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INSTANTONS AND THE VACUUM CONDENSATES OF SUSY-GAUGE THEORIES*

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In the supersymmetric gauge theories the "non-renormalization" theorem guarantees that some quantities which are zero in lowest order remain zero in higher orders of perturbation theory. We show that such quantities get nonvanishing contributions from instanton-induced interactions. Also, no cut-off in the size of instantons is needed.

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1. *Introduction*

The instantons of Euclidean Yang-Mills equations correspond to tunneling between states with different topology and thus lead to a nontrivial vacuum structure of non-abelian gauge theories. In particular $n = 1$ BPST-instantons [1] have been discussed in quantum chromodynamics (QCD) but unfortunately no measurable effects could be pinpointed which should be uniquely due to instantons and not to (higher orders of) ordinary perturbation theory. Quantitative predictions are handicapped by the need for a cut-off in the size of the instantons.

This situation improves in supersymmetric gauge theories (SGT): The "non-renormalization" theorem guarantees that certain quantities which are zero in lowest order remain zero in higher orders of perturbation theory. However, it will be interesting to see that such quantities get nonvanishing contributions from instanton-induced interactions. Also no cut-off in the size of instantons is needed.

Let us consider two such cases which we will need in our discussion later on.
(*i*) The Green functions of lowest components [2, 3] A of chiral superfields [4] ϕ are zero in perturbation theory, e.g. there is no A-A, only a A^*-A propagator. Unbroken super-

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symmetry guarantees that such Green functions are constant:

$$\sigma^{\mu}_{\alpha\dot\alpha}\partial^{x_i}_{\mu}\langle 0|T(A_1(x_1), \dots A_m(x_m))|0\rangle$$
$$= \langle T(A_1(x_1) \dots \{\bar{Q}_{\dot\alpha}\psi_\alpha(x_i)\} \dots A_m(x_m))\rangle \tag{1}$$

(with ψ the second component of the chiral multiplet) because $\bar{Q}$ commutes with A and annihilates the vacuum. We will demonstrate that such Green functions are indeed constant and — different from perturbation theory — nonvanishing in an instanton background field. Since they do not depend on their arguments x_i taking these far apart from each other allows clustering, and the vacuum condensates $\langle A\rangle$ can be calculated.

(*ii*) Perturbation theory does not create new terms in the effective potential of SGT, but such terms can be generated nonperturbatively violating the non-renormalization theorem. They should be in accordance with the unbroken symmetries of the fundamental Lagrangian. Consider e.g. the well-known case [5, 6] of $N_c = 2$ supersymmetric QCD (SQCD) with one flavor, i.e. two ($i = 1, 2$) chiral (with Weyl spinor components) color doublet ($\alpha = 1, 2$) superfields, ϕ^i_α eventually multiplied by $\varepsilon_{ij}{}^{\alpha\beta}$ to give $\phi^\alpha{}_i$. Without superpotential the D-type potential

$$V^D = \frac{1}{2g^2}\left(-\frac{g^2}{2}\sum_i A^{i*}\tau A^i\right)^2 \tag{2}$$

is minimal (zero) for $\langle A^i{}_\alpha\rangle = v\delta^i{}_\alpha$ with undetermined v. For $v \neq 0$ the super Higgs effect makes the three gauge vector multiplets (including a real scalar) heavy and only one of four superfields remains massless. This is the field ϕ in a decomposition $\phi^i{}_\alpha = \delta^i_\alpha\phi + \dots$ in the unitary gauge or the field $T = \phi^i{}_\alpha\phi_i{}^\alpha = \phi^2$ in gauge-invariant language. It is the coordinate in the flat valley of the potential. What could be the effective superpotential compatible with the symmetries of the theory? The symmetries are color (α) and isospin (i) invariance, chiral invariance under

$$\phi \to e^{i\chi}\phi \tag{3}$$

and R-invariance

$$d\theta \to e^{i\varphi}d\theta, \quad \lambda \to e^{i\varphi}\lambda, \quad \phi \to e^{2i\varphi/3} \tag{4}$$

for the gaugino and the "matter" superfields. There is an anomaly free combination of (3) and (4) with $\chi = -5/3\,\psi$. Asking for an effective invariant superpotential containing only the field T, the unique answer is

$$\mathcal{L}^{\text{Nonpert}}_{\text{eff}} = \int d^2\theta\,\frac{c\Lambda^5}{\phi^2}, \tag{5}$$

with Λ the scale of the gauge theory and c some still unknown constant. This gives a potential

$$V^{\text{Nonpert}} \sim \left|\frac{\Lambda^5}{A^3}\right|^2 \tag{6}$$

(this is easily seen expanding ϕ around $\langle A\rangle$) driving $\langle A^2\rangle = \langle T\rangle = v^2 \to \infty$. With a mass term in the superpotential the corresponding potential

$$V = \left| mA - \frac{cA^5}{A^3}\right|^2 \tag{7}$$

has a stable minimum at $\langle A^2\rangle = v^2 \sim \Lambda^{5/2}m^{-1/2}$. The vev calculated in the instanton field according to (*i*) will give the same result with fixed normalization.

