Vol. B19 (1988) ACTA PHYSICA POLONICA No 1

INFRARED ASYMPTOTICS OF THE QUARK PROPAGATOR IN GAUGE THEORIES

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(*Received January 30, 1987; revised version received June 15, 1987*)

Assuming a k^{-L} singularity for the infrared behaviour of the gluon propagator, it is shown that for $L > 3$ in Abelian gauge theories the quark propagator is an entire function of p^2 in the infrared region corresponding to a confining potential. An improved equation is solved for $L = 4$ and the solution is still regular on the mass shell. The four quark Green's function is studied for $L = 4$ in a Bloch-Nordsieck type model and it is shown that the cross section of quark-quark scattering vanishes even if an arbitrary number of soft gluons is included, a phenomenon we interpret as an evidence of confinement.

PACS numbers: 11.15.Tk

1. Introduction

The absolute confinement of quarks can manifest itself in the lack of singularities of the quark propagator in the infrared limit. In various gauges and approximations it has been shown that the quark propagator is vanishing on the mass shell (e.g. [1–6]), while in other approaches the quark propagator is the free one in the infrared limit [7, 8].

In all of these considerations the infrared behaviour of the gluon propagator was described by a more singular term than k^{-2}, namely in several cases a k^{-4} behaviour was used corresponding to a linear confining potential. The infrared behaviour of the gluon propagator (a possible k^{-4} term) was extensively studied by solving the Dyson-Schwinger equations in QCD [3, 9, 10].

The vanishing of the cross section of quark-quark scattering (even including an arbitrary number of soft gluons) is another manifestation of the confinement [11, 12].

As examples other confining theories which are much simpler than a four-dimensional NAGT the three-dimensional QED and the two-dimensional QCD have been studied [13, 14, 15].

In this paper we calculate the infrared asymptotics of the unrenormalized quark propagator S_F' by functional methods [16] (Section 2). Our approximation corresponds

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to a resummation of quark lines with many dressed gluon propagators, both ends of which are attached to the quark line. For the gluon propagator a k^{-L} singularity ($L > 0$) was assumed. This approximation can be interpreted as a result of an effective bilinear Lagrangian with an inverse propagator of the type k^L.

We show that for $L > 3$ the mass shell singularities of the quark propagator are cancelled. These values of L correspond to confining potentials. (Our treatment contains the well known results for three-dimensional QED [12].)

In Section 3 we improve the equation for the operator U used in Section 2 resumming further corrections with small momenta. Looking at $L = 4$ we argue that the quark propagator remains an entire function of p^2 in the infrared region.

In Section 4 we examine the expectation that the cross section of quark-quark scattering vanishes, if the quarks are confined [11, 12]. Assuming a k^{-4} singularity for the infrared behaviour of the gluon propagator it is shown that the four quark Green's function vanishes on the mass shell. The discussion is carried out for a Bloch-Nordsieck-type model in covariant gauges. Similar conclusions are valid for the Green's function with an arbitrary number of soft gluons.

Section 5 contains a discussion of the results.

2. *Infrared limit of the quark propagator*

We are working in axial gauges $n^2 \neq 0$ [17] where ghost loops are absent. In the infrared limit effects of quark loops are neglected. The quark propagator can be expressed by functional derivatives in the following form

$$S'_F(x-y) = N\left[G\left(x, y \,\middle|\, \frac{1}{i}\frac{\delta}{\delta J}\right) Z(J)\right]_{J=0}, \tag{2.1}$$

where

$$Z(J) = \exp\left[i\int d^4x L_I\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right]$$
$$\times \exp\left[-\frac{i}{2}\int d^4x \int d^4y J_{\mu a}(x) G^{\mu\nu}_{0ab}(x-y) J_{\nu b}(y)\right] \tag{2.2}$$

and

$$N^{-1} = [Z(J)]_{J=0}. \tag{2.3}$$

The Lagrangian L_I contains the self-couplings of gluons, $J^\mu_a(x)$ is an external colour current, $G^{\mu\nu}_{0ab}(x-y)$ is the free gluon propagator in axial gauge, and $G(x, y \mid A)$ means the Green's function of the quark moving in the external gluon field A. Under the above assumptions (2.1) is exact in QCD which can be verified by solving the Schwinger's equations for the vacuum functional by functional derivatives (see e.g. [16]). It is also trivial to get (2.1) from the well-known functional integral representation of S'_F. Clearly, (2.1) reproduces the perturbation series of S'_F expanding in g. While (2.1) is exact in axial gauges in the infrared limit for massive quarks, it is only an approximation in covariant gauges where ghosts contribute to S'_F. However, in order to get a feeling what can happen in covariant gauges in the infrared limit of S'_F, and also because of technical complexities we neglect ghosts in S'_F in Landau and Feynman-type gauges.

