ON THE THERMAL PROPERTIES OF NEUTRON MATTER WITH SPIN UP EXCESS

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The schematic model of pure hard core neutron matter proposed by Dąbrowski et al. is generalized to finite temperature, where the attractive part of nuclear forces is treated as a perturbation. We calculate the potential energy, the energy per neutron, the volume and symmetry pressure, the magnetic susceptibility, the effective mass and the velocity of sound as a function of temperature. Our results are compared with previous calculations.

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1. *Introduction*

It is a matter of interest to study the interior of pulsars which are objects emitting electromagnetic pulses at regular intervals between 0.03 and 4 sec. It was pointed out [1] that the most probable candidate to explain pulsars was a rotating neutron star. Today most of our understanding of the structure of neutron stars comes from the study of theoretical models. Neutron matter is one of these models which was studied using different potentials with different techniques. Neutron matter is an infinite homogeneous system of interacting neutrons at densities $\varrho \geqslant \varrho_0$ where ϱ_0 is the typical nuclear matter particle density of nucleons in ordinary nuclei. Neutron stars are considered to consist, to a large extent, of neutrons at densities ranging from ϱ_0 to $20\varrho_0$. As a first approximation the neutron star matter should be neutron matter. The thermal and dynamical properties of neutron matter are of interest for the discussion of heavy and super heavy nuclei as well as for the calculation of equilibrium states of neutron stars.

Some of the first energy calculations for neutron matter were done [2] by using the reaction matrix theory at small densities. It was found that neutron matter is unbound.

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This has been confirmed by all later calculations. The dynamical properties of neutron matter at finite temperature were studied by Walecka [3] and Toki et al. [4]. In their work, Toki et al. calculated the equation of state for pion condensed neutron matter. The equation of state was studied at temperatures up to 50 MeV by using the σ model of Campbell et al. [5]. Most of the studies of neutron matter were done at zero temperature. However, in the center of supernova at the point where neutron star is formed, temperature is of the order of $T \simeq 10$ MeV. Also in high energy heavy ion collisions temperature can be greater than 50 MeV. That makes the calculation of the thermal properties of neutron matter a matter of interest.

In this work we study the thermal properties of neutron matter by generalizing the model of pure hard core neutron matter developed by Dąbrowski et al. [6] to finite temperatures and considering the attractive part as a perturbation [7–10]. Dąbrowski et al. [6] have recommended the model of pure hard core neutron matter because of the fact that at sufficiently high densities, where the short range repulsion is the decisive part of nuclear forces [11, 12], the hard core model should approximately describe real neutron matter. This method has been applied by Hassan and Montasser [13] in the case of nuclear matter with neutron excess.

In Sect. 2, we explain the theory where we show how the entropy, the pressure, the effective mass, the magnetic susceptibility and the velocity of sound can be calculated. In Sect. 3, the computational procedure is explained together with the result and discussion of our work.

2. *Theory*

The binding energy of neutron matter with pure hard core interaction with spin-up excess is given by [6, 12]

$$E_R/N = E_v/N + \tfrac{1}{2}(E_s/N)\alpha^2, \tag{1}$$

where

$$E_v/N = \frac{\hbar^2 k_F^2}{2m}\left[\tfrac{3}{5} + \frac{2}{3\pi}X_c + \frac{4}{35\pi^2}(11 - \ln 2)X_c^2 + \left(\frac{2}{3\pi} + 0.0165\right)X_c^3\right], \tag{2}$$

$$E_s/N = \frac{\hbar^2 k_F^2}{2m}\left[\tfrac{2}{3} - \frac{4}{3\pi}X_c - \frac{32}{45\pi^2}(2 + \ln 2)X_c^2 + 0.3185\, X_c^3\right], \tag{3}$$

$$X_c = k_F r_c,$$

r_c is the hard core radius and k_F is the Fermi momentum for nucleons in the neutron matter. k_F is related to the density of nucleons ϱ by

$$k_F^3 = 3\pi^2\varrho, \tag{4}$$

α is the spin-up excess parameter, i.e.

