LETTERS TO THE EDITOR

ON THE TRANSPORT COEFFICIENTS OF A QUARK PLASMA

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The gauge covariant set of kinetic equations of a quark plasma is discussed. The collision term is included using the relaxation time approximation. The color conductivity coefficient is studied.

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The knowledge of transport coefficients of a quark-gluon plasma allows one to include dissipative effects in the plasma hydrodynamics which has been widely applied, see the review [1], to study of the evolution of the plasma generated (if indeed generated) in ultra-relativistic heavy-ion collisions. These coefficients have been recently discussed in papers [2], where the mean field (Vlasov) terms have been neglected in the kinetic equations and the color forces have entered the equations through the parton-parton cross sections only. Such an approach does not take into account the characteristic features of the plasma as a system of colored quarks interacting via non-Abelian fields, the interaction range of which can be much greater than the average inter-quark distance. Further, the collective effects in the plasma are lost and one is not able to calculate the transport coefficients involving color like the color conductivity since the Vlasov terms are absent in the kinetic equations.

Recently much has been done towards the construction of the transport theory of a quark-gluon plasma with the Vlasov terms included [3, 4]. Particular attention has been paid to the system of colored quarks interacting with one another through the classical non-Abelian potentials. This system is called the quark plasma since the existence of thermal (nonvirtual) gluons is neglected here. The pioneer contribution has been done by Heinz [3] who proposed, the latter derived [4], transport equations. However, as explained in our previous paper [5], the variant of Heinz's [3, 6] with the color treated as a continuous classi-

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cal variable is gauge dependent or incomplete. Therefore we find not quite satisfactory the papers [7], where the plasma properties have been discussed in the framework of this approach. Very recently Dyrek and Florkowski [8] have calculated the transport coefficients of the quark plasma using the gauge covariant kinetic equations.

In this paper we briefly discuss the earlier used transport equations with the collision terms in the relaxation time approximation. Then we study in detail the color conductivity coefficient, which is calculated in two ways.

Let us start with the presentation of the transport theory of a quark plasma [3, 4]. The colored quarks interact via the classical non-Abelian SU(3) potential $A^{\mu}{}_a(x)$, $a = 1, \ldots, 8$. The (anti-) quark distribution function is a two-color-index matrix $f_{ij}(p, x)$ $(\underline{f}_{ij}(p, x))$ $i, j = 1, 2, 3$, which transforms under local gauge transformations as an octet i.e.

$$f(p, x) \to U(x) f(p, x) U^{+}(x), \tag{1}$$

where the color indices have been suppressed. The trace of the distribution function is, of course, gauge invariant. The distribution functions satisfy the transport equations

$$p^{\mu} D_{\mu} f(p, x) - \frac{g}{2} p^{\mu} \frac{\partial}{\partial p_{\nu}} \{F_{\mu\nu}(x), f(p,x)\} = \eta p^{\mu} u_{\mu} [f^{eq}(p) - f(p, x)], \tag{2a}$$

$$p^{\mu} D_{\mu} \underline{f}(p, x) + \frac{g}{2} p^{\mu} \frac{\partial}{\partial p_{\nu}} \{F_{\mu\nu}(x), \underline{f}(p, x)\} = \underline{\eta} p^{\mu} u_{\mu} [\underline{f}^{eq}(p) - \underline{f}(p, x)], \tag{2b}$$

where $p^{\mu} \equiv p \equiv (E, \bar{p})$; u^{μ} is the hydrodynamical velocity; $\eta^{-1} = \tau$ $(\underline{\eta}^{-1} = \underline{\tau})$ is the relaxation time of the (anti-) quarks plasma component; f^{eq} $(\underline{f}^{eq})$ is the (anti-) quark equilibrium distribution function

$$f^{eq}_{ij}(p) = \delta_{ij} n(p); \quad \underline{f}^{eq}_{ij}(p) = \delta_{ij} \underline{n}(p)$$

D_{μ} is the covariant derivative in adjoint representation which acts as $\partial_{\mu} + ig[A_{\mu}(x), \ldots]$; $F_{\mu\nu}(x)$ is the stress tensor generated by the color current

$$D_{\mu} F^{\mu\nu}(x) = j^{\nu}(x)$$

with

$$j^{\nu}_{ij}(x) = g \int \frac{d^3 \bar{p}}{2E(2\pi)^3} p^{\nu} [f_{ij} - \underline{f}_{ij} - \tfrac{1}{3} \delta_{ij} (f_{kk} - \underline{f}_{kk})]. \tag{3}$$

The set of kinetic equations (2), (3) is gauge covariant due to the transformation law (1). The equilibrium distribution function is gauge invariant.