The dressed gluon Green's functions are given by the functional derivatives of $Z(J)$ at $J=0$ multiplied by N.

The Green's function $G(x, y \mid A)$ satisfies the equation

$$\left[i\gamma_\mu\left(\partial_x^\mu - ig\frac{\lambda_a}{2}A_a^\mu(x)\right) - m\right]G(x, y|A) = \delta^{(4)}(x-y), \tag{2.4}$$

m means the mass parameter of the quark, λ_a is the colour matrix. Let us introduce the functional $H(x, y \mid A)$ by the definition

$$G(x, y|A) = \left[i\gamma_\mu\left(\partial_x^\mu - ig\frac{\lambda_a}{2}A_a^\mu(x)\right) + m\right]H(x, y|A) \tag{2.5}$$

and from (2.4) we get

$$\left[-\partial_x^2 - m^2 + ig\frac{\lambda_a}{2}\gamma_\mu\gamma_\nu\partial_x^\mu A_a^\nu(x) + ig\lambda_a \right.$$

$$\left. \times A_{a\mu}(x)\partial_x^\mu + \frac{g^2}{4}(\gamma_\mu\lambda_a A_a^\mu(x))^2\right]H(x, y|A) = \delta^{(4)}(x-y). \tag{2.6}$$

The Fourier transform of $H(x, y \mid A)$, $\tilde{H}(p, q \mid A)$ determines the quark propagator in momentum space as follows

$$(2\pi)^4 S'_F(p)\delta^{(4)}(p-q) = S^{\mathrm{I}} + S^{\mathrm{II}},$$

$$S^{\mathrm{I}} = ((\gamma_\mu p^\mu + m)(\tilde{H}(p, q|A)NZ(J))_{J=0},$$

$$S^{\mathrm{II}} = \frac{g}{2(2\pi)^4}\gamma_\mu\lambda_a\left(\int d^4q'\tilde{A}_a^\mu(q')\tilde{H}(p-q', q|A)NZ(J)\right)_{J=0}, \tag{2.7}$$

here $\tilde{A}_a^\mu(q')$ is the Fourier transform of $A_a^\mu(x) \to \dfrac{1}{i}\dfrac{\delta}{\delta J_{a\mu}(x)}$ and $\tilde{H}$ satisfies the equation

$$(p^2-m^2)\tilde{H}(p, q|A) + \frac{1}{(2\pi)^4}\int d^4k\left[g\lambda_a p^\mu\tilde{A}_{a\mu}(k) - g\frac{\lambda_a}{2}(g_{\mu\nu} + i\sigma_{\mu\nu})k^\mu\tilde{A}_a^\nu(k)\right.$$

$$\left. + \frac{g^2}{4(2\pi)^4}\gamma_\mu\gamma_\nu\lambda_a\lambda_b\int d^4k'\tilde{A}_a^\mu(k')\tilde{A}_b^\nu(k-k')\right]\tilde{H}(p-k, q|A) = (2\pi)^4\delta^{(4)}(p-q). \tag{2.8}$$

Following Ref. [13] (the fifth parameter method of Fock) we represent $H(p, q \mid A)$ as the integral

$$\tilde{H}(p, q|A) = -i\int_0^\infty dv U(p, q; v|A)\exp\left[i(p^2-m^2+i\varepsilon)v\right], \tag{2.9}$$

where the new functional $U(p, q; v \mid A)$ obeys the normalisation

$$U(p, q; 0|A) = (2\pi)^4 \delta^{(4)}(p-q). \tag{2.10}$$

This can be shown by taking the Fourier transform of (2.6) in $x-y$ and writing the corresponding Fourier transform of H as the integral of the exponental function of its inverse with respect to v.

