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DYNAMICAL DIMENSIONAL REDUCTION IN MULTIDIMENSIONAL BIANCHI I MODELS*

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We discuss some mechanisms of isotropization in the class of n-dimensional Bianchi I models. We show that these models can isotropize but this process leads to the breakdown of dimensional reduction. Thus we ought to look for such a mechanism of dimensional reduction that isotropizes the space.

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1. *Introduction*

Chodos and Detweiler [1] considered vacuum Einstein equations in the case of 5-dimensional spacetime. Assuming the Kasner type of metric

$$ds^2 = dt^2 - \sum_{i=1}^{4} \left(\frac{t}{t_0}\right)^{2p_i} (dx^i)^2, \tag{1}$$

where

$$\sum_{i=1}^{4} p_i = \sum_{i=1}^{4} p_i^2 = 1,$$

and imposing the condition that the total space was isotropic in 4 dimensions, they found the following solution: $p_1 = p_2 = p_3 = -p_4 = 1/2$. This solution represents the model with expanding macrospace and contracting microspace, i.e. the dimensional reduction takes place herein. More systematic approach to the problem of dimensional reduction

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was proposed by Demiański et al. [2] when the full classification of homogeneous multidimensional cosmologies was performed. Many models from this classification have been investigated in the literature and were found to possess the property of dynamical dimensional reduction [3]. However, there still exists the problem of choice of physically interesting homogeneous multidimensional models. For instance, those models which can isotropize are physically interesting. In the case of classical Bianchi models, only those containing the Friedman models can isotropize [4]. Because the space has in multidimensional case the structure of the product $M^3 \times B$ where M^3 is the homogeneous macrospace and B is a compact homogeneous microspace, the models which isotropize in macro and microspace are physically interesting. Because macro and microspace are dynamically coupled the choice of FRW models as candidates for M^3 does not guarantee isotropization of macro and microspace. Therefore there is no simple criterion of choice for models interesting from the point of view of isotropization within the class of multidimensional homogeneous models [5].

In constructing their solution Chodos and Detweiler assume the isotropy of both macro and microspace. It is, however, unlikely that the universe starts from such special initial conditions. In a more realistic description we give up the assumption of isotropy and we take into account some mechanisms isotropizing the space — the matter for example. Considering an n-dimensional Bianchi I model with matter, it turns out that there exist models in the neighbourhood of the point determined by Chodos and Detweiler which tend to isotropy when dimensional reduction takes place.

Many vacuum multidimensional models with dynamical dimensional reduction are known in the literature. In general dimensional reduction ends when the microspace reaches constant or zero size. The investigation of the influence of matter upon this process is in fact the investigation of stability of dynamical compactification with respect to perturbations caused by matter. A qualitatively new feature of multidimensional cosmological models is the existence of Kasner asymptotic behaviour near the singularity for a wide class of homogeneous multidimensional models. This problem was investigated by Demaret [6] in the context of vacuum Einstein equations and its significance in cosmology was pointed out by Ishihara [7] and Halpern [7]. Therefore the investigation of dynamical reduction in Bianchi type I models is very important and this model can serve as a typical description near the singularity.

In the present work we investigate the dimensional reduction in a Bianchi I multidimensional model and the influence of matter upon this process. Its typical character is also discussed.

2. *Kasner asymptotic behaviour in homogeneous multidimensional models*

The problem of classification of homogeneous multidimensional models has been solved in [2]. Table I illustrates this classification in the case of 11-dimensional models.

We assume the metric in the form:

$$ds^2 = dt^2 - g_{ij}(t)\omega^i\omega^j, \quad i, j = 1, \ldots, n,$$

TABLE I

The classification of 10-dimensional homogeneous spaces $M^{10} = M^3 \times B^7$. M^3 is one of nine Bianchi types, B^7 is a compact homogeneous space. L_1 is a one-dimensional algebra, $L_3(IX)$ is isomorphic to SO(3)