The effective Lagrangian approach, however, is not unique since it depends on the choice of the low energy effective fields. Introducing also a field $S = \frac{g^2}{16\pi^2}\lambda\lambda(\lambda^\alpha_{A\beta}$ the gaugino spinor field) one can even reproduce the anomalous symmetry [7]. With

$$\mathscr{L}_{\rm eff} = \int d^4\theta(\log(1+SS^*)+\log(1+TT^*)+\int d^2\theta(S\log(ST)-\alpha S) \tag{8}$$

the potential is

$$V = (1+SS^*)^2|\log ST|^2+(1+TT^*)\left|\frac{S}{T}-m\right|^2 \tag{9}$$

leading to

$$\langle S\rangle = \langle T^{-1}\rangle, \quad \left\langle\frac{T}{S}\right\rangle = \langle T^2\rangle \sim \frac{1}{m}, \tag{10}$$

i.e. the same result as above. But this is not always the case, in particular $N_f > N_c$ is excluded in the approach without field S because of Bose statistics of the fundamental fields ϕ building up the composite fields of the effective Lagrangian in a strictly local way (which seems to be rather artificial [8]) and also the case $N_f = N_c$ cannot be described with such a Lagrangian [5]. Since the construction of effective Lagrangians requires educated guesses and since the appearance of terms breaking the nonrenormalization theorem has to be substantiated, let us pass now to more concrete instanton calculations.

2. *SUSY instanton calculus*

The well-known $n = 1$ BPST instanton field [1]

$$A^a_m = \frac{2}{g}\eta_{amn}x^n f(x-x_0) \quad \text{with} \quad f(x) = (x^2+\varrho^2)^{-1} \tag{11}$$

($SU_c(2)$ index $a = 1, 2, 3$, $m = 1, \ldots, 4$) is a self-dual solution of Euclidean (imaginary time) classical SU(2) Yang-Mills equations. η_{amn} is projecting one SU(2) out of O(4) $\sim$ SU(2)$\times$SU(2). Inserting the field strength

$$G^a_{mn} = -\frac{4}{g}\eta_{amn}\varrho^2 f^2 = \tilde{G}^a_{mn} \tag{12}$$

into the action integral one obtains a topological charge

$$S = \int d^4x \, \tfrac{1}{4} \, G^a_{mn} G^a_{mn} = \int d^4x \, \tfrac{1}{4} \, G^a_{mn} \tilde{G}^a_{mn} = n \, \frac{8\pi^2}{g^2} \tag{13}$$

with $n = 1$. Instanton contributions to the vacuum functional correspond to tunneling between degenerate minima differing by unit topological charge. Quantum corrections around the instanton background field can be calculated after renormalization. Zero modes of the operator bilinear in the quantum fields are related to translation (x_0), dilatation (ϱ), and color rotation of the instanton and are absorbed into collective coordinates x_{0_m}, ϱ — the position and size of the instanton — and the direction in color space. In a pure gauge theory the $n = 1$ instanton contribution to the vacuum functional is given by [9, 16]

$$f \int d^4 x_0 \, \frac{d\varrho}{\varrho^5} \left(\frac{8\pi}{g^2(\mu)} \right)^4 \exp\left(\frac{-8\pi^2}{g^2(\mu)} + 8 \log \mu\varrho + \phi \right), \tag{14}$$

with constant f and function ϕ including the effects of non-zero modes. (14) is renormalization group (μ) invariant and can be expressed in terms of the invariant $\Lambda = \mu \exp \int\limits_{g(\Lambda)}^{g(\mu)} \frac{dg}{\beta(g)}$.
In more complicated cases renormalization group invariant expressions like (14) allow to determine the β-function [2, 10]. This is particular relevant for SUSY gauge theories where the bosonic and fermionic non-zero modes are degenerate and cancel in the determinant of the vacuum functional.

The zero modes of fermionic fields — these are matter fields and also gaugino fields — in SGT require particular care: The (Weyl spinor) kinetic term $-i\bar{\psi}\bar{\sigma}^m D_m^{\text{inst}} \psi$ contains the covariant derivative in the instanton background field. General spinor fields can be expanded in the normalized eigenfunctions ψ_i of the operator $\gamma_\mu D^{\mu\,\text{inst}}$ with eigenvalues E_i

$$\psi(x) = \gamma^\lambda \psi_0^\lambda + \sum_{i \neq 0} c_i \psi_i(x), \tag{15}$$

where zero modes ψ_0^λ are written separately. The path integral of the vacuum functional then is in the Grassman-valued variables γ^λ, c_i and in particular for zero modes we have

$$[D\psi]_0 \Rightarrow \prod_\lambda (N\mu)^{-1/2} d\gamma_\lambda^\lambda \tag{16}$$

(with normalization factor N in case of ψ_0 not properly normalized and with μ entering after renormalization). Since $\int d\gamma\gamma^n = \delta_{n1}$, one has to calculate the Green functions with at least one ψ per zero mode in order not to obtain zero if there are no ψ-mass terms. Thus there is a new nonperturbative interaction of fermions in the instanton background field [9]. It is an important observation [2, 4] that in pure SUSY Yang-Mills theory the gaugino modes are solutions of the equations of motion in an instanton background[1]

$$(D^{m\,\text{inst}}G_{mn})^a = ig\varepsilon^{abc}\bar{\lambda}^b\bar{\sigma}_n\lambda^c,$$

$$\gamma_\mu D^{\mu\,\text{inst}}\lambda = \gamma_\mu D^{\mu\,\text{inst}}\bar{\lambda} = 0, \tag{17}$$

with $\bar{\lambda} = 0$ the instanton field G^a_{mn} of Eq. (12) is again a solution and λ is its superpartner obtained by a SUSY transformation since the Eqs (17) are supersymmetric. Indeed they are even invariant under superconformal (SUCO) transformations. SUSY and SUCO transformations with Grassmann-valued spinor parameters θ_0 and $\bar{\vartheta}$

$$\xi(x) = -\theta_0 - x_m\sigma^m\bar{\vartheta} \tag{18}$$

applied to the instanton field lead to gaugino zero modes ($\delta_\xi\lambda = \sigma^{mn}\xi G_{mn}$)

$$\lambda^{a(\text{SUSY})}_{0\alpha}(x,\beta) = \frac{2}{\pi}\,\sigma^{a\beta}_\alpha\varrho^2 f^2(x-x_0),$$