Substituting (2.9) into (2.8), using (2.10), leads to the definition equation of $U(p, q; v \mid A)$

$$\frac{d}{dv} U(p, q; v|A) - \frac{i}{(2\pi)^4} \int d^4k \left[g\lambda_a p^\mu \tilde{A}_{a\mu}(k) - g\frac{\lambda_a}{2} k^\mu (g_{\mu\nu} + i\sigma_{\mu\nu}) \tilde{A}^\nu_a(k) + \frac{g^2}{4(2\pi)^4} \right.$$
$$\left. \times \gamma_\mu \gamma_\nu \lambda_a \lambda_b \int d^4k' \tilde{A}^\mu_a(k') \tilde{A}^\nu_b(k-k') \right] \exp\left[i(k^2-2kp)v\right] U(p-k, q; v|A) = 0. \tag{2.11}$$

We need the functional U for p^2's around m^2. Since $\tilde{A}_{a\mu}(k)$ and the bracket in the integral act as gluon operators in the gluon Green's function in S^{I}, S^{II}, we can approximate the integrand in (2.11) by its value at $k \to 0$. Actually, assuming $\tilde{A}_{a\mu}(k) \approx k^{-L}$, $L < 4$, for $k \to 0$ the most singular term is $p\tilde{A}$ at $k \ll p \approx m$ in the bracket. Finally, we replace (2.11) in the infrared limit as follows

$$i\frac{d}{dv} U(p, q; v|A) + g\lambda_a p^\mu A_{a\mu}(2pv) U(p, q; v|A) = 0. \tag{2.12}$$

(Corrections to (2.12) will be studied in Section 3.) (2.12) has the usual time ordered operator solution, which making use of (2.7), (2.9), (2.10) yields

$$S^{\mathrm{I}} = -i(2\pi)^4 \delta^{(4)}(p-q)(\gamma_\mu p^\mu + m) \int_0^\infty dv \exp\left[i(p^2 - m^2 + i\varepsilon)v\right]$$
$$\times \left[1 + \sum_{n=1}^\infty (ig)^n \prod_{i=1}^n \lambda_{a_i} p^{\mu_i} \int_0^v dv_1 \ldots \int_0^{v_{n-1}} dv_n \langle T A_{a_1\mu_1}(2pv_1) \ldots A_{a_n\mu_n}(2pv_n)\rangle_0\right], \tag{2.13}$$

where $A_{a\mu}(2pv)$ is the interacting gluon field.

For S^{II} one obtains in the above approximation

$$S^{\mathrm{II}} = -\frac{ig}{2} \int_0^\infty dv \exp\left[i(p^2 - m^2 + i\varepsilon)v\right] \int d^4x \gamma_\mu \lambda_a \exp\left[i(p-q)x\right]$$
$$\times \left[\langle A^\mu_a(x)\rangle_0 + \sum_{n=1}^\infty (ig)^n \prod_{i=1}^n \lambda_{a_i} p^{\mu_i} \int_0^v dv_1 \ldots \int_0^{v_{n-1}} dv_n \langle T A_{a\mu}(x) \right.$$
$$\left. \times A_{a_1\mu_1}(2pv_1) \ldots A_{a_n\mu_n}(2pv_n)\rangle_0\right]. \tag{2.14}$$

Now we calculate (2.13) and (2.14) in such an approximation where gluons starting from the quark line are absorbed by the same line corresponding to keeping the propagators in the dressed gluon Green's function in (2.13), (2.14). (This is the only possibility in QED.)

For a k^{-4}-type gluon propagator, the summation has been carried out for the SU(2) gauge group [5], and also for SU(3) [6]. The result is that in the presence of the colour factors S_F' has a confining behaviour similar to the Abelian case with a k^{-4}-type gluon propagator. The summation of the remaining colour factors is extremely complicated even for an SU(2) gauge group in case of an arbitrary L, therefore we confine ourselves to an Abelian gauge group. Hence

$$\frac{1}{(2\pi)^4}\int d^4q S^{\mathrm{I}} = -i(\gamma_\mu p^\mu + m)\int_0^\infty dv \exp\left[i(p^2-m^2+i\varepsilon)v - ig^2\int_0^v dv_1 \int_0^{v_1} dv_2 f(v_1-v_2)\right], \tag{2.15}$$

$$\frac{1}{(2\pi)^4}\int d^4q S^{\mathrm{II}} = i\frac{g^2}{2}\int_0^\infty dv \exp\left[i(p^2-m^2+i\varepsilon)v\right]$$

$$\times \int_0^v \gamma^\mu p^\nu G_{\mu\nu}(2pv_0)dv_0 \exp\left[-ig^2\int_0^{v_1} dv_1 \int_0^{v_2} dv_2 f(v_1-v_2)\right], \tag{2.16}$$

where $f(v_1-v_2) = p_\alpha p_\beta G_0^{\alpha\beta}(2pv_1-2pv_2)$.