Type of algebra generating a symmetry group on B^7
$\bigoplus_{i=1}^{7} L_1^{(i)}$
$\bigoplus_{i=1}^{4} L_1^{(i)} \oplus L_3(IX)$
$L_1 \oplus L_3(IX) \oplus L_3(IX)$

where ω^i, $i = 1, \ldots, n$ are basis one-forms. The Ricci tensor components are the following:

$$R_0^0 = -1/2\dot{\kappa}_i^i - 1/4\kappa_j^i\kappa_i^j$$

$$R_i^0 = -1/2\kappa_k^j(C_{ji}^k - \delta_i^k C_{lj}^l)$$

$$R_i^j = P_i^j - \frac{1}{2\sqrt{|g|}}\frac{d}{dt}(\sqrt{|g|}\,\kappa_i^j),$$

where $\kappa_k^j = \dot{g}_{ki}g^{ij}$, $g = \det g_{ij}$ and

$$P_{ij} = -\Gamma_{il}^k\Gamma_{jk}^l - C_{lk}^l\Gamma_{ij}^k$$

$$\Gamma_{ij}^k = 1/2(C_{ij}^k + C_{ij}^m g_{ml}g^{kl} + C_{jl}^m g_{mi}g^{kl}).$$

The Einstein equations have the following form:

$$R_\nu^\mu - 1/2\delta_\nu^\mu R + \Lambda\delta_\nu^\mu = T_\nu^\mu, \quad \mu, \nu = 0, 1, \ldots, n,$$

where $T_\nu^\mu = \mathrm{diag}\,(\varrho, -p, \ldots, -p)$. By using the standard methods [8] we can show that the metric of the product $M^3 \times B$ can be made diagonal in the case of Bianchi I, II, VI_0, VII_0, VIII, IX type macrospace, whereas in the other cases (i.e. Bianchi V, IV, VI_h, VII_h) it can be reduced to the form with one nondiagonal component g_{12}. In the classical 3-dimensional Bianchi models the curvature terms cannot be neglected near the singularity. In the case of multidimensional homogeneous cosmological models there is a wide class of models for which the effect of curvature is negligible near the singularity i.e. there exists Kasner asymptotic behaviour: $r_j \sim t^{p_j}$, $j = 1, \ldots, n$ and $\sum_{j=1}^{n} p_j = \sum_{j=1}^{n} p_j^2 = 1$. Figs 1, 2 and 3 illustrate sets of exponents on the plane (P, p_1), where $P = \sum_{j=4}^{n} p_j$, for which curvature effects are negligible.

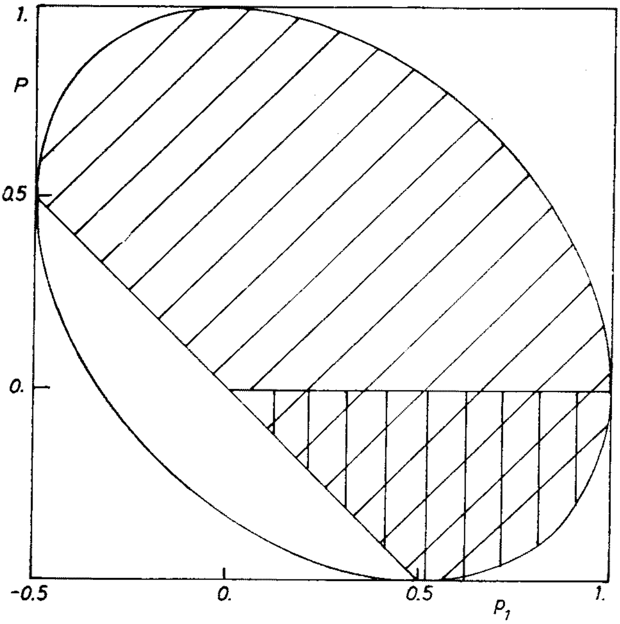

Fig. 1. Shaded area represents the set of exponents for which Kasner asymptotic behaviour occurs in $B(II)\times T^D$ model (T^D is D-torus). The region with $P = \sum_{j=4}^{n} p_j < 0$ corresponds to the situation when the microspace contracts (double shading)