$$\lambda^{a(\text{SUCO})}_{0\alpha}(x,\dot{\beta}) = \frac{\sqrt{2}}{\pi}\,(\sigma^a\bar{\sigma}^m)_{\alpha\dot{\beta}}(x-x_0)_m\varrho f^2(x-x_0), \tag{19}$$

multiplied by $\theta_{0\beta}$ and $\bar{\vartheta}^{\dot{\beta}}$, respectively. Hence these zero mode parameters are generalized (Grassmann-valued) collective coordinates of the instanton. The introduction of superfield language [2, 11, 12] turns out to be very profitable: The most general gauge superfield is obtained from a superfield with the instanton in the one-θ-component by SUCO ($\bar{\vartheta}$) and SUSY (θ_0) transformations:

$$W_\alpha(y,\theta) = -i\sigma^{mn\beta}_\alpha\tilde{\theta}G^{\text{inst}}_{mn}(y) \qquad (y = x + i\theta\sigma\bar{\theta}), \tag{20}$$

with

$$\tilde{\theta} = \theta - \theta_0 - (y-x_0)_m\sigma^m\bar{\vartheta}$$

(remember $W_\alpha = -i\lambda - i\sigma^{mn\beta}_\alpha\theta_\beta G_{mn} + \ldots$). This gives the gauge-invariant field

$$\frac{g^2}{32\pi^2}W^2 = -\frac{6}{\pi^2}\varrho^4 f^4(y-x_0)\tilde{\theta}^2. \tag{21}$$

Note that SUSY is not broken automatically by instantons since there is no D-component in W_α.

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[1] These are equations in Euclidean space. The definition of chiral fermions in the Euclidean requires some care. The fermionic components of chiral (anti-chiral) superfields ϕ and $\bar{\phi}$ are not related by complex conjugation and thus represent independent field variables. Hermiticity has to be replaced by Osterwalder-Schrader positivity and supersymmetry is realized in a non-unitary way [16]. An alternative way presented in the Appendix A of Ref. [11] is to stay with Minkowski space tensors and only work with imaginary time in the path integral.

For chiral "matter" fermions we have further zero modes which enter the instanton-induced interaction. Knowing the number of zero modes, it is very easy to read off the anomaly-free current (e.g. the one mentioned following Eq. (4), see Fig. 1) from this effective vertex. More general the anomalies of the anomaly supermultiplet (SUCO, dilatation, $U_A(1)$) follow from the noninvariance of the integration measure due to zero modes [2].

In the case of chiral matter fields $\phi = A + \theta\psi + \ldots$ the fermion is not in the lowest component which we need later on in the Green functions. The most straightforward possibility for a scalar field A in a Green function is to couple to fermionic zero modes λ_0, ψ_0 through perturbative gauge interactions [2, 3] (Fig. 2) which is equivalent to insertion of a solution of

$$(D^{2\,\text{inst}}A)_a = ig\psi^A_{0\beta}\varepsilon_{AB}\lambda^{aB}_0\left(\frac{\tau^a}{2}\right)^\beta_\alpha, \tag{22}$$

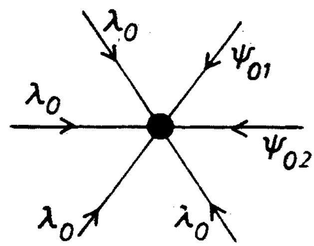

Fig. 1. Instanton-induced interaction in the case $N_c = 2$, $N_{\text{chiral flavor}} = 2$

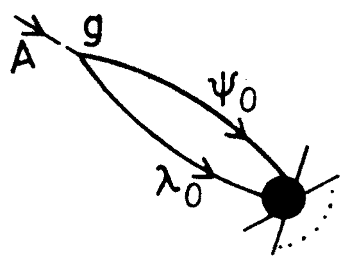

Fig. 2. Perturbative coupling of scalars to the instanton-induced vertex

where ψ_0, the solution of

$$\gamma_\mu D^{\mu\,\text{inst}}\psi_0 = 0 \tag{23}$$

for ψ in the fundamental representation is well known [9, 1]:

$$\psi^A_{0\alpha}(x) = \frac{1}{\pi}\,\varrho\delta^A_\alpha f^{3/2}(x-x_0) \tag{24}$$

(Lorentz spinor indices A, B are suppressed occasionally). The λ_0 are given in Eq. (19). In the superfield formalism ψ^i_0 is accompanied by a spinor Grassmann parameter $\chi^{(i)}$ and appears in the θ component of the chiral superfield. The scalar component discussed above is obtained substituting θ by $\bar{\theta}$ (Eq. 20) since (22) follows from (23) (multiplied by $\gamma_\mu D^\mu$) through a SUSY-SUCO transformation with parameters θ_0 and ϑ, respectively [2, 11, 12]

$$\phi^i_\alpha(y, \theta) = 2\bar{\theta}_A \psi^A_{0\alpha} \chi^{(i)}, \tag{25}$$

$$\phi^2(y, \theta) = \frac{4}{\pi^2} \varrho^2 f^3(y-x_0)\bar{\theta}^2 \chi^2. \tag{25a}$$

It is crucial to note that the solution of (22) has no homogeneous part if $|A| \to 0$ for $|x| \to \infty$, i.e. if there is no classical nonzero vev of the scalar fields A^i_α. Also only in this case the (QCD-type) instanton stays a solution of the field equations. Since in this approach one assumes that no big ($\langle A \rangle > \Lambda$) vevs appear, the gauge theory should confine. Instantons are the dominant nonperturbative contributions only at small distances. Hence reliable calculations can be only performed in this region. One calculates the Green functions at small distances and one has to assume that small instantons dominate in this case [3]. The latter is not generally accepted and we will discuss a different approach soon, where a Higgs phase with large $\langle A \rangle > \Lambda$ guarantees weak coupling.