A similar type of exponential v dependence has been shown with a dipole gluon field too [18].

In covariant and axial gauges one has for $G_0^{\alpha\beta}(k)$ in d dimensions

$$G_{0c}^{\alpha\beta}(k) = -\frac{\Omega^{L-2}}{k^L}\left[g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2} + \alpha\frac{k^\alpha k^\beta}{k^2}\right], \tag{2.17}$$

$$G_{0a}^{\alpha\beta}(k) = -\frac{\Omega^{L-2}}{k^L}\left[g^{\alpha\beta} - \frac{k^\alpha n^\beta + k^\beta n^\alpha}{kn} + \frac{k^\alpha k^\beta}{(kn)^2}n^2 + (1+\delta)(4-d)\left(g^{\alpha\beta} - \frac{n^\alpha n^\beta}{n^2}\right)\right], \tag{2.18}$$

where $iG_0^{\alpha\beta}(x-y) = \langle TA^\alpha(x)A^\beta(y)\rangle_0$; $\alpha = 0$ (1) corresponds to the Landau (Feynman) gauge. Ω is a constant and δ is a parameter. The choice $\delta = 0$ is used in [8], $\delta = -1$ reproduces the usual axial gauge. $f(v_1-v_2)$ can be calculated in general covariant (f_c) and axial (f_a) gauges with the result

$$f_c(v_1-v_2) = i(-1)^{L/2+1}\frac{\Omega^{L-2}}{8\pi^{5/2}}\left[2(v_1-v_2)\right]^{L-d}$$

$$\times\Gamma(d-L)\left[\frac{L\Gamma(L/2-3/2)+2(\alpha-1)\Gamma(L/2-1/2)}{2\Gamma(L/2+1)}\right](p^2)^{L/2+1-d/2}, \tag{2.19}$$

$$f_A(v_1-v_L) = i\frac{\Omega^{L-2}}{8\pi^2}(v_1-v_2)^{L-d}\left\{\frac{2^{L-d}(-1)^{L/2+1}L\Gamma(L/2-3/2)\Gamma(d-L)(p^2)^{\frac{L-d}{2}}}{2\sqrt{\pi}\,\Gamma(L/2+1)}\right.$$

$$\times\left[p^2+(1+\delta)(4-d)\left(p^2-\frac{(pn)^2}{n^2}\right)\right]-\frac{\Gamma(d/2-L/2)\,(L-d+1)}{\Gamma(L/2)}(p^2)^{\frac{L-d}{2}}$$

$$\times\left[p^2F\left(1,d/2-L/2;\tfrac{3}{2};\frac{(pn)^2}{p^2n^2}\right)+2\frac{(pn)^2}{p^2n^2}\left(p^2-\frac{(pn)^2}{n^2}\right)\tfrac{4}{3}(d/2-L/2)\right.$$

$$\left.\times F\left(2,d/2-L/2+1;\tfrac{5}{2};\frac{(pn)^2}{p^2n^2}\right)\right]\Bigg\}, \qquad (2.20)$$

where $F(a, b; c; z)$ is the hypergeometric function [19]. For $\int_0^v dv_1\gamma^\mu p^\nu G_{0\mu\nu}(2pv_1)$ in S^{II} one obtains

$$\int_0^v dv_0\gamma^\mu p^\nu G_{\mu\nu}(2pv_0) = -iB\frac{v^{L-d+1}}{L-d+1}, \qquad (2.21)$$

with

$$B_c = (-1)^{L/2}(\gamma^\mu p_\mu)\frac{\Omega^{L-2}}{8\pi^{5/2}}(p^2)^{\frac{L-d}{2}}\Gamma(d-L)\frac{4^{L/2-d/2}}{2\Gamma(L/2+1)}$$

$$\times[L\Gamma(L/2-3/2)+2(\alpha-1)\Gamma(L/2-1/2)] \qquad (2.22)$$

$$B_A = \frac{\Omega^{L-2}}{8\pi^2}\left\{\frac{4^{L/2-d/2}(-1)^{L/2}L\Gamma(L/2-3/2)\,(p^2)^{L/2-d/2}}{2\sqrt{\pi}\,\Gamma(L/2+1)}\right.$$