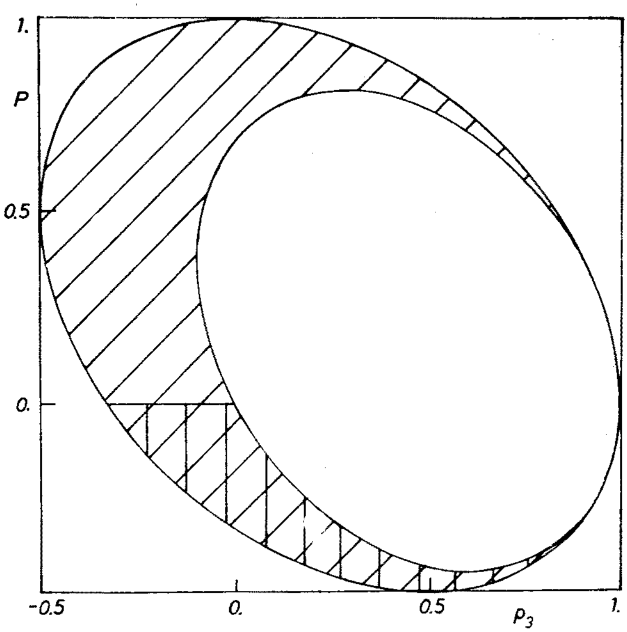

Fig. 2. The set of exponents for which Kasner asymptotic behaviour exists (shaded area) and additionally the microspace contracts (double shaded area) is shown in case of $B(IX)\times T^D$ or $B(VIII)\times T^D$ models (T^D is D-torus)

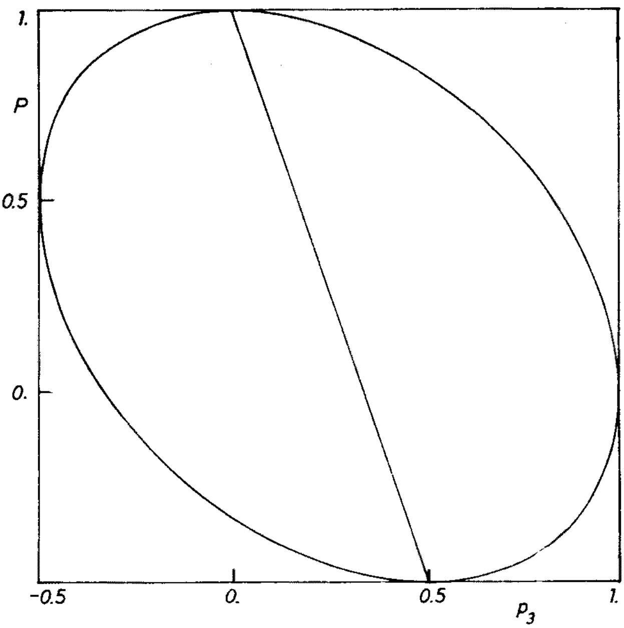

Fig. 3. In case of the B(V)$\times$T^D model (T^D is D-torus) the exponents leading to Kasner asymptotic behaviour lie on the line $P = 1-3p_3$

3. Dimensional reduction in a generalized Bianchi I model — a vacuum case

Assuming the diagonal form of the metric tensor with an abelian group of isometries acting on n-dimensional spacelike sections $\{t = \text{const.}\}$:

$$ds^2 = dt^2 - \sum_{j=1}^{n} r_j^2(t)\,(dx^j)^2 \tag{2}$$

Einstein equations take the form:

$$R_{\mu\nu} = 8\pi\bar{G}\left(T_{\mu\nu} - \frac{1}{n-1}\,g_{\mu\nu}T\right) - \frac{2}{n-1}\,\Lambda \tag{3}$$

$$\mu, \nu = 0, 1, \ldots, n,$$

where $T_{\mu\nu} = (\varrho+p)u_\mu u_\nu - pg_{\mu\nu}$ is the perfect fluid energy-momentum tensor, ϱ is energy density, p is pressure, $\bar{G}$ is the generalized gravitational constant, $\bar{\Lambda} = \dfrac{2\Lambda}{n-1}$ is the cosmological constant in $n+1$ dimensions. The equations of motion $T^{\mu\nu}{}_{;\mu} = 0$ for the state equation $p = p(\varrho)$ read

$$\frac{\dot{\varrho}}{\varrho+p} + \frac{\dot{V}}{V} = 0, \tag{4}$$

where $V = \prod_{j=1}^{n} r_j$ is the spatial volume, and the dot denotes differentiation with respect to the cosmological time t.