3. *Calculation of the Green functions in the instanton background*

Let us have a look now to some Green functions [2, 3, 8] containing gauge-invariant $W^2 = \lambda^a_\alpha \lambda^{\alpha a}$ and $\phi^i_\alpha \phi^\alpha_i$ ($\alpha = 1, 2$; $i = 1, ..., M$). Whereas in the component approach there is a lot of combinatoric, the calculation is very elegant in the superfield formalism [2, 12], e.g. we have for the pure Yang-Mills theory ($N = 2$, $M = 0$) (see Fig. 3)

$$g^{(M=0)}_{00} = \left\langle T\left(\frac{g^2}{32\pi^2} W^2(y_1, \theta_1) \frac{g^2}{32\pi^2} W^2(y_2, \theta_2)\right)\right\rangle$$

$$= 4\pi C \int d^4x_0 d\varrho^2 d^2\theta_0 d^2\vartheta \left(-\frac{6}{\pi^2} \varrho^4 f^4(y_1 - x_0)\bar{\theta}^2_1\right) \times (1 \leftrightarrow 2) \tag{26}$$

which gives a nonzero, finite Feynman integral for the lowest components

$$= 4\pi C \int d^4x_0 d\varrho^2 \left(\frac{6}{\pi^2}\right)^2 \varrho^8 f^4(x_1 - x_0) f^4(x_2 - x_0) 4(x_2 - x_1)^2 = \frac{4}{5\pi^2} 4\pi C \Lambda^6, \tag{27}$$

C is a well-defined constant related to the definition [12] of Λ. No cut-off in ϱ is needed like in QCD and indeed the lowest component G_{00} is a constant as derived from SUSY in the introduction. Accepting that for x_1 close to x_2 small instantons dominate and that therefore the resulting constant is correct, the same constant should come out (because of SUSY, Eq. (1)) for large distances where instantons play no role [3]. Taking x_1, x_2 far apart from each other the Green functions cluster into a product of vev's $\langle \lambda\lambda \rangle^2$ and one obtains $\left\langle \frac{g^2}{32\pi^2} \lambda\lambda \right\rangle \sim \Lambda^3$ in agreement with the effective Lagrangian approach but with the prefactor fixed.

For $M_{fl} \neq 0$ calculations run through similarly. A perturbative mass term insertion [8] is needed for some Green functions in order to give a nonzero results, e.g. for $M_{fl} = 1$ G_{00} corresponds to the graph of Fig. 4 and is calculated as $\sim m\Lambda^5$. $G_{10} = \langle T(\phi^2(x_1)W^2(x_2))\rangle$ corresponds to the graph of Fig. 5 and is calculated as $\sim \Lambda^5$. Cluster decomposition gives

$$\langle\lambda\lambda\rangle^2 \sim m\Lambda^5, \quad \langle\lambda\lambda\rangle\langle A^2\rangle \sim \Lambda^5, \tag{28}$$

and hence

$$\langle A^2\rangle \sim \Lambda^{5/2}/\sqrt{m}, \quad \langle\lambda\lambda\rangle \sim \Lambda^{5/2}\sqrt{m}. \tag{29}$$

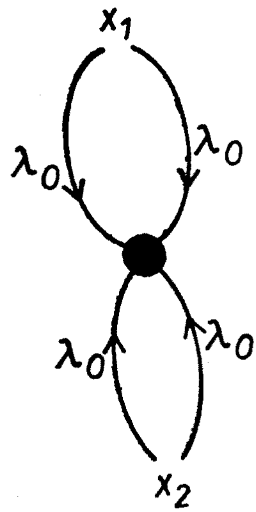

Fig. 3. Graph for $G_{00}^{(M=0)}$

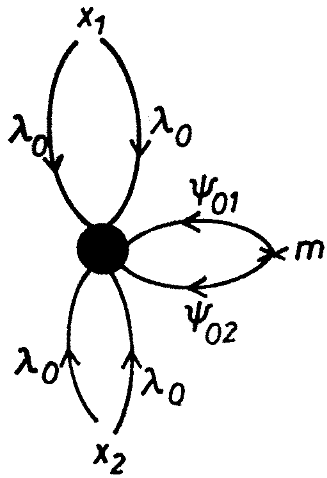

Fig. 4. Graph for $G_{00}^{(M=1)}$

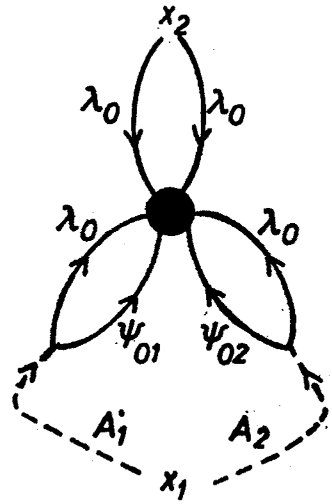

Fig. 5. Graph for $G_{10}^{(M=1)}$

This is in qualitative agreement with the results of the effective potential approach, but quantitatively the prefactors in (29) are in disagreement with the "Konishi anomaly relation" [13]

$$\{\bar{Q}_1 \bar{\psi}_1^{\dot{\alpha}}(x) A_1(x)\}/2\sqrt{2} = -m A_2(x) A_1(x) + \frac{g^2}{32\pi^2} \lambda\lambda \tag{30}$$

