$m = 0$ LIMIT OF NONMINIMAL DESCRIPTION OF SPIN $\frac{3}{2}$*

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It is shown that the theories of spin $\frac{3}{2}$, equivalent in the massive case, are not equivalent in the $m = 0$ limit. The Townsend description with the help of the antisymmetric tensor-bispinor is obtained as the $m = 0$ limit of the nonminimal theory of the spin $\frac{3}{2}$.

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1. Introduction

In the previous paper [1] the nonminimal description of a massive field carrying spin $\frac{3}{2}$ (using an antisymmetric tensor-bispinor $\psi^{\mu\nu}$) has been obtained. This description is equivalent to the minimal one of Rarita and Schwinger (using a vector-bispinor ψ^{μ}). It is well known that theories equivalent for $m \neq 0$ need not to be equivalent in the $m = 0$ limit (e.g. the notoph [2–4] and the notivarg [5–7]). The Rarita-Schwinger theory in the zero mass limit describes particles with the helicities $\pm\frac{3}{2}$.

In the present paper we give the analysis of the zero mass limit of the nonminimal description [1]. The only nontrivial result is the Townsend theory [8] describing particles with the helicity $\pm\frac{1}{2}$ and dipole ghosts with the helicity $\pm\frac{3}{2}$.

2. The $m = 0$ actions

2.1. The $m = 0$ limits of the nonminimal theory

Let us start with the action [1]

$$I = \int dx[\tfrac{1}{2}\,\bar{\psi}_{\beta\lambda}\partial^{\beta}\psi^{\lambda}+\tfrac{1}{2}\,\partial^{\beta}\bar{\psi}^{\lambda}\psi_{\beta\lambda}-m(\bar{\psi}^{\alpha}\psi_{\alpha}-\bar{\psi}_{\alpha}\gamma^{\alpha}\gamma_{\beta}\psi^{\beta})$$
$$-\tfrac{1}{4}\,m(\bar{\psi}_{\beta\lambda}-\bar{\psi}^{\alpha}\varepsilon_{\alpha\beta\lambda\kappa}\gamma_{5}\gamma^{\kappa})\phi^{\beta\lambda}$$
$$-\tfrac{1}{2}\,m\bar{\phi}_{\beta\lambda}(\psi^{\beta\lambda}-\varepsilon^{\alpha\beta\lambda\kappa}\gamma_{5}\gamma_{\kappa}\psi_{\alpha})], \tag{1}$$

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where ψ^α is a vector-bispinor and $\psi^{\alpha\beta}$, $\phi^{\alpha\beta}$ are antisymmetric tensor-bispinors $\psi^{\alpha\beta} = -\psi^{\beta\alpha}$, $\phi^{\alpha\beta} = -\phi^{\beta\alpha}$. Eliminating the Lagrange multipliers $\psi^{\alpha\beta}$, $\bar{\psi}^{\alpha\beta}$, $\phi^{\alpha\beta}$ and $\bar{\phi}^{\alpha\beta}$ we get the well known Rarita–Schwinger action. To obtain the nonminimal description of the spin $\frac{3}{2}$ we perform integration by parts in the action (1) (ψ^α and $\bar{\psi}^\alpha$ are converted into the Lagrange multipliers).

The possible massless theories, resulting from the nonminimal description in the $m = 0$ limit, are obtained as follows:
(*i*) we put $m = 0$ in the action (1); we get

$$I = \int dx(-\tfrac{1}{2}\,\partial^\beta\bar{\psi}_{\beta\lambda}\psi^\lambda - \tfrac{1}{2}\,\bar{\psi}^\lambda\partial^\beta\psi_{\beta\lambda}); \tag{2}$$

(*ii*) we put $m = 0$ after elimination of the Lagrange multipliers ψ^α and $\bar{\psi}^\alpha$; we get

$$I = \int dx\{-\tfrac{1}{4}\,\partial^\beta\bar{\psi}_{\beta\lambda}[\tfrac{1}{3}\,\gamma^\lambda(\sigma\psi) + i\gamma_\sigma\psi^{\sigma\lambda}] - \tfrac{1}{4}\,[\tfrac{1}{3}\,(\bar{\psi}\sigma)\gamma^\lambda + i\bar{\psi}^{\lambda\sigma}\gamma_\sigma]\partial^\beta\psi_{\beta\lambda}\}, \tag{3}$$

where $(\sigma\psi) \equiv \sigma_{\mu\nu}\psi^{\mu\nu}$;
(*iii*) we put $m = 0$ after elimination of all the Lagrange multipliers (first ψ^α, $\bar{\psi}^\alpha$ and then $\phi^{\alpha\beta}$, $\bar{\phi}^{\alpha\beta}$); we get

$$I = \int dx[\tfrac{1}{12}\,\partial_\beta\bar{\psi}^{\beta\lambda}\gamma_\lambda(\sigma\psi) + \tfrac{1}{12}\,(\bar{\psi}\sigma)\gamma_\lambda\partial_\beta\psi^{\beta\lambda} - \frac{i}{4}\,\bar{\psi}^{\beta\mu}\gamma_\mu(\gamma\partial)\gamma^\lambda\psi_{\lambda\beta} + \frac{i}{12}\,(\bar{\psi}\sigma)\,(\gamma\partial)\,(\sigma\psi)], \tag{4}$$

where $(\gamma\partial) \equiv \gamma_\mu\partial^\mu$.