$$\times\Gamma(d-L)\left[(\gamma p)+(1+\delta)(4-d)\left((\gamma p)-\frac{(\gamma n)\,(np)}{n^2}\right)\right]-\frac{\Gamma(d/2-L/2)\,(p^2)^{L/2-d/2}}{\Gamma(L/2)}$$

$$\times\left[\left((\gamma p)+\frac{(n\gamma)\,(np)}{n^2}(L-d+1)+\frac{(p\gamma)\,(np)^2}{p^2n^2}(L-d)\right)\right.$$

$$\times F\left(1,d/2-L/2;3/2;\frac{(pn)^2}{p^2n^2}\right)+\tfrac{4}{3}(L-d)\left(\frac{(pn)^2(p\gamma)}{p^2n^2}\right.$$

$$\left.\left.-(np)^4\frac{(p\gamma)}{p^4n^4}\right)F\left(2,d/2-L/2+1;3/2;\frac{(pn)^2}{p^2n^2}\right)\right]\Bigg\}. \qquad (2.23)$$

In the above approximation S^{I} and S^{II} become entire functions of p^2 in the infrared region if $2 \geqslant L-d+2 > 0$ in covariant and $3 > L-d+2 > 0$ in axial gauges. The upper bounds come from the existence of (2.15), (2.16). For instance in covariant gauges the violation of $0 < L-d$ can induce both regular (vanishing or nonvanishing) and singular S'_{F} depending on the value of L.

In covariant gauges at $d \to 4$ $S'_{\mathrm{F}} = 0$ for $L = 3, 4$; otherwise for $4 > L > 2S'_{\mathrm{F}}$ is nonvanishing and regular.

For these values of d and L the infrared singularities of the quark propagator are cancelled, thus no quark can appear asymptotically. These results contain the well known result for the three-dimensional QED [12].

A static potential can be defined from the gluon propagator Ω^{L-2}/k^L by the equation

$$V(x) = \int_{-\infty}^{\infty} dx_0 \int d^d k \exp(-ikx)\left[-\frac{\Omega^{L-2}}{k^L}\right]. \tag{2.24}$$

One can carry out the integrations [20] and gets

$$V(x) = \Omega^{L-2}\pi^2 \frac{\Gamma(L/2-d/2+1/2)\Gamma(d/2-1/2-L/2)\Gamma(d-L)}{\Gamma(L/2)\Gamma(d/2-L/2)\,|x|^{d-L-1}} \quad \text{if} \quad d-L-1 \neq 0,$$

$$V(x) \propto \ln |x| \quad \text{if} \quad d-L-1 = 0. \tag{2.25}$$

Hence $L-d+1 \geqslant 0$ leads to confining potential.

In four dimensions and in axial (covariant) gauges $5 > L \geqslant 3$ $(4 > L \geqslant 3)$ S'_F is regular in the infrared region so it corresponds to confining static potentials.

In both gauges for $3 > L > 2$ (nonconfining potentials) the singularities of S'_F are verified to depend on the regularization chosen.

For $L = 2$ one obtains the well-known result of QED, independently of the regularization. For $L < 2 S'_F$ is singular and these L's lead to nonconfining potentials.

3 Corrections of small momenta

Instead of (2.12) let the definition of U be

$$\frac{d}{dv} U(p, q; v|A) - \frac{i}{(2\pi)^4} \int d^4k g\lambda_a p_\mu \tilde{A}^\mu_a(k) \exp[i(k^2-2pk)v] U(p-k, q, v|A) = 0. \tag{3.1}$$

Using (3.1), (2.13) and (2.14) become

$$\frac{1}{(2\pi)^4}\int d^4q S^{\mathrm{I}} = -i(\gamma_\mu p^\mu + m)\int_0^\infty dv_0 \exp[i(p^2-m^2+i\varepsilon)]$$

$$\times\{1+\sum_{n=1}^{\infty}[\prod_{j=1}^{n}\int d^4k_j ig(p-\sum_{l=1}^{j-1}k_l)_{\mu j}\int_0^{v_{j-1}} dv_j \exp[(k_j^2-2$$

$$\times(p-\sum_{l=1}^{j-1}k_l)k_j)iv_j]\int d^4x_j \exp(ik_jx_j)]\,\langle TA^{\mu_1}_{a_1}(x_1)\ldots A^{\mu_n}_{a_n}(x_n)\rangle_0\}, \tag{3.2}$$

$$\frac{1}{(2\pi)^4}\int d^4q S^{\mathrm{II}} = -\frac{ig}{2}\int_0^\infty dv_0 \exp[iv_0(p^2-m^2+i\varepsilon)]\gamma_\mu\lambda_a\{\langle A^\mu_a(0)\rangle_0$$