which predicts a relation between lowest component Green functions containing A^2 and $\lambda\lambda$. Further quantitative inconsistencies appear for $M_{\mathrm{fl}} \geqslant 2$ [12, 14, 11]. There are more relations than condensates now, and the prefactors (not the powers in m) for the same condensates obtained from different Green functions are not consistent. Even worse, a Green function like $\langle \phi_1^\alpha \phi_\alpha^2(x_1) \phi_2^\alpha \phi_\alpha^1(x_2)\rangle$ is also nonvanishing and clustering leads to condensates which do not make sense. This is related to the point that the instanton calculation preserves flavor symmetry. A mass term for matter fields breaks this symmetry, but it only plays a role in certain Green functions (e.g. not in the 4-ϕ Green function mentioned before). In Ref. [14] it was proposed that in the case of massive ϕ-fields, nonzero modes and related infrared effects $\left(\frac{\text{"}m\text{"}}{m}\right)$ are important in order to avoid such inconsistencies. The determination of higher modes for general $m \neq 0$ is prohibitively difficult but for large m the calculation is simple since chiral field zero modes become unimportant and the massive field propagator can be substituted by the free one in leading order. Nonleading contributions to lowest component Green functions calculated that way have an x-dependence forbidden by SUSY, but they vanish for $m \to \infty$. The limit $m \to \infty$ might not look very appealing but indeed it can be proved that the power in m of such Green functions is also a constant. This follows [15, 14] from the anomaly-free U(1) built out of R and $U_{\mathrm{fl}}(1)$. Thus one can calculate for large m and continue to small m. For a consistent calculation the arguments of the Green functions should fulfil the conditions [14]

$$\frac{1}{m} \ll |x-y| \ll \frac{1}{\Lambda}, \tag{31}$$

which allow a perturbative calculation (g small) and still make the mass cut-off effective. With such infrared effects everything becomes consistent for SQCD type theories with N colors and M flavors. E. g. for the case $N = 2$, $M_f = 1$ considered before G_{00} is unchanged, but G_{01} is given by a contribution corresponding to Fig. 6 for $m \to \infty$. In theories

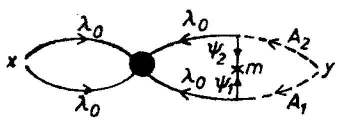

Fig. 6. Leading graph for $G_{10}^{(M=1)}$ for $m \to \infty$

with chiral fermions there is no mass parameter m available and the above method cannot be applied. Fortunately enough in an SU(6) gauge theory with chiral flavor (one a.s. tensor, two fundamental representations) consistency [17] comes out without infrared effects. However, in Ref. [18] it was shown by very effective superspace methods that in SU(6) with K ($K > 1$) a.s. and $2K$ fundamental representations and in other chiral theories indeed inconsistencies appear. Thus further effects have to be searched for[2].

4. *Instanton effects in the Higgs phase*

There is another approach [4, 11] to instanton calculations which is rather different in spirit: One is looking for gauge theories in the Higgs phase, i.e. with a scalar vev of a chiral superfield $\langle A \rangle \gg \Lambda$ which fixes the gauge coupling and prevents strong coupling and confinement à la QCD. As already observed [9] by 't Hooft a Higgs vev $\langle A \rangle = v$ still allows for an approximate instanton solution for $\varrho < v$:

$$D^{m\,\text{inst}} G_{mn}^{\text{inst}} = igA^* D_n^{\text{inst}} A \sim 0 \tag{32}$$

together with a zero mode solution

$$D^{2\,\text{inst}} A_0 = 0, \quad (A_0^2 \to v^2 \quad \text{for} \quad |x| \to \infty) \tag{33}$$

for the scalar field:

$$A_{0\alpha}^{i} = iv\sigma_{i\alpha}^{m}(x-x_0)_m f^{1/2}(x-x_0).$$

[2] Writing up this talk I was kindly informed by K. Konishi that in a forthcoming preprint he will present methods how to project out a Green function in a vacuum with broken flavor symmetry out of the symmetric result of the instanton calculation which should be interpreted as a sum of amplitudes in different vacua. This remark also applies to SQCD-type theories, but there infrared effects are also needed for consistency.

"Approximate" means that the action $\frac{8\pi^2}{g^2} + 4\pi^2 v^2 \varrho^2$ is *not* really minimized for $\varrho \neq 0$ and that the integration over the collective parameter ϱ is done for such a nonminimal contribution to the path integral.

SUSY (SUCO) transformations [11, 5, 12] ($\bar{Q}$ on A_0 then $Q, \bar{S}$ on the resulting fermionic component lead to a superfield up to normalization which indeed has the fermionic zero mode discussed before (Eq. (24)). The color singlet ϕ^2 is given by

$$\phi^2(y, \theta) = 2v^2(\tilde{y}-x_0)^2 f(\tilde{y}-x_0), \tag{34}$$

with

$$\tilde{y} = y + 2i\bar{\theta}\sigma\theta,$$

where $\bar{\theta}_0$, the parameter of $\bar{Q}$-transformations, corresponds to χ in Eq. (25). The $\lambda\psi A$ gauge coupling term in the action requires a substitution

$$\varrho^2 v^2 \to \varrho^2(1+4i\bar{\vartheta}\bar{\theta}_0)v^2 = \tilde{\varrho}^2 v^2 \tag{35}$$

in the action. Calculations of the Green functions again are performed very economically in the superfield formalism though they are somewhat more involved than in the first approach.