$$\alpha = (N_\uparrow - N_\downarrow)/N, \tag{5}$$

where $N_\uparrow$ and $N_\downarrow$ are the number of neutrons with spin up and spin down respectively, and

$$N = N_\uparrow + N_\downarrow. \tag{6}$$

We will use for the attractive part of the interaction the form

$$V_A = \begin{cases} -\frac{1}{2}(1+P^M)V_0 & \text{for} \quad r_c < r < r_c + b, \\ 0 & \text{for} \quad r > r_c + b. \end{cases} \tag{7}$$

P^M is the Majorana exchange operator. The intrinsic range b is determined from the relation [14]

$$2r_c + b = 2.7 \text{ fm}. \tag{8}$$

The potential depth V_0 is fixed by the relation [14]

$$\frac{MV_0}{h^2} b^2 = \frac{\pi^2}{4}. \tag{9}$$

The contribution of the attractive part to the binding energy in the first order is

$$E_A/N = \frac{-3V_0}{4}\left[\frac{k_F^3}{27}(X_b^3 - X_c^3)\gamma_\uparrow^6\gamma_\downarrow^6 + k_F\gamma_\uparrow^2\gamma_\downarrow^2 I_{\uparrow\downarrow}\right], \tag{10}$$

where $I_{\uparrow\downarrow} = \int_{r_c}^{r_c+b} dr j_1(\kappa r) j_1(\lambda r)$, $X_b = k_F(r_c+b)$, $\kappa = \gamma_\uparrow k_F$, $\lambda = \gamma_\downarrow k_F$, $\gamma_\uparrow = (1+\alpha)^{1/3}$ and $\gamma_\downarrow = (1-\alpha)^{1/3}$.

The above model has been generalized in the case of nuclear matter to finite temperature by Stocker [15]. This generalization was extended to include nuclear matter with neutron excess by Hassan and Montasser [13] and for polarized nuclear matter by Hassan et al. [16]. For neutron matter with spin up excess we use the method discussed by Küpper et al. [17]. Keeping terms up to T^4, we get for the entropy per neutron [18]

$$\frac{S(T,\varrho)}{N} = S_1 + S_2 = \frac{1}{\mu_0 T}\left(\frac{\pi}{2}\right)^2 (KT)^2\left[\frac{\gamma_\uparrow}{\beta_\uparrow} + \frac{\gamma_\downarrow}{\beta_\downarrow}\right]$$
$$+ \frac{1}{2\mu_0^3 T}\left(-\tfrac{4}{5}\right)\left(\frac{\pi}{2}\right)^4 (KT)^4[\beta_\uparrow^{-3}\gamma_\uparrow^{-3} + \beta_\downarrow^{-3}\gamma_\downarrow^{-3}], \tag{11}$$

where

$$\beta_\uparrow = m/m_\uparrow^*, \quad \beta_\downarrow = m/m_\downarrow^* \quad \text{and} \quad \mu_0 = h^2k_F^2/2m, \tag{12}$$

$m_\uparrow^*$ and $m_\downarrow^*$ are the effective masses of neutrons with spin up and spin down respectively and m is the non-interacting neutron mass. T is the absolute temperature (in MeV) and K is Boltzmann's constant.

By calculating the single particle potential for neutrons with spin up and spin down, taking into account the above potential and the first order K-matrix (see Appendix), we get

$$\beta_\uparrow = 1 + \frac{2m}{\hbar^2}\left\{\frac{1}{3\pi}\cdot\frac{\hbar^2 r_c^3}{m}[\kappa^3+\tfrac{2}{3}\lambda^3] + \frac{V_0\lambda}{6\pi k_F^3}[X_b^3 j_2(\gamma_\downarrow X_b) - X_c^3 j_2(\gamma_\downarrow X_c)]\right\}, \tag{13}$$

$$\beta_\downarrow = 1 + \frac{2m}{\hbar^2}\left\{\frac{1}{3\pi}\cdot\frac{\hbar^2 r_c^3}{m}[\lambda^3+\tfrac{2}{3}\kappa^3] + \frac{V_0\kappa}{6\pi k_F^3}[X_b^3 j_2(\gamma_\uparrow X_b) - X_c^3 j_2(\gamma_\uparrow X_c)]\right\}, \tag{14}$$