$$+\sum_{n=1}^{\infty}\Big[\prod_{j=1}^{n}\int d^4k_j ig\Big(p-\sum_{l=1}^{j-1}k_l\Big)_{\mu_j}\int_0^{v_{j-1}}dv_j\exp\Big[\Big(k_j^2-2\Big(p\sum_{l=1}^{j-1}k_l\Big)k_j\Big)iv_j\Big]$$

$$\times\int d^4x_j\exp(ik_jv_j)\Big]\langle TA_a^{\mu}(0)A_{a_1}^{\mu_1}(x_1)\ldots A_{a_n}^{\mu_n}(x_n)\rangle_0\}. \tag{3.3}$$

Examine now the $L = 4$, $d = 4+\zeta$ case (linearly confining potential). Following the approximation of Section 2, one sees that in covariant gauges S_F' does not change as compared to the result on the uncorrected U.

In axial gauge we apply another approximation, namely

$$\Big(p-\sum_{l=1}^{j-1}k_l\Big)_{\mu}A_a^{\mu}\approx p_{\mu}A_a^{\mu}. \tag{3.4}$$

In the present case one can carry out the integrations for an SU(2) gauge group [18]. The formulas for S^{I}, S^{II} are complicated but still entire functions of p^2 in the infrared region. The correction terms with small momenta do not influence the conclusions of Section 2. Thus assuming a k^{-4} singularity for the infrared behaviour of the gluon propagator, the quark propagator is an entire function of p^2 in the infrared region.

4. *Quark-quark scattering in a Bloch-Nordsieck type model*

We examine the four-quark Green's function in a Bloch-Nordsieck type model

$$G_4(x_1, x_2, x_3, x_4) = \langle T\psi(x_1)\psi(x_2)\bar\psi(x_3)\bar\psi(x_4)\rangle_0. \tag{4.1}$$

It is known that for the four-quark Green's function for the case of an external gluon field A is

$$G_4(x_1, x_2, x_3, x_4|A) = G_2(x_1, x_4|A)G_2(x_2, x_3|A)-G_2(x_1, x_3|A)G_2(x_2, x_4|A) \tag{4.2}$$

(4.2) provides (4.1) in the same manner as $G\left(x, y\,\middle|\,\frac{1}{i}\frac{\delta}{\delta J}\right)$ determines $S_F'(x-y)$.

Let us introduce $U(v, x\,|\,A)$ by the definition

$$\tilde G_2(p, x) = -i\int_0^{\infty}dvU(v, x|A)\exp[-iv(m-up-i\varepsilon)], \tag{4.3}$$

where $\tilde G_2(p, x)$ means the Fourier transform of $G(x, y)$ with respect to $x-y$. For $U(v, x\,|\,A)$ we obtain

$$U(0, x|A) = 1,$$

$$\left[i\frac{\partial}{\partial v}+u_{\varrho}\left(i\partial^{\varrho}+g\frac{\lambda_a}{2}A_a^{\varrho}(x)\right)\right]U(v, x|A) = 0. \tag{4.4}$$

As it is known [20] this model corresponds to taking the Dirac matrices as c numbers u_{ν}, $u^2 = 1$, and we may assume $u_0 > 0$. Then it follows that contributions of closed quark loops are vanishing. Fourier transforming (4.1) and using (4.2) we get

$$\tilde{G}_4(p_1, p_2, p_3, p_4) = \int \exp[ix(p_1-p_3)]\tilde{G}_2(p_3, x)dx$$
$$\times \int \exp[ix'(p_2-p_4)]\tilde{G}_2(p_4, x')dx' NZ(J)|_{J=0} - \{3 \leftrightarrow 4\} \tag{4.5}$$

where N and Z are given in (2.2) and in (2.3). $\{3 \leftrightarrow 4\}$ means the interchange of the third and fourth variables in the first term. Being the gluon Green's function translation invariant, we can take $U(v, x \mid A)$ at $x - uv = 0$. (4.4) has the usual time ordered operator solution which making use of (4.3) and (4.5) yields for an Abelian theory (motivated by eliminating the extremely complicated colour summations and still keeping a confining S_F')