In the case $N_c = 2$, $N_f = 1$ discussed above one obtains after some substitutions [11, 12]

$$g_{01}^{(M=1)} = \frac{CA^5}{v^2}\int d^4x_0 \frac{d\varrho^2}{\varrho^2} d^2\theta_0 d\vartheta d^2\bar{\theta}_0 \exp(-4\pi v^2\tilde{\varrho}^2)$$
$$\times\left(-\frac{6}{\pi^2}\right)\varrho^4 f^4(\tilde{y}_1-x_0)\theta_1^2 2v^2(\tilde{y}_2-x_0)^2 f(\tilde{y}_2-x_0) = -8CA^5. \tag{36}$$

Also "big" instantons with size v contribute in this integral. Contrary to the philosophy of Ref. [3, 14] it is not dominated by instantons of size $|x_2-x_1|$. The Green function $g_0^{(M=1)} = \left\langle \frac{g^2}{32\pi^2} W^2 \right\rangle_{1-\text{inst}}$ corresponding to Fig. 7 can be calculated as

$$g_0^{(M=1)} = -\frac{4CA^5}{v^2} \tag{37}$$

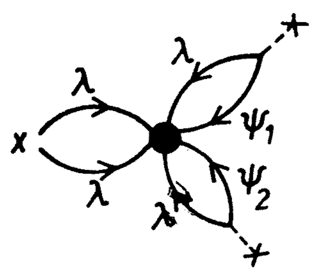

Fig. 7. $G_0^{(M=1)}$ in the Higgs phase approach

and with $g_1^{(M=1)} = 2v^2$ (no instanton) one has consistently $G_{01} = G_0 G_1$. Putting in the "Konishi" relation $G_0 = -\frac{m}{2} G_1$ it follows

$$v^2 = 2\sqrt{2}\frac{\Lambda^{3/2}}{\sqrt{m}} \tag{38}$$

in agreement with the result of effective Lagrangians. Instead of using the Konishi relation one could also calculate the fermionic part of the effective superpotential [11, 19] induced by the instanton (Fig. 8) and obtain (38), i.e. consistency with the Konishi relation.

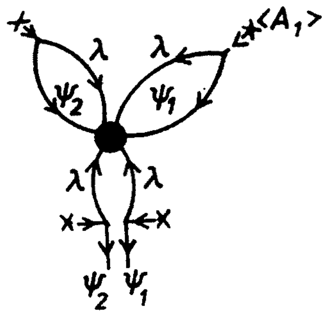

Fig. 8. Fermionic part of the effective superpotential in the case $M_{fl} = 1$ discussed in Section 2

$g_{00(1\text{-instanton})}^{(M=1)}$ turns out to be x dependent, i.e. it violates SUSY, but for $v \to \infty$ (or $m \to 0$) this contribution vanishes [12] faster than m and is dominated by the two instanton contribution which clusters into $(g_{0(1\text{-inst.})}^{(M=1)})^2$. Hence the limit $m \to 0$, $v \to \infty$ is needed! In this limit everything is consistent also for $M_{fl} \geqslant 2$. Recently consistency for all SQCD type theories has been proved [20] using powerful superfield methods.

Since scalar zero modes which are basic in this approach only exist for $m = 0$, the limit $m \to 0$ is necessary anyway, though perturbation in m around the case $m = 0$ which is topologically different from the case $m \neq 0$ still is dangerous [21]. One should be able to show that the topological untwisting at distances $|x| > \frac{1}{m}$ from the instanton center necessary to obtain a trivial topological configuration, does not effect the calculation.

The two approaches with $v_{cl} = 0$ and $v_{cl} \neq 0$ are based on two different phases of the gauge systems—confinement in one case, Higgs phase in the other case. The normalization of the instanton induced the Green functions expressed in powers of Λ is different, but both pictures lead to consistency for the Green functions. Of course the consistency of a hypothesis does not prove that it is right. In the first case the calculation is done for $m \to \infty$ ($v \to 0$), in the other case for $m \to 0$ ($v \to \infty$). The powers in m are universal, but apparently there is no analytic connection between the two cases; they are disconnected. Still the results for the vacuum condensates in both approaches agree qualitatively. Very often they can be read off from simple diagrams including an instanton-induced effective vertex.

Leaving aside these questions, very impressive effects induced by instantons can be demonstrated, in particular the breaking of SUSY in gauge theories with chiral matter, discussed in Ref. [22]. In the last part of these lectures we will use the information about the vacuum condensates of SQCD-type theories in order to discuss the BPY model [23].

5. *The vacuum structure and the spectrum of the BPY-model [24]*

Consider [23] a supersymmetric SU(2) gauge theory with six doublets of chiral superfields χ_α^I and hence with a flavor group $U_f(6) \times U_R(1)$. If the anomaly-free flavor group $SU(6) \times U_\chi(1)$ is broken spontaneously, Goldstone bosons appear accompanied by massless fermions because of SUSY. The latter might be candidates for composite l.h. quarks and leptons. The 't Hooft anomaly matching between the fundamental and the effective theory including the fermionic fields of the Goldstone supermultiplet fit for two cases:

(*i*) the breaking

$$SU(6) \times U_\chi(1) \rightarrow SU(4) \times SU(2) \times U_{\tilde{\chi}}(1), \tag{39}$$

where $U_{\tilde{\chi}}(1)$ is anomaly-free like $U_\chi(1)$.

(*ii*) Unbroken flavor symmetry.

The case (*i*) requires $36-(15+3+1) = 17$ Goldstone bosons in at least $8+1$ chiral supermultiplets. The latter are conjectured to correspond to the underlined interpolating composite superfields in the a.s. 15-representation of SU(6):

$$\varphi_{[IJ]} = \varepsilon^{\alpha\beta}\chi_\alpha^I\chi_\beta^J \tag{40}$$

decomposed according to (39):

$$(15)_{SU(6)} = \underline{(4,2)} + \underline{(1,1)} + (6,\, 1). \tag{41}$$

In such a model one is tempted to argue that there are also composite W-bosons made out of the scalars of the chiral superfields. In this picture the weak interaction should originate as an effective coupling between composite quarks/leptons and W-bosons like e.g. the coupling N-N-ϱ of composite hadrons. Contrary to QCD, however, the effective coupling should be weak! Universality (here inside one family, generalizations are possible) comes out because of the conserved weak isospin and because of the spectator character of one of the constituents. An effective Weinberg angle appears as a result of elementary γ-composite W-mixing [25]. The question arises if one can understand the binding of the W if there are further composite vector bosons mediating further interactions, and last not least, why the weak coupling is weak in such a model.