where j_2 is the spherical Bessel function of the second degree. The internal and free energies per neutron can be calculated using Eq. (11) and the known expression for the specific heat per unit volume C_v, namely

$$C_v = \varrho T\frac{\partial S}{\partial T} = \varrho(S_1+3S_2)$$

i.e.,

$$e(T,\varrho) = e(0,\varrho) + \frac{1}{\varrho}\int C_v dT = e(0,\varrho) + \tfrac{1}{2}S_1 T + \tfrac{3}{4}S_2 T, \tag{15}$$

$$F(T,\varrho) = e(0,\varrho) - TS = e(0,\varrho) - \tfrac{1}{2}S_1 T - \tfrac{1}{4}S_2 T. \tag{16}$$

The pressure can be obtained from the relation

$$P = \varrho^2\left(\frac{\partial F}{\partial\varrho}\right)_T. \tag{17}$$

If we expand the entropy per particle up to second order in α^2, we get

$$S = S_{1v} + S_{2v} + \tfrac{1}{2}(S_{1s}+S_{2s})\alpha^2. \tag{18}$$

Therefore, the internal energy and the pressure per neutron can be written as

$$e(T,\varrho) = e_v(T,\varrho) + \tfrac{1}{2}\alpha^2 e_s(T,\varrho), \tag{19}$$

$$P(T,\varrho) = P_v(T,\varrho) + \tfrac{1}{2}\alpha^2 P_s(T,\varrho), \tag{20}$$

where

$$e_v(T,\varrho) = e_v(0,\varrho) + \frac{T}{2}S_{1v} + \tfrac{3}{4}S_{2v}T, \tag{21}$$

$$e_s(T,\varrho) = e_s(0,\varrho) + \frac{T}{2}S_{1s} + \tfrac{3}{4}S_{2s}T, \tag{22}$$

$$e_v(0,\varrho) = E_v(k_F) = \tfrac{3}{5}\frac{\hbar^2 k_F^2}{2m}\left[1 + \frac{10}{9\pi}X_c + \frac{4}{21\pi^2}(11-2\ln 2)X_c^2\right.$$

$$\left. + \left(\frac{10}{9\pi}+0.275\right)X_c^3\right] - \frac{3v_0}{4\pi}\left[\tfrac{1}{27}(X_b^3 - X_c^3) + \int_{X_c}^{X_b} j_1^2(X)dX\right], \tag{23}$$

$$e_s(0, \varrho) = E_s(k_F) = \frac{2}{3}\frac{\hbar^2 k_F^2}{2m}\Bigg[1 - \frac{2}{\pi}X_c - \frac{16}{15\pi^2}(2+\ln 2)X_c^2$$

$$+0.062X_c^3\Bigg] - \frac{3v_0}{4\pi}\left[-\tfrac{1}{27}(X_b^3 - X_c^3) + \tfrac{1}{9}\{X_b j_0(X_b) - X_c j_0(X_c)\}\right], \quad (24)$$

$_0(X)$ and $j_1(X)$ are the spherical Bessel functions of the zero and first degree, respectively, $e_v(T, \varrho)$ is the total energy per neutron in the case of $\alpha = 0$ and $e_s(T, \varrho)$ is the spin symmetry energy per neutron of the neutron matter.

The magnetic susceptibility of neutron matter can be written in terms of $e_s(T, \varrho)$ as [19]

$$\chi(T, \varrho) = \mu_n^2 \varrho / e_s(T, \varrho), \quad (25)$$

where μ_n is the neutron magnetic moment.