We shall assume the equation of state in the form: $p = \gamma \cdot \varrho$, $0 \leqslant \gamma \leqslant 1$, where $\gamma = 0$ for dust and $\gamma = 1/n$ for radiation. Then (4) can be integrated and gives:

$$\varrho = \varrho_0 V^{-(1+\gamma)}. \tag{5}$$

The only non-zero components of the Ricci tensor for the metric (2) are:

$$-R_0^0 = \sum_{j=1}^{n} \left(\frac{\dot{r}_j}{r_j}\right)^{\cdot} + \sum_{j=1}^{n} \left(\frac{\dot{r}_j}{r_j}\right)^2,$$

$$-R_j^j = \left(\frac{\dot{r}_j}{r_j}\right)^{\cdot} + \frac{\dot{r}_j}{r_j} \sum_{i=1}^{n} \frac{\dot{r}_i}{r_i}, \quad j = 1, \ldots, n. \tag{6}$$

The solutions of equations (3) in the vacuum case with Λ are:
for $\Lambda < 0$:

$$r_j = r_{0j} \sin^{1/n} \beta t \cdot \mathrm{tg}^{\beta_j/\beta} \beta t/2,$$

$$\sum_{i=1}^{n} \beta_i, = 0 \ \sum_{i=1}^{n} \beta_i^2 = (1-n)\bar{\Lambda}, \ \beta^2 = -n\bar{\Lambda} \tag{7}$$

for $\Lambda < 0$:

$$r_j = r_{0j} t^{p_j}, \quad \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} p_i^2 = 1, \tag{8}$$

for $\Lambda > 0$:

$$r_j = r_{0j} \sinh^{1/n} \alpha t \cdot \mathrm{th}^{\alpha_j/\alpha} \alpha t/2,$$

$$\sum_{i=1}^{n} \alpha_i = 0, \quad \sum_{i=1}^{n} \alpha_i^2 = (1+n)\bar{\Lambda}, \quad \alpha^2 = n\bar{\Lambda}. \tag{9}$$

The solutions (7)–(9) have been known for a long time and can be found in Petrov [9]. (8) is a generalized Kasner solution.

We shall now investigate the typical cases of dynamical reduction for the models with $\Lambda = 0$, $\Lambda < 0$ and $\Lambda > 0$ subsequently.

A. The case with $\Lambda = 0$

Every model described by the solution (8) is uniquely determined by the system of $n-2$ independent exponents p_j. Let us assume that two of them, say p_1, p_2 are eliminated by using the constraint condition i.e.

$$p_1 = 1 - p_2 - p_3 - P,$$

$$p_2 = 1/2[1 - P - p_3 + (1 - 3p_3^2 - P^2 - 2p_3 P + 2p_3 + 2P - 2 \sum_{j=4}^{n} p_j^2)^{1/2}], \tag{10}$$

where

$$P = \sum_{j=4}^{n} p_j.$$

Then the space $U = \{(p_3, \ldots, p_n)\}$ would define a Kasner solution if there exists a real exponent p_2 i.e. if:

$$1-3p_3^2-P^2-2p_3P+2p_3+2P-2\sum_{j=4}^{n} p_j^2 \geqslant 0. \tag{11}$$

The condition (11) determines the set of models within U which represents the Kasner solution (8). We see from the condition (10) that if the microspace contracts ($P < 0$ for $\dot{V}_m < 0$) then the macrospace has to expand ($\sum_{j=1}^{3} p_j = 1-P$). The converse need not to be true. Thus $U_1 = \{(p_3, \ldots, p_n)$: (11) holds and $P < 0\} \subset U$ defines the subset in which dimensional reduction takes place. It can be shown that (11) defines the interior of certain hyper-ellipsoid in the space U. The discussion becomes particularly simple if we limit ourselves to the case $\sum_{j=4}^{n} p_j^2 = \alpha \cdot P^2$. Then U_1 is reduced to the subset of the plane:

$$U_1^* = \{(p_3, P) : P < 0 \text{ and } 1-3p_3^2-(1+2\alpha)P^2-2p_3P+2p_3+2P \geqslant 0\}.$$

In the case $\alpha = 1$ we have a generalization of the case considered by Chodos and Detweiler in which $P = p_4$, see Fig. 4. The point described by Chodos and Detweiler lies on the line $P = 1-3p_3$ as it is illustrated in Fig. 4. It does not possess any open neighbourhood, however there exist nearby models belonging to U_1. For the n-dimensional space we choose $\alpha = \dfrac{1}{n-1}$. Then we get a similar picture as in Fig. 4. The isotropic solution of Ishihara