The analysis of the spectrum of hadrons in QCD is conveniently performed with SVZ sum rules [26] for gauge-invariant composite operator two-point functions. These sum rules are based on asymptotic freedom and analyticity and some knowledge about the vacuum structure of QCD. With some ansatz for the intermediate state spectrum (e.g. one particle + high energy background) the parameters, masses and couplings can be calculated. In QCD the vacuum structure is not so well known, but of course one knows the particle spectrum which should come out from experiments. In the case of the BPY model the latter is just in question. Fortunately the vacuum structure can be fixed to a large extent. Contrary to QCD scalar condensates turn out to be most important and indeed the sum rules will look completely different from QCD.

The first point to realize [24] is that an $SU_c(2)$ gauge theory with six chiral flavors corresponds to $SQCD_2$ with three flavors. Mass terms

$$m_1\varepsilon^{ij}\varphi_{ij}+m_2\varepsilon^{a_1b_1}\varphi_{a_1b_1}+m_3\varepsilon^{a_2b_2}\varphi_{a_2b_2} \tag{42}$$

coupling pairs of χ^I_α ($i, j = 1, 2$; $\underset{2\;2}{a_1 b_1} = 1, 2$ corresponding to $I = 1 \ldots 6$) gauge-invariantly break the original flavor symmetry:

$$\mathrm{SU}(6) \to \mathrm{SU}_V(3) \times \mathrm{SU}(1) \tag{43}$$

explicitly and condensates break it spontaneously:

$$\begin{aligned}\langle\varphi_{ij}\rangle &= \langle\varepsilon^{\alpha\beta}\chi^i_\alpha\chi^j_\beta\rangle = v_1^2\varepsilon_{ij}\\ \langle\varphi_{\substack{a_1b_1\\a_2b_2}}\rangle &= v^2_{\substack{2\\3}}\varepsilon_{\substack{a_1b_1\\a_2b_2}}.\end{aligned} \tag{44}$$

As discussed in previous chapters, instanton-induced interaction leads to vacuum condensates

$$\begin{aligned}\langle\lambda\lambda\rangle &\sim \Lambda^{3/2}(m_1m_2m_3)^{1/2},\\ v_i^2 &\sim \frac{\Lambda^{3/2}}{m_i}(m_1m_2m_3)^{1/2}\end{aligned} \tag{45}$$

if the Green functions including φ_{ij} and $\lambda\lambda$ are clustered (Fig. 9).

Unbroken SUSY implies for fermionic vacuum condensates:

$$\langle\psi\psi\rangle \sim m\langle\chi^*\chi\rangle. \tag{46}$$

Hence with finite $\langle\chi^*\chi\rangle$ (see below) for $m \to 0$ fermionic "matter" condensates vanish.

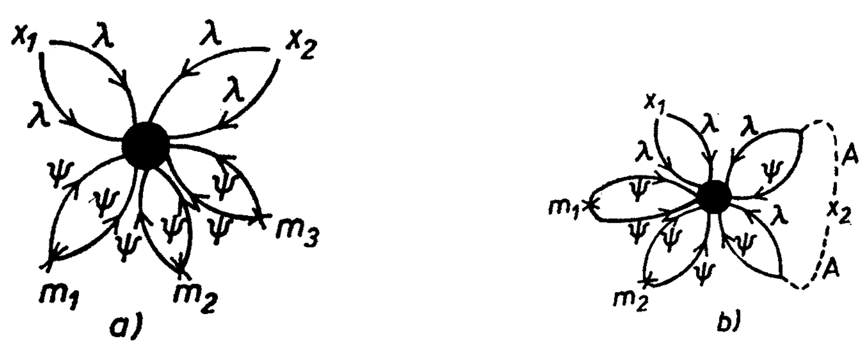

Fig. 9. Graphs for a) $G_0^{(M=3)}$, b) $G_{10}^{(M=3)}$ in the confinement picture

From (45) we read off [24] two possible limiting cases for $m_i \to 0$ with finite condensates:

$$(i)\ m_2 \sim m_3,\ m_1 \sim m_2 m_3 \to 0:\quad v_1^2 = 0,\ v_2^2 = v_3^2 = 0, \tag{47}$$

$$(ii)\ m_i \sim m \to 0:\quad v_1^2 = v_2^2 = v_3^2 = 0. \tag{48}$$

These are exactly the two cases of $SU_{fl}(6)$ breaking/nonbreaking mentioned before. This invites for an interpretation which is not quite in the spirit of composite models of strongly interacting subparticles: $\chi_\alpha^{1,2}$ are the two Higgs fields H, H' and $\chi_\alpha^{3,\dots,6}$ the quark/lepton chiral superfields of one family in the supersymmetric standard model. As a consequence there would be Higgs condensates in the gauge theory without Higgs Lagrangian. The condensates discussed here do not break the gauge symmetry, but it is well known that the so-called spontaneous breaking of the gauge symmetry and the Higs effect leading to massive vector bosons can (and perhaps even must) be treated in a gauge-invariant picture.

What about $\langle \chi^* \chi \rangle = \tilde{v}^2$ condensates? ($\langle \chi_1^{*\alpha} \chi_{1\alpha} \rangle = \langle \chi_2^{*\alpha} \chi_{2\alpha} \rangle = \tilde{v}_1^2$ etc. similar to (44)) $\chi^*\chi$ is not the lowest component of a chiral field. Still the condensates are not unconstrained as one might suppose: They can be determined using (generalized) SUSY-Dashen sum rules [24]. Neglecting effects of gluon condensates one obtains

$$\tilde{v}_1^2 = v_1^2 = f_1^2, \quad \tilde{v}_{\substack{2\\2}}^2 = v_{\substack{2\\3}}^2 = 0, \quad f_2 = f_3. \tag{49}$$

Here the f_1, f_2 and f_3 are the Goldstone boson couplings to the currents (1, 1) and (4, 2), respectively, of spontaneously broken symmetries (39).

