\gamma^R, \qquad p_t = k_2\alpha + p_2(1-\alpha),$$

$$k_3 = k_1\alpha + k_2(1-\alpha), \qquad p_{st} = k_1\alpha + p_2(1-\alpha), \tag{B16}$$

and β_i are the velocities corresponding to p_i, k_i in the R_γ-system. In deriving (B12–B15) one takes advantage of the fact that $T^A(x, t)$ does not depend on the photon angles and that all momenta used may be chosen to depend on only one of the photon angles, $\cos\theta_\gamma^R$. The Feynman parameter integral over α has been introduced to simplify the dependence of numerators on the photon momentum. Eqs (B13–B14) have been obtained after integrating over $(n-3)$ angles in the n-dimensional space-time and after restricting the photon momentum p_0^R to be smaller than the infinitesimal parameter $\bar{\omega}$:

$$\bar{\omega} \ll m_e, m_f \ll s. \tag{B17}$$

This, in term, leads to the restriction for x to the interval $x \in (s-2m_f\cdot\bar{\omega}, s)$, containing at $x = s$ the infrared singularity and allows one to separate the Born cross-section factor. In fact the integral over ω is the limited by $\bar{\omega}$ x-integral (see Appendix C). Some further details on the method used may be taken from [11–13]. The kinecmatis is the same as in [13].

By straightforward integration one gets

$$\delta_{\text{soft}} = \left(p_{\text{IR}} + \ln\frac{2\bar{\omega}}{\eta}\right) \cdot \delta_0^s + \delta_1^s,$$

$$\delta_0^s = -2(1-L_e) - 4QL_i - 2Q^2(1-L_f),$$

$$\delta_1^s = \left[-\frac{\pi^2}{6} + (1-L_e)\cdot(L_e + L_f + L_+ + L_-) + \tfrac{1}{2}L_e^2 - \tfrac{1}{2}L_i^2\right]$$

$$+ 2QL_f \cdot L_i + Q^2 \cdot \left[1 + L_f - L_f^2 - \frac{\pi^2}{6}\right],$$

$$p_{\text{IR}} = \frac{1}{n-4} + \frac{\gamma_E}{2} - \ln(2\sqrt{\pi}). \tag{B18}$$

APPENDIX C

The matrix element M_β^{br} corresponding to Fig. 2 is the same as in [13] and has been integrated as follows:

$$\frac{d\sigma^{\text{br}}}{d\cos\theta} = \frac{\alpha^3}{\pi^2 s}\int d\Gamma \overline{\sum_{\text{spins}}} |M_\beta^{\text{br}}|^2, \tag{C1}$$

$$\int d\Gamma = \frac{\pi^2}{4S}\int_0^s x\,dx \frac{s-x}{4\pi(s-x+m_f^2)}\int_{-1}^{+1} d\cos\theta_\gamma^{\text{R}} \int_0^{2\pi} d\varphi_\gamma^{\text{R}}. \tag{C2}$$

Because of the infrared singularity we cut the x-integral into two parts: $i_1 = (0, s-2m_f\bar{\omega})$, $i_2 = (s-2m_f\bar{\omega}, s)$. In App. B we split up the singular part of the squared matrix element and integrated it over i_2. What remains is

$$\int_{(i_1+i_2)} d\Gamma\Big[\overline{\sum_{\text{spins}}} |M_\beta^{\text{br}}|^2 - 64T^A(x,t)\cdot B^{\text{IR}}\Big] + \int_{(i_1)} d\Gamma 64 T^A B^{\text{IR}}, \tag{C3}$$

where T^{A} and B^{IR} are defined in (B7). The first integral over the unrestricted phase space is finite. The second one is finite too and even well-defined in the limit $\bar{\omega}\to 0$ with the exception of terms containing the numerator (S−X) for which the exclusion of i_2 is important:

$$\int_0^{s-2m_f\bar{\omega}} \frac{dx}{s-x} = \ln\frac{s}{2m_f\bar{\omega}}. \tag{C4}$$

Effectively, one may use (C1)–(C2) and take into account (C4) whenever necessary. This has been done using the system of analytic manipulation SCHOONSCHIP [14], heavily relying on approximations of calculations and tables of integrals described in [12]. The result is

$$\frac{d\sigma^{\mathrm{br}}}{d\cos\theta}=\frac{d\sigma_{\mathrm{Born}}}{d\cos\theta}\,\frac{\alpha}{\pi}\,\delta_{\mathrm{soft}}+\frac{d\sigma_{\mathrm{hard}}}{d\cos\theta},\tag{C5}$$

$$\frac{d\sigma^{\mathrm{hard}}}{d\cos\theta}=\frac{\pi\alpha^2}{2s}\,\frac{\alpha}{\pi}\,Q^2\delta_{\mathrm{hard}},\tag{C6}$$

$$\delta_{\mathrm{hard}}=\delta^{\mathrm{i}}_{\mathrm{hard}}-Q\delta^{\mathrm{int}}_{\mathrm{hard}}+Q^2\delta^{\mathrm{f}}_{\mathrm{hard}},\tag{C7}$$

$$\begin{aligned}\delta^{\mathrm{i}}_{\mathrm{hard}}(\cos\theta)=&\Big[\tfrac{10}{9}+\tfrac{4}{9}L_{\mathrm{e}}+(L_{\mathrm{e}}-1)(1+\cos^2\theta)\Big(-\tfrac{11}{6}-\ln\frac{2\varpi}{m_{\mathrm{f}}}+L_{\mathrm{f}}\Big)\\&-\tfrac{2}{3}L_{\mathrm{f}}+\tfrac{8}{3}C_-L_-+\frac{1}{C_-^2}(L_+L_{\mathrm{e}}-2L_+-\Phi(C_-))-\tfrac{2}{3}L_-\\&+\frac{2}{C_+}L_-+\frac{1}{2C_+^2}L_-^2+\frac{1}{C_-}(\tfrac{4}{3}L_--\tfrac{19}{9}+\tfrac{5}{3}L_{\mathrm{e}}+\tfrac{2}{3}L_{\mathrm{f}})\Big]\\&+[\cos\theta\leftrightarrow-\cos\theta],\end{aligned}\tag{C8}$$

$$\begin{aligned}\delta^{\mathrm{int}}_{\mathrm{hard}}(\cos\theta)=&\Big[2(1+\cos^2\theta)L_-\Big(-2\ln\frac{2\varpi}{m_{\mathrm{f}}}+2L_{\mathrm{f}}-3\Big)\\&+\cos\theta\Big(3+\frac{\pi^2}{6}+\tfrac{1}{2}L_{\mathrm{i}}^2\Big)+\frac{3}{C_-}+\frac{3}{C_-^2}L_++8L_-\Big]\\&-[\cos\theta\leftrightarrow-\cos\theta],\end{aligned}\tag{C9}$$

$$\begin{aligned}\delta^{\mathrm{f}}_{\mathrm{hard}}(\cos\theta)=&\Big\{1+(1+\cos^2\theta)\Big[(1-L_{\mathrm{f}})\ln\frac{2\varpi}{m_{\mathrm{f}}}+\tfrac{1}{8}-\tfrac{5}{4}L_{\mathrm{f}}\\&+\tfrac{3}{4}L_{\mathrm{f}}^2-\frac{\pi^2}{4}\Big]\Big\}+\{\cos\theta\leftrightarrow-\cos\theta\}.\end{aligned}\tag{C10}$$

From the sum of δ_{soft} (with Born factor) and δ_{hard} one gets the cut-off independent F^a_{br}, $a=$ i, int, f, the IR-singularity of which will be compensated by the corresponding virtual corrections.

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