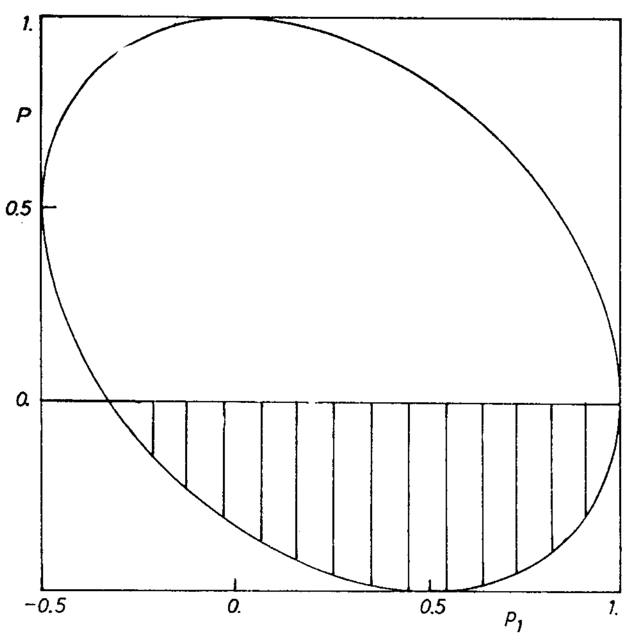

Fig. 4. The large ellipse surrounds the region of Kasner exponents P, p_1 identifying the model. Shaded area represents the set of exponents for which dimensional reduction takes place

lies then also on the boundary of this region:

$$p_1 = p_2 = p_3 = p^* = 1/n\left[1-\sqrt{\frac{3(n-1)}{n-3}}\right],$$

$$p_4 = \ldots = p_n = p = 1/n\left[1+\sqrt{\frac{(n-3)(n-1)}{3}}\right]. \tag{12}$$

B. The cases $\Lambda < 0$, $\Lambda > 0$

At the beginning of the analysis of dimensional reduction in this class of models we shall determine the space of independent parameters describing the solution. It is a subspace of the space

$$U = \{(\beta_1, \ldots, \beta_n) : \beta_1 + \ldots + \beta_n = 0 \wedge \beta_1^2 + \ldots + \beta_n^2 = (1-n)\bar{\Lambda}\}.$$

Proceeding the same way as in the previous case we eliminate two parameters β_1, β_2 which describe the dynamics of macrospace. The condition that real β_2 exists determines the interior of a certain hyperellipsoid:

$$\beta^2+3\beta_3^2+2\beta\beta_3+2\sum_{j=4}^{n}\beta_j^2-2(1-n)\bar{\Lambda} \leqslant 0 \tag{13}$$

for $\Lambda < 0$, and:

$$\alpha^2+3\alpha_3^2+2\alpha\alpha_3+2\sum_{j=4}^{n}\alpha_j^2-2(\ \ +n)\bar{\Lambda} \leqslant 0 \tag{14}$$

for $\Lambda > 0$, respectively. Assuming $\sum_{j=4}^{n}\alpha_j^2 = \kappa\alpha^2$, $\sum_{j=4}^{n}\beta_j^2 = \chi\beta^2$ we can reduce this to the 2-dimensional case. The condition that a point lies on the boundary of the ellipse implies $\alpha_1 = \alpha_2$ ($\beta_1 = \beta_2$) respectively. Thus the model with isotropic micro and macrospace lies on the line $\alpha = -3\alpha_3$ ($\beta = -3\beta_3$). This point has the following coordinates:

$$\beta_1 = \beta_2 = \beta_3 = \beta^* = \tfrac{1}{3}\sqrt{(1-n)\bar{\Lambda}(n-3)}, \quad \beta_4 = \ldots = \beta_n = -\sqrt{\frac{(1-n)\bar{\Lambda}}{(n-3)n}},$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha^* = \tfrac{1}{3}\sqrt{(1+n)\bar{\Lambda}(n-3)}, \quad \alpha_4 = \ldots = \alpha_n = -\sqrt{\frac{(1+n)\bar{\Lambda}}{(n-3)n}}. \tag{15}$$