It is more convenient to introduce the ratio of χ to the magnetic susceptibility χ_F of the Fermi gas of non-interacting neutron which is

$$\chi_F = (3/2)\mu_n^2 \varrho / \mu_0. \quad (26)$$

This ratio can be written as

$$\chi_F / \chi = \frac{3}{2}\frac{e_s(T, \varrho)}{\mu_0}. \quad (27)$$

The sound velocity v_s in neutron matter at zero temperature can be calculated by using the formula [20]

$$v_s = \sqrt{(\partial P / \partial \varrho)}. \quad (28)$$

3. *Computational procedure and results*

The potential energy per neutron $E_{pot}(k_F)/N$ at $\alpha = 0$ and $T = 0$ can be calculated from the relation

$$E_{pot}(k_F)/N = E_v/N - \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}, \quad (29)$$

where E_v/N can be obtained from Eqs (2) and (10).

We calculated $E_{pot}(\varrho)/N$ at different r_c and we found that ϱ_m (the value of ϱ which gives minimum $E_{pot}(\varrho)/N$) varies with r_c. The value of $r_c = 0.3$ fm gives $\varrho_m = 0.3\ \text{fm}^{-3}$ (see Fig. 1). This value is comparable with other calculations (see for example Ref. [21]). For this reason we choose the value of $r_c = 0.3$ fm for the rest of our computations.

The energy per neutron at $\alpha = 0$ and $T = 0$ namely E_v/N was calculated as discussed before and the results are shown in Fig. 2 together with some of the previous calculations [22–24]. From this figure we notice that our calculations are in reasonable agreement with that of Owen [22] and Friedman and Pandharipande [23] till $\varrho \simeq 1.8\ \text{fm}^{-3}$. Above that

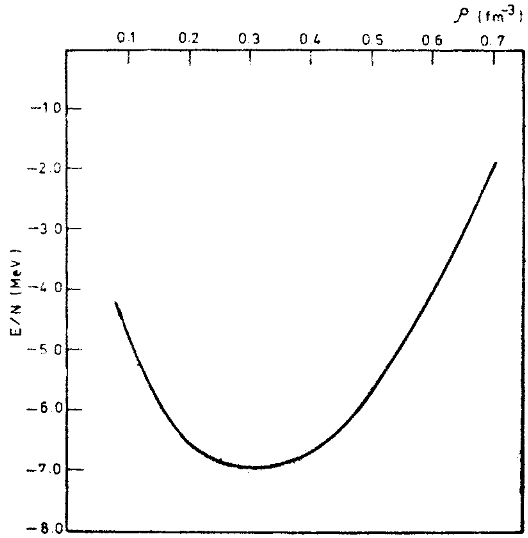

Fig. 1. The potential energy per neutron E_{pot}/N as a function of density ϱ at $r_c = 0.3$ fm

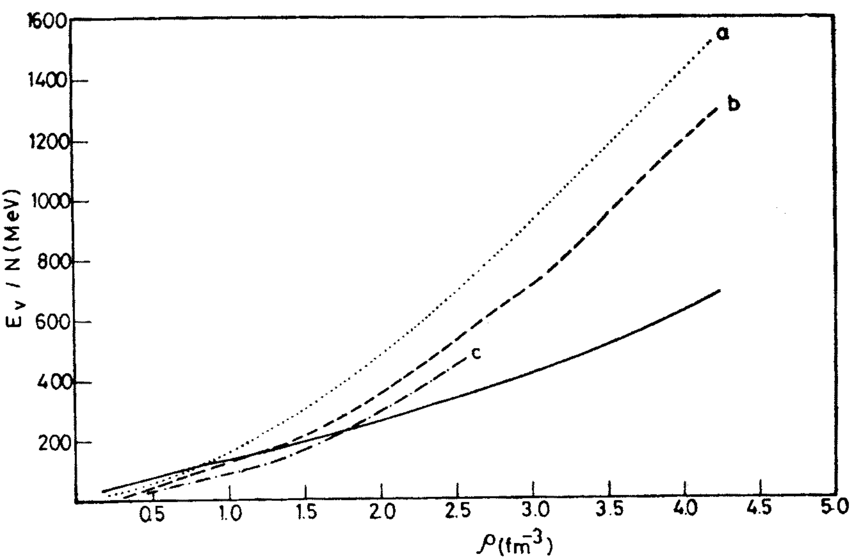

Fig. 2. The binding energy per neutron E_v/N at $\alpha = 0$ as a function of density ϱ together with the results of the previous calculations (a [24], b [22] and c [23])

density all other results are more repulsive than ours. This can be attributed to the fact that our calculation is reliable only for $\varrho < 1.25$. Above this value ϱ makes $k_F r_c > 1$ which is in contradiction with the validity of the expansion used in our calculations for the repulsive part of the binding energy.