$$\tilde{G}_4(p_1, p_2, p_3, p_4) = -\int \exp[ix(p_1-p_3)]dx \int \exp[ix'(p_2-p_4)]dx' \int_0^\infty dv$$
$$\times \exp[-iv(m-up_3-i\varepsilon)] \int_0^\infty dv' \exp[-iv'(m-up_4-i\varepsilon')]$$
$$\times \sum_{n,m} (ig)^{n+m} \frac{1}{n!m!} \int_0^v dv_1$$
$$\ldots dv_n \int_0^{v'} dv_1' \ldots dv_m' \langle TB(x-uv+uv_1) \ldots B(x-uv+uv_n)B(x'$$
$$-uv'+uv_1') \ldots B(x'-uv'+uv_m')\rangle_0 - \{3 \leftrightarrow 4\}, \tag{4.6}$$

where $u_\varrho \frac{\lambda_a}{2} A_a = B$. One can carry out the resummation

$$\sum_{n,m}^{\infty} (ig)^{n+m} \frac{1}{n!m!} \int_0^v dv_1 \ldots dv_n \int_0^{v'} dv_1' \ldots dv_m' \left\langle T \prod_{i=1}^n B(i) \prod_{i=1}^m B(j') \right\rangle_0$$
$$= \exp(-g^2 bvv') \left[\exp\left(-\frac{g^2}{2} v^2 a - \frac{g^2}{2} v'^2 a'\right) - 1\right], \tag{4.7}$$

where $\langle B(i)B(j)\rangle_0 = a$, $\langle B(i')B(j')\rangle_0 = a'$, $\langle B(i)B(j')\rangle_0 = b$, $i = x-uv+uv_i$, $j = x-uv+uv_j$, $i' = x'-uv'+uv_i'$, $j' = x'-uv'+uv_j'$.

It turns out that if we add up all the virtual gluon exchanges (minimum one gluon exchange) combined with the terms where gluons starting from the quark line are absorbed by the same line the result exponentiates. Assuming a k^{-4} singularity for the infrared behaviour of the gluon propagator we get for a, a' and b in covariant gauges

$$a = a' = b = \frac{\Omega^2}{32\pi^2}(3+\alpha)\Gamma(\zeta), \tag{4.8}$$

where the dimension number is $d = 4+\zeta$. For $\tilde{G}_4$ we have

$$\tilde{G}_4(p_1, p_2, p_3, p_4) = -(2\pi)^8\delta^{(4)}(p_1-p_3)\delta^{(4)}(p_2-p_4)\int_0^\infty dv$$
$$\times \exp[-iv(m-up_3-i\varepsilon)]\int dv' \exp[-iv'(m-up_4-i\varepsilon')]$$
$$\times \exp\left[-\frac{g^2}{2}b(v^2+v'^2)\right][\exp(-g^2bvv')-1]-\{3\leftrightarrow 4\}. \tag{4.9}$$

The Taylor expansion of the integral is

$$\tilde{G}_4(p_1, p_2, p_3, p_4) = -(2\pi)^8\delta^{(4)}(p_1-p_3)\delta^{(4)}(p_2-p_4)\,\{\beta(2-\pi)+\beta^{3/2}\sqrt{\pi}$$
$$\times [\gamma+\gamma']+\beta^2\left[(4/3-\pi)(\gamma^2+\gamma'^2)+\left(\tfrac{4}{3}-\frac{4}{\pi}\right)\gamma\gamma'\right]+\beta^{5/2}\sqrt{\pi}$$
$$\times[3/2(\gamma^3+\gamma'^3)+1/2(\gamma'^2\gamma+\gamma'\gamma^2)]+\ \ldots\}-\{3\leftrightarrow 4\}, \tag{4.10}$$

where

$$\beta = \frac{4\pi^2}{g^2\Omega^2\Gamma(\zeta)}, \qquad \gamma = m-up_3, \qquad \gamma' = m-up_4. \tag{4.11}$$

G_4 is proportional to the constant $\beta(2-\pi)$, as γ and $\gamma' \to 0$. On the mass-shell only the pole term survives.

G_4 vanishes as $d \to 4$ ($\zeta \to 0$).

This is not surprising we encounter a similar result in QED: one cannot observe the charged particles separated from their long range electromagnetic field. In QED, however, the probability of observing charged particles plus an indefinite number of soft photons is nonvanishing [22]. What can we say about the cross section of soft gluon emission in quark-quark scattering?