When microspace contracts and total space expands i.e. $\dot{V}_m < 0$, $\dot{V} > 0$, then the macrospace expands. Dimensional reduction takes place when the microspace contracts. This happens if:

$$\sum_{j=4}^{n}\alpha_j = \alpha < -\sqrt{n\bar{\Lambda}}\,\frac{n-3}{n}\cosh\sqrt{n\bar{\Lambda}}\,t \tag{16}$$

for $\Lambda > 0$, and:

$$\sum_{j=4}^{n} \beta_j = \beta < -\frac{\sqrt{-n\bar{\Lambda}}}{n}(n-3)\cos\sqrt{-n\bar{\Lambda}}\,t \tag{17}$$

for $\Lambda < 0$. The condition (16) is valid only for $t < t_0$, where t_0 is such that

$$\alpha = -\sqrt{n\bar{\Lambda}}\,\frac{n-3}{n}\cosh \alpha t_0$$

and $V_m(t_0) = \text{const}$.

Near the singularity ($t \to 0$) (16) takes the form $\alpha < -\sqrt{n\bar{\Lambda}}\,\frac{n-3}{n}$ and the corresponding subset of models undergoing dimensional reduction is typical in the space of independent parameters. The condition (17) implies that models with dimensional reduction have to fulfill the condition $\beta < \sqrt{-n\bar{\Lambda}}\,\frac{n-3}{n}$ near the singularity, for all $n > 3$.

In the case of models with $\Lambda < 0$ it is not, in general, sufficient to determine the class of spaces with contracting microspace since the total space expands and then contracts to the final singularity, i.e.

$$\frac{\dot{V}}{V} = \frac{\dot{V}_m}{V_m} + \frac{\dot{V}_M}{V_M}\begin{cases} > 0 & \text{for} \quad t \in \left[0, \dfrac{\pi}{2\sqrt{-n\bar{\Lambda}}}\right], \\ < 0 & \text{for} \quad t \in \left[\dfrac{\pi}{2\sqrt{-n\bar{\Lambda}}}, \dfrac{\pi}{\sqrt{-n\bar{\Lambda}}}\right] \end{cases}$$

and

$$(\ln V_M)^{\cdot} = \frac{3}{n}\sqrt{-n\bar{\Lambda}}\operatorname{ctg}\sqrt{-n\bar{\Lambda}}\,t + \frac{\sum_{i=1}^{3}\beta_i}{\sin\sqrt{-n\bar{\Lambda}}\,t},$$

$$(\ln V_m)^{\cdot} = \frac{n-3}{n}\sqrt{-n\bar{\Lambda}}\operatorname{ctg}\sqrt{-n\bar{\Lambda}}\,t + \frac{\sum_{j=1}^{3}\beta_j}{\sin\sqrt{-n\bar{\Lambda}}\,t}. \tag{18}$$

Thus the contraction of the microspace is generally accompanied by the expansion of the macrospace for $t \in \left[0, \frac{\pi}{2\sqrt{-n\bar{\Lambda}}}\right]$. The necessary condition of macrospace contraction is:

$$\beta < -\frac{n-3}{n}\sqrt{-n\bar{\Lambda}}.$$

For $t \in \left[\frac{\pi}{2\sqrt{-n\bar{\Lambda}}}, \frac{\pi}{\sqrt{-n\bar{\Lambda}}}\right]$ it is possible that macrospace expands further or contracts to the final singularity. It is sufficient that $\beta < 0$ for the microspace to contract. The sufficient condition of macrospace expansion ($\dot{V}_M > 0$) is:

$$\beta < -\frac{3}{n}\sqrt{-n\bar{\Lambda}}.$$

It is easy to see that the following sets are typical in the space U:

$$U_2 = \left\{(\beta, \beta_3) : \beta < -\frac{n-3}{n}\sqrt{-n\bar{\Lambda}}\right\},$$

$$U_3 = \left\{(\beta, \beta_3) : \beta < -\frac{3}{n}\sqrt{-n\bar{\Lambda}}\right\}, \tag{19}$$

where U_2 is the set of models with contracting macrospace and U_3 is the set of models with expanding macrospace. For $n < 6$ if microspace contracts then the macrospace expands to infinity or contracts after expansion phase. For $n > 6$ the contraction of microspace is equivalent to the expansion of macrospace. Near the singularity i.e. for $t \to 0$ the total spatial volume is:

$$V(t) \sim \beta t - \frac{\beta^3}{3!}t^3, \quad \text{for} \quad \Lambda < 0$$

$$V(t) \sim \beta t + \frac{\beta^3 t^3}{3!}, \quad \text{for} \quad \Lambda > 0.$$