The volume and symmetry pressure $P_v(\varrho, T)$ and $P_s(\varrho, T)$ can be calculated using Eqs (16), (17) and (20). The results of these calculations are shown in Fig. 3 for $P_v(\varrho, T)$ together

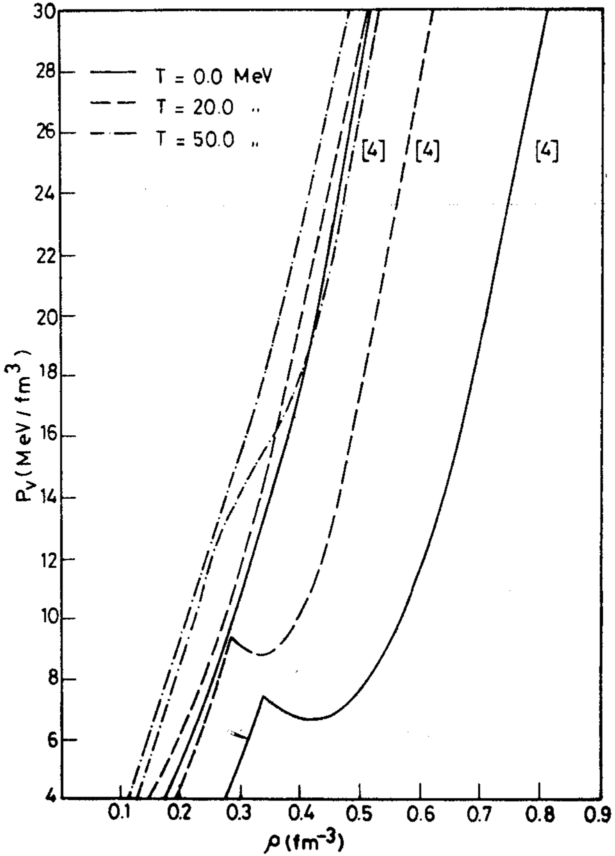

Fig. 3

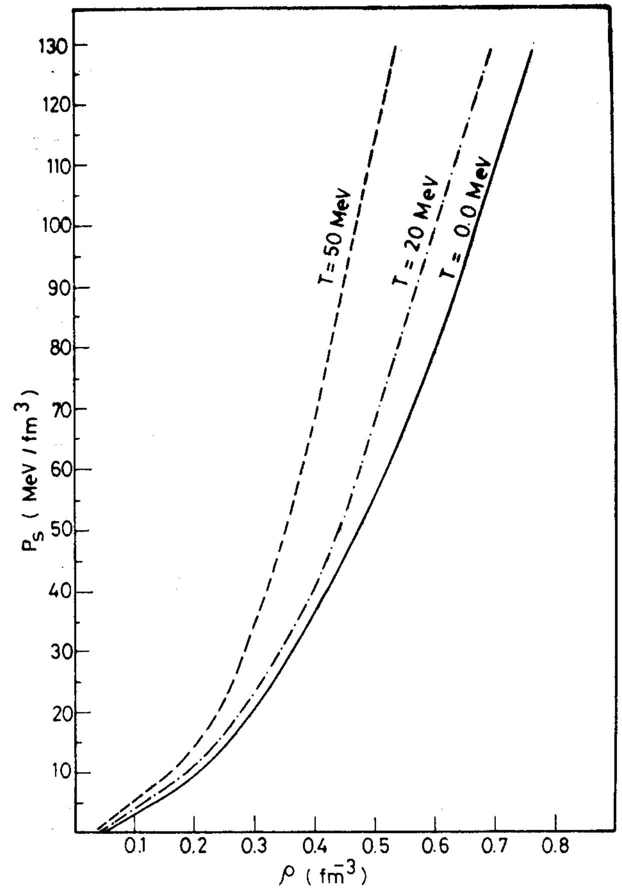

Fig. 4

Fig. 3. The pressure per neutron P_v at $\alpha = 0$ at different T together with that of Toki [4]