Let us calculate the Green's function of four quarks plus a definite (h) number of outgoing soft gluons.

$$G = \left\{N\int \delta\psi\delta\bar{\psi}\exp[iS_0(\psi)]\right\} G_4\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\prod_{j=1}^{h}\frac{1}{i}\frac{\delta}{\delta J_{\mu j}(S_j)}$$
$$\times S_0\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\exp\left[iS_1\left(\frac{1}{i}\frac{\delta}{\delta J}\right)\right]\exp(-i/2JG_0J)|_{J=0}, \tag{4.12}$$

where S_0, J, G_0, N were given in Section 2. S_0^K is a term containing exclusively closed quark loop effects.

The physical gluon emission is correlated with the amputated Green's function

$$\tilde{G}(p_1, p_2, p_3, p_4, k_1 \ldots k_h)\varepsilon_{\mu 1}(k_1) \ldots \varepsilon_{\mu_h}(k_h)k_1^2 \ldots k_h^2, \tag{4.13}$$

where k_i is the momentum and ε_{μ_i} is the polarisation vector of the i^{th} outgoing gluon. Using the properties of U we can calculate $\tilde{G}_4(p_1, p_2, p_3, p_4, k_1, \dots k_h)$

$$\tilde{G}(p_1, p_2, p_3, p_4, k_1 \dots k_h) = -\int \exp\left(i \sum_{j=1}^{h} k_j y_j\right) \prod_{j=1}^{h} dy_j \int_0^{\infty} dv$$

$$dv' \exp\left[-iv(m-up_3-i\varepsilon)\right] \exp\left[-iv'(m-up_4-i\varepsilon')\right] \sum_{n,m} \frac{1}{n!m!}$$

$$\times \int dv_1 \dots dv_n dv'_1 \dots dv'_m \langle TA_1(y_1) \dots A_h(y_h) B(1)$$

$$\dots B(n) B(1') \dots B(m') \rangle_0 (2\pi)^8 \delta^{(4)}(p_1-p_3) \delta^{(4)}(p_2-p_4) - \{3 \leftrightarrow 4\}. \tag{4.14}$$

If we add up all the gluon exchanges (minimum one) combined with terms where gluons starting from the quark line are absorbed by the same line, for a fix number of outgoing gluons the result exponentiates

$$\tilde{G}(p_1, p_2, p_3, p_4, k_1, \dots k_h) \propto \prod c_i \int_0^{\infty} dv dv' \left\{ \exp\left[-\frac{g^2}{2} a(v+v')^2\right]\right.$$

$$\left. -\exp\left[-\frac{g^2}{2} a(v^2+v'^2)\right]\right\}, \tag{4.15}$$

where a is given above and

$$C_i = \langle A(i) B(j) \rangle_0.$$

The on-shell Green's function is

$$(2\pi)^8 \delta^{(4)}(p_1-p_3) \delta^{(4)}(p_2-p_4) \prod_{i=1}^{h} \left[\delta^{(4)}(k_i) c_i k_i^2 \varepsilon_{\mu_i}(k_i) (2\pi)^4\right]$$

$$\times \left[\beta(2-\pi) + \beta^{3/2} \sqrt{\pi} (\gamma+\gamma') + \dots\right] - \{3 \leftrightarrow 4\}.$$

(4.17) vanishes for a fix number of outgoing gluons. Hence the cross section of quark--quark scattering is zero even if the emission due to an indefinite number of soft gluons is taken into account. This is in accordance with the theorem of Kinoshita, Lee and Neuenberg [23, 24].

5. Discussion

In the present paper we have calculated the quark propagator in the infrared limit by functional methods for different types of (k^{-L}) infrared gluon propagators. Using the same approximation in different gauges it was shown, that special values of L correspond to confining quark propagators (in four dimensions and in axial gauge for

$5 > L > 3$ S_F' is regular in the infrared region). For $L > 5(4)$ and $d = 4$ in axial (covariant) gauge the method does not work because the integrals do not exist ($A > 0$).

For $L = 4$ an improved equation was examined. Under the ν integrals in (2.15) and (2.16) exclusively analytic expression of g^2 take place. After the ν integration this is no longer true, S_F' cannot be expanded around $g^2 = 0$.

It has been argued that the cross section of quark-quark scattering vanishes even including an indefinite number of soft gluons. We interpret this result as a signal that quarks and gluons are never produced in asymptotic states, i.e. for confinement.

Due to algebraic complications mainly an Abelian or in Sect. 3 for $L = 4$ an SU(2) gauge group and in Sect. 4 a Bloch-Nordsieck-type model was assumed.

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