The calculation of higher condensates including $\chi^*\chi$, if possible at all, is much more difficult. One can make the very plausible assumption that higher condensates including $\chi^*\chi$ also factorize, i.e. are equal to a product of smaller condensates of gauge-invariant quantities into which they can be decomposed. This fixes all scalar condensates, and sum rules can be written down for the two-point function of gauge-invariant composite operators of type $\chi_\alpha \chi_\beta \varepsilon^{\alpha\beta}$ with quantum numbers (1, 1), (4, 2) and (6, 1) of unbroken ($m \to 0$) flavor SU(4) × SU(2). In the (6, 1) channel there is a nonperturbative, instanton induced interaction in a vacuum with $\langle \chi_1 \varepsilon \chi_2 \rangle = v_1^2 \neq 0$ leading to a mass $\sim \Lambda$ (Fig. 10). The sum rules in the two first channels can be saturated by massless (pseudo) Goldstone superfields. These channels also appear in the two-point functions of flavor currents $\bar{\chi}_i^\alpha (e^{2gV})_\alpha^\beta \chi_{j\beta} = J_{ij}$ but in this case there are also the channels (1, 3) and (15, 1).

The invariant part of the perturbative side of the sum rule for $\int d^4x e^{iqx} \langle T(J(x)J(0)) \rangle$

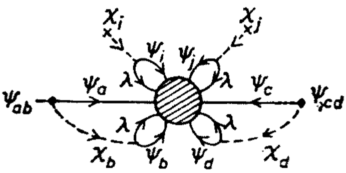

Fig. 10. Instanton process which generates a mass for the SU(4) sextet ϕ_{ab}

in the (1, 3) channel has the form [24] (in lowest order in $g^2/4\pi = \alpha_G$ with $\alpha_G\langle\chi^*\chi\rangle \sim O(1)$, see also Fig. 11)

$$-\frac{1}{8\pi^2}\log\frac{q^2}{\mu^2}+\frac{1}{q^2}\langle\chi^*\chi\rangle-\frac{2\pi\alpha_G}{(q^2)^2}\langle(\chi^*\vec{\tau}\chi)^2\rangle$$

$$+\frac{(2\pi\alpha_G)^2}{(q^2)^3}\langle(\chi^*\vec{\tau}\chi)^2\chi^*\chi\rangle+\dots \qquad (50)$$

($\vec{\tau}$ are the Pauli matrices in SU(2) isospin space, χ is the $\chi_{1/2}$ -isospin doublet). Using factorization the third and the following terms give a geometrical series which sums to a pole at a vector boson (supermultiplet) mass $m_W^2 = 2\pi\alpha\langle\chi^*\chi\rangle$. The first term corresponds

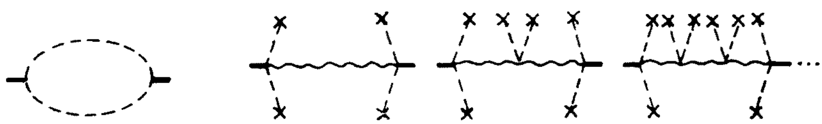

Fig. 11. Graphs corresponding to the expression (45)

to a loop graph and is most important for QCD sum rules. Here it can be interpreted [27] as part of a two-particle (W-Higgs) cut. In the (1, 15) channel there is only this kind of singularity and there is no indication for further vector bosons. Thus with the vacuum structure discussed above one just reproduces the spectrum of the SUSY standard model in a gauge invariant language. Composite quark/lepton and Higgs superfields are the standard elementary superfields dressed by the vacuum containing the fields χ^1 and χ^2. The "Novino" of the BPY model is a massless Higgsino.

Thus there is no hint for genuine composite vector bosons in the BPY model though of course our assumption about the higher condensates is only highly plausible and not proved and the ansatz for the spectrum in sum rules allows much freedom. SUSY gauge theories lead to scalar condensates. This gives sum rules completely different from QCD. Analogies with the QCD case are not allowed and we may be led back to a Higgs-standard picture. Inspection of the case (*ii*) with unbroken U(6) symmetry leads to the conclusion that such a phase is unlikely to exist at all.

A similar sum rule for the vector channel has been discussed before [28, 27] for the Abbot-Farhi model, a composite model with the field content of the standard model but with an unbroken SU(2) gauge symmetry supposed to lead to strong interactions and to QCD-type composites. Also in this case the standard model is a solution of the sum rules for a factorizing vacuum structure, as one would expect from a gauge-invariant formulation of the standard model. The effective weak coupling [28] (Fig. 12) then is equal to the gauge coupling α_G and *not* small in a strongly interacting system. In Ref. [28] we speculated about a more unconventional vacuum structure leading to $\alpha_{\rm eff} \ll \alpha_G$. Another possibility is that the scalar condensates are very large: $\langle\chi^*\chi\rangle \gg \Lambda$ such that α_G does not get big. This is technically possible with the limit (42) but not very plausible. Besides these difficulties there remains also the question how SUSY is broken. This has to be definitely answered if one wants to

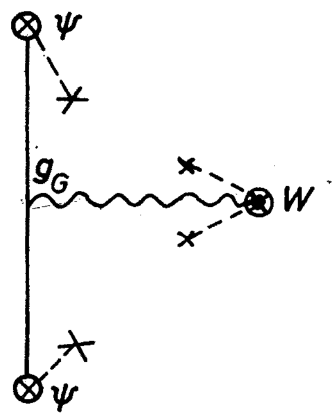

Fig. 12. Effective weak coupling of composite W to composite fermions in the case of dominance of χ-condensates

make contact with phenomenology. Still the idea that instanton effects generate the (Higgs) condensate of weak interactions is intriguing.

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