Fig. 4. The spin symmetry pressure per neutron P_s as a function of density at different temperature T

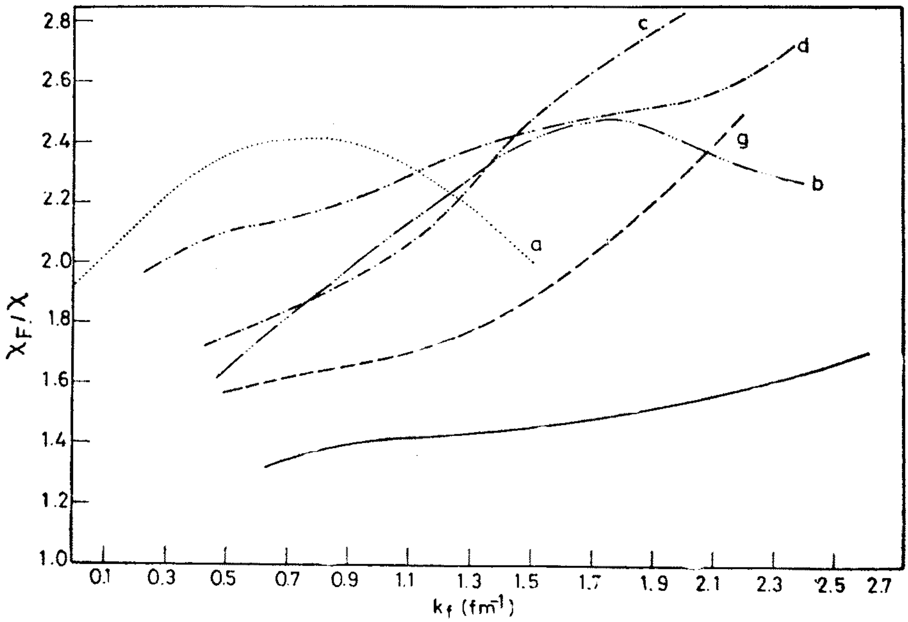

Fig. 5. The ratio χ_F/χ (see text) as a function of k_F together with the result of the previous calculations (a [25], b [26], c [27], d [28] and g [20]).

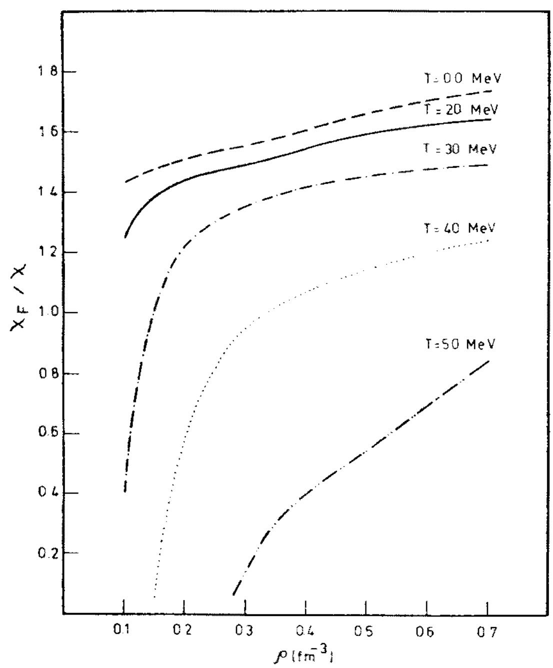

Fig. 6. Temperature dependence of χ_F/χ as a function of density ϱ

with that of Toki et al. [4] and in Fig. 4 for $P_s(\varrho, T)$. From Fig. 3 we can see that, unlike that of Toki et al., our results do not give any sign of phase transition since it gives always a positive compressibility. The symmetry pressure $P_s(\varrho, T)$ has a similar behaviour as that of the volume pressure where it increases monotonically with ϱ and T.