We first discuss the plasma where the collisions dominate. The Vlasov terms should be neglected in Eq. (2). However it cannot be done by substitution $A^{\mu}(x) = 0$ since, in this case, the gauge covariance of the transport equations is spoiled (the covariant derivative D^{μ} is replaced by the ordinary one ∂^{μ}). Otherwise we assume that

$$A_{\mu}(x) = -ig^{-1} \Lambda(x) \partial_{\mu} \Lambda^{+}(x), \tag{4}$$

where $A(x)$ is an arbitrary unitary matrix. Eq. (4) provides the well-known pure gauge field which gives $F^{\mu\nu}(x) = 0$.

Because the quantities which are color independent such as the energy-momentum tensor, baryon current etc., are expressed through the traces of distribution functions f_{ii} and $\underline{f}_{ii}$, the knowledge of these traces is sufficient to determine the viscosity or heat conductivity coefficients. Therefore we take the trace of the matrices from the left-hand and right-hand sides of Eq. (2). Introducing the gauge invariant distribution functions $\psi(p, x) = f_{ii}(p, x)$ and $\underline{\psi}(p, x) = \underline{f}_{ii}(p, x)$ which are color independent we get

$$p_\mu \partial^\mu \psi(p, x) = \eta p_\mu u^\mu (\psi^{eq}(p) - \psi(p, x)), \tag{5a}$$

$$p_\mu \partial^\mu \underline{\psi}(p, x) = \underline{\eta} p_\mu u^\mu (\underline{\psi}^{eq}(p) - \underline{\psi}(p, x)), \tag{5b}$$

where $\psi^{eq}(p) = 3n(p)$, $\underline{\psi}^{eq}(p) = 3\underline{n}(p)$. In this way we have arrived to the well-known kinetic equations where color enters through the quantities η and $\underline{\eta}$ and additionally it plays a role of an internal degree of freedom of the quarks. The transport coefficients of the quark-gluon-plasma following from Eq. (5) have been calculated in the papers [2].

Let us now return to Eqs. (2), (3) and to discuss the color conductivity coefficient $\sigma^{\alpha\beta}_{ab}(k)$ which is defined, in analogy to electrodynamics, as

$$j^\alpha_a(k) = \sigma^{\alpha\beta}_{ab}(k) E^\beta_b(k), \qquad \begin{array}{l} \alpha, \beta = 1, 2, 3 \\ a, b = 1, \dots 8 \end{array}$$

where $j^\alpha_a(k)$ is the (Fourier transformed) current induced by the (Fourier transformed) chromoelectric field $E^\beta_b(k)$; $k \equiv (\omega, \bar{k})$. The indices α, β label the Cartesian coordinate axes. Because the above definition is nonlocal in x-space there are very complicated gauge properties of the color conductivity coefficient.

In our previous paper [5] it has been argued that at a certain choice of gauge, only the (mean) fields E_3 and E_8 are nonzero in the plasma near equilibrium. These fields are expressed through the potentials A_3 and A_8, respectively, just like the electrodynamic field is expressed through the electrodynamic potential. Therefore it is possible, as in the electrodynamics, to introduce the chromoelectric polarization and induction vectors. Further, one can express the color conductivity through the chromoelectric permeability $\varepsilon^{\alpha\beta}_{ab}(k)$

$$\sigma^{\alpha\beta}_{ab}(k) = -i\omega[\varepsilon^{\alpha\beta}_{ab}(k) - \delta^{\alpha\beta}\delta_{ab}]. \tag{6}$$

Trivially modifying the calculations from the paper [5] one finds the chromoelectric permeability of the isotropic quark plasma with collisions described in the relaxation time approximation

$$\varepsilon^{\alpha\beta}_{ab}(k) = \delta^{\alpha\beta}\delta_{ab} - g^2\delta_{ab} \int \frac{d^3\bar{k}}{2\omega(2\pi)^3} \left\{ \frac{v^\alpha \partial n/\partial p^\beta}{\bar{k}\bar{v} - \omega - i\eta} + \frac{v^\alpha \partial \underline{n}/\partial p^\beta}{\bar{k}\bar{v} - \omega - i\underline{\eta}} \right\}, \quad a, b = 3,8 \tag{7}$$

where $v = p/E$. Substituting Eq. (7) in the formula (6) one gets the chromoelectric color conductivity.

Let us discuss in more detail the "static" conductivity $\sigma(k = 0)$. We use the terminology from the electrodynamics where the static conductivity connects the constant, or rather slowly varying in space-time, fields and currents. As discussed below this terminology is somewhat inadequate for the chromodynamics. From Eq. (7) one finds the static color conductivity of the isotropic quasiequilibrium plasma

$$\sigma_{ab}^{\alpha\beta} = \sigma\delta^{\alpha\beta}\delta_{ab} \tag{8}$$

with

$$\sigma = -g^2\int_0^\infty \frac{d|\bar{p}|\bar{p}^4}{6E\pi^2}\left\{\tau\frac{\partial n}{\partial(\bar{p}^2)} + \underline{\tau}\frac{\partial \underline{n}}{\partial(\bar{p}^2)}\right\}.$$

The color conductivity coefficient (8) (with trivial modifications) coincides with the electric conductivity of the electron-positron plasma. It is not surprising since the gluon contribution to the color conductivity is neglected in the above formulas.