It corresponds to Kasner vacuum solutions up to linear terms. This means that the cosmological constant is negligible near $t = 0$. The asymptotic behaviour of scale factors is described as follows:

$$r_j \sim t^{\beta_j/\beta + 1/n} \quad \text{for} \quad \Lambda < 0,$$

$$r_j \sim t^{\alpha_j/\alpha + 1/n} \quad \text{for} \quad \Lambda > 0. \tag{20}$$

This is Kasner asymptotic behaviour i.e.:

$$\sum_{j=1}^{n}\left(\frac{\beta_j}{\beta} + \frac{1}{n}\right) = \sum_{j=1}^{n}\left(\frac{\beta_j}{\beta} + \frac{1}{n}\right)^2 = 1,$$

$$\sum_{j=1}^{n}\left(\frac{\alpha_j}{\alpha} + \frac{1}{n}\right) = \sum_{j=1}^{n}\left(\frac{\alpha_j}{\alpha} + \frac{1}{n}\right)^2 = 1.$$

Models with $\Lambda > 0$ which isotropize for large times t possess de Sitter asymptotic behaviour $r_j \sim \exp\sqrt{\bar{\Lambda}/n}\, t$. For $\Lambda < 0$ and $t \to \frac{\bar{\Lambda}}{\beta}$ the asymptotic behaviour is the following: $r_j \sim (t_0 - t)^{\beta_j/\beta + 1/n}$ (Kasner). In case of models with dimensional reduction (both $\Lambda > 0$ or $\Lambda < 0$) $V_m \to \infty$, $V_M \to 0$, $V \to 0$ in the initial singularity.

4. Dimensional reduction in Bianchi I models filled with dust

In the case of dust i.e. $p = 0$ and $\Lambda = 0$ equations (6), (3) and (5) give the following formulae:

$$\left(\frac{\dot{r}_j}{r_j}\right)^{\cdot} + \frac{\dot{r}_j}{r_j}\frac{\dot{V}}{V} = 8\pi\bar{G}\frac{\varrho}{n-1},$$

$$\sum_{j=1}^{n}\frac{\ddot{r}_j}{r_j} = 8\pi\bar{G}\varrho\frac{n-2}{n-1},$$

$$\varrho = \frac{\varrho_0}{V}. \tag{21}$$

The solutions of (21) are the following:

$$V = \frac{8\pi\bar{G}\varrho_0}{2(n-1)}t^2+bt, \qquad b = \text{const.} > 0.$$

$$r_j = r_{0j}t^{p_j}\left(\frac{a}{2}t+b\right)^{\frac{2}{n}-p_j}, \tag{22}$$

where

$$a = \frac{8\pi\bar{G}\varrho_0}{n-1}, \qquad \sum_{i=1}^{n}p_i = \sum_{i=1}^{n}p_i^2 = 1,$$

$$r_{0j} = \text{const.}$$

Near the singularity i.e. when $t \to 0$ we have Kasner asymptotic behaviour: $r_j \sim t^{p_j}$, while for large t: $r_j \sim t^{2/n}$. Dimensional reduction takes place when the microspace contracts. This happens if

$$\frac{\dot{V}}{V} = \frac{\dot{V}_M}{V_M}+\frac{\dot{V}_m}{V_m} > 0,$$

$$\frac{\dot{V}_M}{V_M} = \frac{\frac{a}{n}3t+b\sum_{j=1}^{3}p_j}{\frac{a}{2}t^2+bt},$$

$$\frac{\dot{V}_m}{V_m} = \frac{\frac{a}{n}(n-3)t+b\sum_{j=1}^{3}p_j}{\frac{a}{2}t^2+bt} < 0,$$

and the condition of microspace contraction is equivalent to the following one. $\sum_{i=1}^{n} p_i < -\frac{a}{n}\frac{(n-3)t}{b}$. Obviously it is true for $t < t_0 = -\frac{nb}{a(n-3)}\sum_{j=4}^{n} p_j$ only. Until this moment the microspace contracts and then begins to expand. The asymptotic behaviour for large t does not depend on particular choice of the parameters identifying the model.