The ratio χ_F/χ, Eq. (27) is displayed as a function of k_F in Fig. 5 together with some of the previous calculations [20, 25–28]. We notice that ours show similar behaviour as that of Holinde [28] except for a gap of about 0.7 between them. The behaviour of χ_F/χ as a function of k_F and T is displayed in Fig. 6. It can be shown that χ_F/χ decreases with increasing temperature.

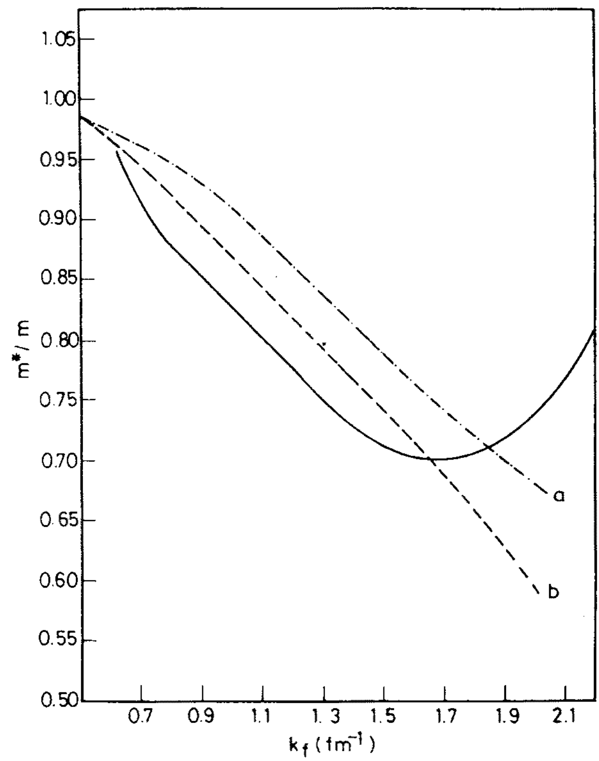

Fig. 7. The effective mass m^*/m ratio as a function of k_F together with the result of two of the previous calculations (a [25] and b [20])

The effective mass ratio of the nucleons m^*/m for equal number of protons and neutrons has been calculated using Eqs (13) and (14) and putting $\alpha = 0$. The result of this calculation is shown in Fig. 7 together with that of Behara [25] and Nitsch [20]. We can see that our results have the same behaviour as the others except for the minimum value at $k_F = 1.7$ fm. This minimum feature was reproduced by Horowitz [29].

We calculate the sound velocity in neutron matter as a function of k_F at zero temperature (almost zero temperature) by using Eq. (28) and taking $\alpha = 0$. The result is shown in Fig. 8 together with that of Nitsch [20]. We can see that our results are near to that of

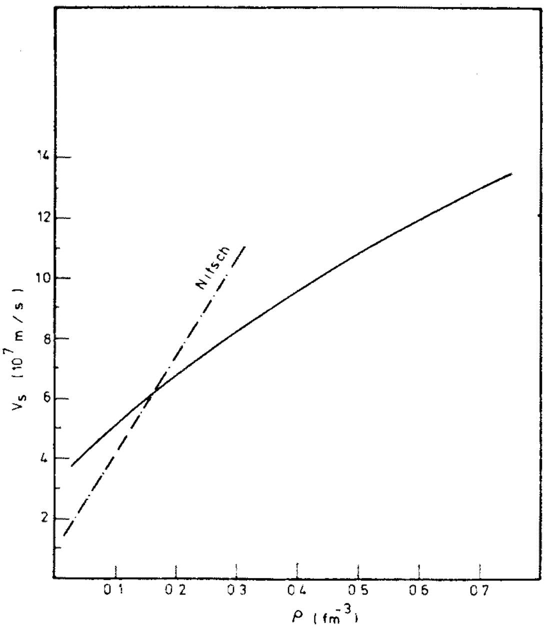

Fig. 8. The velocity of sound in neutron matter as a function of the density ϱ together with that of Nitsch [20]

Nitsch at lower k_F but have smaller values at higher k_F. We can notice also that the velocity of sound in neutron matter is near to the velocity of light. It is therefore recommended to use a relativistic approach to deal with it (see for example Kistler et al. [30] and Friedman and Pandharipande [23]).