We estimate σ for the zero-baryon charge plasma ($\tau = \underline{\tau}$) of massless quarks of N_f flavours. Then Eq. (8) gives

$$\sigma = \omega_0^2\tau, \tag{9}$$

where ω_0 is the plasma frequency and $\omega_0^2 = N_f g^2T^2/18$ for the quark plasma with the Fermi--Dirac equilibrium distribution function [5]. We can effectively take into account the thermal gluons with their self-interaction if one identifies, as it has been done by Heinz [7], the plasma frequency from Eq. (9) with the one from the finite-temperature field theory calculations, see, e.g., [9]. Then, the plasma frequency (in one-loop approximation) reads $\omega_0^2 = (N_f+6)g^2T^2/18$ [9].

The relaxation time is usually identified with the gas particle mean free time. Therefore τ can be estimated from the formula

$$\tau = (\varrho\sigma_t)^{-1},$$

where ϱ is the parton density and σ_t is the quark-parton transport cross section. The rough estimation of σ_t for $T \gg \Lambda$ (where T is the temperature and Λ is the QCD scale parameter) is given by Danielewicz and Gyulassy [2]

$$\sigma_t = \frac{5g^4\ln g^{-2}}{136\pi T^2}$$

where $g^2/4\pi$ is the temperature dependent running coupling constant ($4\pi g^{-2} \gg 1$ for $T \gg \Lambda$). Because $\varrho = (9N_f+16)\zeta(3)T^3/\pi^2$ (with thermal gluons included) one estimates

$$\tau = \frac{136}{5\pi(9N_f+16)\zeta(3)g^4\ln g^{-2}}. \tag{10}$$

Substituting (10) in (9) we get the rough estimation of the color conductivity of the very hot ($T \gg \Lambda$) plasma

$$\sigma = \frac{68(N_f+6)T}{45\pi(9N_f+16)\zeta(3)g^2 \ln g^{-2}}.$$

An obvious disadvantage of the derivation of the plasma conductivity presented above is that we have used a certain gauge. However the static conductivity (8) can be found in a gauge covariant manner. We linearize Eqs. (2) assuming that the equilibrium distribution functions f^{eq} and $\underline{f}^{eq}$ can be substituted (instead of f and $\underline{f}$) in the left-hand sides of Eq. (2). Then one finds the distribution functions in the plasma rest frame ($u^\mu = (1, 0, 0, 0)$)

$$f(p, x) = f^{eq}(p) + g\tau \frac{p^\mu}{E} \frac{\partial f^{eq}}{\partial p_\nu} F_{\mu\nu}(x),$$

$$\underline{f}(p, x) = \underline{f}^{eq}(p) - g\underline{\tau} \frac{p^\mu}{E} \frac{\partial \underline{f}^{eq}}{\partial p_\nu} F_{\mu\nu}(x).$$

Putting the above distribution functions in the formula of current from Eq. (3) we get the static conductivity coefficient (8). This method has been applied by Dyrek and Florkowski [8].

Let us return to the definition of the static color conductivity

$$j_a = \sigma_{ab} E_b, \tag{11}$$

where j_a and E_b are the currents and fields in the x-space (coordinate indices are suppressed). In the analogous electrodynamic definition there are constant (in space-time) fields and currents. In the case of chromodynamics the notion of constant field or current is not gauge invariant. To make the definition (11) gauge covariant one has to assume that the color conductivity transforms under local infinitesimal gauge transformations as

$$\sigma_{ab} \to \sigma_{ab} + f_{acd}\omega_c(x)\sigma_{db} + f_{bcd}\omega_c(x)\sigma_{ad}$$

where f_{abc} is the gauge group structure constant and $\omega_c(x)$ is the infinitesimal transformation parameter. Therefore the definition (11) should be rewritten as

$$j_a(x) = \sigma_{ab}(x) E_b(x),$$

to expose the x-dependence of all quantities which enter the definition. One sees that, in the case of chromodynamics, it is more reasonable to say the "local" conductivity instead of the "static" conductivity. It is also notable that the diagonal color conductivity coefficient as that one from Eq. (8) is gauge invariant.

Let us recapitulate our considerations.

Firstly we have demonstrated how to arrive to the well-known transport equations (5) of relativistic gases starting from the gauge covariant Vlasov-Boltzmann equations (2) of the quark plasma. Then we have discussed the color conductivity of the plasma using the earlier found [5] color permeability tensor. Further we have concentrated on the static, or local, conductivity coefficient, which for the quasiequilibrium plasma is diagonal in the color indices, and consequently is gauge invariant. We have also given an estimate of the. color conductivity of a very hot (perturbative) plasma.

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