5. Dimensional reduction in models with radiation $p = 1/n \cdot \varrho$ — a unified form of the metric for the equation of state $p = \gamma\varrho$

Using the unified form of the metric for a classical Bianchi I model [10] we see that it is worthwhile to seek for solutions of the n-dimensional model in the form

$$ds^2 = A^{2\gamma}dt^2 - \sum_{i=1}^{n} t^{2p_i}A^{2q_i}(dx^i)^2, \tag{23}$$

where

$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} p_i^2 = \sum_{i=1}^{n} q_i^2 = 1.$$

Then solutions of (6) are the following:

$$q_i = \frac{2}{n} - p_i, \quad \varrho = \frac{\varrho_0}{(tA)^{\gamma+1}}, \quad \varrho_0 = \frac{2(n-1)}{8\pi G n}\alpha_m,$$

$$A^{1-\gamma} = \alpha_s + \alpha_m t^{1-\gamma}, \quad \alpha_s, \alpha_m = \text{const.} \tag{24}$$

For given γ the solution depends on three parameters: p_i, α_s, α_m and only two of them are important because we are always able to rescale α_s or α_m. If $\alpha_m = 0$ we can choose $\alpha_s = 1$ and get the generalized Kasner solution. If $\alpha_s = 0$ we rescale $\alpha_m = 1$ and obtain a generalized flat Friedman model:

$$ds^2 = dt^2 - t^{\frac{4}{n(1+\gamma)}}\sum_{i=1}^{n}(dx^i)^2. \tag{25}$$

If $\alpha_m\alpha_s \neq 0$ we have the generalized Jacobs solution [11]. If $\alpha_s > 0$ then $A > 0$ for all $t \geqslant 0$ and a singularity appears in this class of models for $t = 0$. If $\alpha_s < 0$ then a singularity appears at a certain positive moment t_s such that $A(t_s) = 0$. Near the singularity we have Kasner asymptotic behaviour while for large t isotropic behaviour occurs.

From the point of view of dimensional reduction these models exhibit the so-called dimensional reduction breaking which is a characteristic feature of dust models. These models never isotropize to the point described by Chodos and Detweiler. In the generalized Bianchi I model we have always certain anisotropy until the moment of full contraction of the microspace, while in the classical case the influence of matter isotropizes the model.

6. Conclusions

We have considered certain mechanisms isotropizing the space in an n-dimensional Bianchi I model. Although these models can isotropize, it leads to the breakdown of dimensional reduction. In particular, the solution given by Chodos and Detweiler (isotropic macro and microspace) is a very special one in the sense that there is no simple mechanism of isotropization which can lead to this solution.

We conclude that while considering the connection between isotropization and dimensional reduction we ought to look for a mechanism of dimensional reduction leading to isotropization of the macro and microspace. The converse dependence does not exist.

The classical Kaluza-Klein model is usually treated as a ground state of the quantum version of the Kaluza-Klein theory. It is usually assumed that the space possesses the structure of $M^4 \times B^D$ where M^4 is the 4-dimensional Minkowskian spacetime and B^D is a certain compact D-dimensional space, most often it is a sphere S^D [12].

This assumption about M^4 is not valid in the cosmological context and should be replaced by $R^1 \times S^3$ for closed models and $R^1 \times Q^3$ for open ones [13]. We can in general assume that the ground state possesses the structure of $R^1 \times M^3 \times B^D$ where M^3 and B^D are homogeneous spaces. The symmetries of the ground state are not as large as the symmetries of the theory i.e. spontaneous symmetry breaking exists. If we take the classical cosmological solutions as a ground state then those solutions which lead to isotropic macro and microspace are the most interesting from the physical point of view. One can show that the set of models leading to an isotropic microspace is of measure zero in the class of homogeneous Kaluza-Klein theories (for details see: J. Szczęsny, M. Biesiada and M. Szydłowski, submitted to *Phys. Lett.* **B**). This result corresponds to the theorem formulated by Collins and Hawking for classical Bianchi models [4].

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