In conclusion, we believe that in spite of its simplicity the model we used can give plenty of information about the neutron matter at finite temperature. However, the agreement between our results and those of previous calculations can be improved by including higher order terms in the attractive part of the potential.

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APPENDIX

Single particle potential energy

The single particle potential energy can be written as

$$U(\vec{m}_1) = \sum_{\vec{m}_2} (\vec{m}_1\vec{m}_2|K|\vec{m}_1\vec{m}_2 - \vec{m}_2\vec{m}_1),$$

where

$$K \simeq V_A + K_s.$$

K_s is the repulsive reaction matrix which can be approximated by the free space repulsive reaction matrix K_s^0. The matrix element of the latter [31] is

$$(\vec{m}_1'\vec{m}_2'|K_s^0|\vec{m}_1\vec{m}_2) = \delta_{\vec{K}\vec{K}'}(\vec{k}'|K_s^0|\vec{k}),$$

where $\vec{k} = \frac{1}{2}(\vec{m}_1-\vec{m}_2)$, $\vec{k}' = \frac{1}{2}(\vec{m}_1'+\vec{m}_2')$, $\vec{K} = (\vec{m}_1+\vec{m}_2)$, $\vec{K}' = (\vec{m}_1'+\vec{m}_2')$ and

$$(\vec{k}'|K_s^0|\vec{k}) = (\vec{k}'|K_s^0|\vec{k})_s+(\vec{k}'|K_s^0|\vec{k})_p$$

$$= \frac{4\pi}{\Omega}\frac{\hbar^2}{m}(r_c+\tfrac{1}{3}k^2r_c^3) \pm \frac{4\pi}{\Omega}\frac{\hbar^2}{m}k^2r_c^3, \quad \text{for} \quad \vec{k} = \pm\vec{k}'.$$

The matrix element for V_A is

$$(\vec{m}_1\vec{m}_2|V_A|\vec{m}_1\vec{m}_2) = (\vec{m}_2\vec{m}_1|V_A|\vec{m}_1\vec{m}_2)$$

$$= -\frac{v_0}{2\Omega}\int_{r_c}^{r_c+b} d\vec{r}(1+e^{i(\vec{m}_2-\vec{m}_1)\cdot\vec{r}}).$$

Keeping terms up to m_1^2 the single particle potential for spin up particle can then be written as

$$U_\uparrow(\vec{m}_1) = \frac{2}{3}\frac{\hbar^2 r_c}{\pi m}\{[\lambda^3+\tfrac{1}{5}\lambda^5r_c^2+\tfrac{3}{10}\kappa^3r_c^2]+m_1^2r_c^3[\tfrac{1}{3}\lambda^3+\tfrac{1}{2}\kappa^3]\}$$

$$-\frac{v_0}{\pi}\left\{\frac{\lambda^3}{4}[(r_c+b)^3-r_c^3]+\lambda^2\int rj_1(\lambda r)dr - \frac{m_1^2}{6}\int r^3j_1(\lambda r)dr\right\}.$$

Putting

$$U_\uparrow(\vec{m}_1) = U_{0\uparrow}(\vec{m}_1)+U_{1\uparrow}(\vec{m}_1)m_1^2$$

we have

$$\beta_\uparrow = m^*/m_\uparrow^* = 1+\frac{2m}{\hbar^2}U_{1\uparrow}(\vec{m}_1),$$

where

$$U_{1\uparrow}(\vec{m}_1) = \frac{\hbar^2r_c^3}{3\pi m}[\kappa^3+\tfrac{2}{3}\lambda^3]+\frac{v_0}{6\pi}\int r^3j_1(\lambda r)dr.$$

Solving the integral in the above equation we can get Eq. (3). Eq. (14) can be obtained in the similar manner.

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