2.2. The action (2)

The field equations are

$$\partial^\alpha\psi^\beta - \partial^\beta\psi^\alpha = 0, \tag{5a}$$

$$\partial_\beta\psi^{\beta\alpha} = 0. \tag{5b}$$

Let us analyse Eq. (5a). It can be regarded as a constraint on the field ψ^α. We deduce the general form of ψ^α

$$\psi^\alpha = \partial^\alpha\psi \tag{6}$$

where ψ is a bispinor. Substituting this solution to the action (2) we get (after integration by parts) $I = 0$. We conclude that the action (2) does not describe any physical degrees of freedom.

Let us look at the action (2) from another point of view. The action (2) is invariant under the following gauge transformation:

$$\delta\psi^\alpha = \partial^\alpha\varepsilon, \qquad \delta\psi^{\alpha\beta} = \varepsilon^{\alpha\beta\mu\nu}\partial_\mu\varepsilon_\nu,$$

where ε and ε_ν are a bispinor and a vector-bispinor respectively. From Eqs. (5) we obtain the general form of solutions

$$\psi^\alpha = \partial^\alpha\psi, \qquad \psi^{\alpha\beta} = \varepsilon^{\alpha\beta\mu\nu}\partial_\mu\phi_\nu,$$

where ϕ_ν is a vector-bispinor. So, we have the case of a pure gauge theory.

We have been solving time dependent constraints in the above analysis. To confirm the result we have obtained, we perform the canonical analysis of the action (2).

Let us describe the action (2) in the form

$$I = \int dx \mathscr{L}$$

where

$$\mathscr{L} = \tfrac{1}{2}\,\bar{\psi}_{\beta\lambda}\partial^{\beta}\psi^{\lambda} - \tfrac{1}{2}\,\bar{\psi}^{\lambda}\partial^{\beta}\psi_{\beta\lambda}.$$

Then, introducing $\phi^k \equiv \psi^{0k}$, we can rewrite the action in the form ($k = 1, 2, 3$):

$$I = \int dx(\mathscr{P}_k\partial^0\psi^k + \Pi_k\partial^0\phi^k - \mathscr{H}) \tag{7}$$

where

$$\mathscr{P}_k = \frac{\partial\mathscr{L}}{\partial\partial^0\psi^k} = \tfrac{1}{2}\,\bar{\phi}_k,$$

$$\Pi_k = \frac{\partial\mathscr{L}}{\partial\partial^0\phi^k} = -\tfrac{1}{2}\,\bar{\psi}_k$$

and

$$\mathscr{H} = -\partial^i\mathscr{P}_i\psi^0 - \tfrac{1}{2}\,\bar{\psi}^0\partial_i\phi^i - \tfrac{1}{2}\,\bar{\psi}_{ij}\partial^i\psi^j + \partial^i\Pi^j\psi_{ij}. \tag{8}$$

We observe that ψ^0, $\bar{\psi}^0$, ψ^{ij} and $\bar{\psi}^{ij}$ are the Lagrange multipliers. Varying the action with respect to them we get the following constraints

$$\partial_i\phi^i = 0, \qquad \partial_i\mathscr{P}^i = 0,$$

$$\partial^i\psi^j - \partial^j\psi^i = 0, \qquad \partial^i\Pi^j - \partial^j\Pi^i = 0.$$

They are consistent with the canonical equations. To solve the constraints we use the formulae of Appendix. We obtain

$$\psi^k(\pm\tfrac{3}{2}) = 0, \qquad \psi^k(\pm\tfrac{1}{2}) = -\left(\gamma^k + 3\,\frac{\gamma_m\partial^m\partial^k}{\Delta}\right)\psi,$$

$$\mathscr{P}^k(\pm\tfrac{1}{2}) = \tfrac{1}{2}\left(3\,\frac{\partial^k\partial^m}{\Delta}\,\mathscr{P}\gamma_m + \mathscr{P}\gamma^k\right),$$

$$\Pi^k(\pm\tfrac{3}{2}) = 0, \qquad \Pi^k(\pm\tfrac{1}{2}) = -\left(\Pi\gamma^k + 3\,\frac{\partial^m\partial^k}{\Delta}\,\Pi\gamma_m\right),$$

$$\phi^k(\pm\tfrac{1}{2}) = \tfrac{1}{2}\left(3\,\frac{\partial^k\partial_i\gamma^i}{\Delta} + \gamma^k\right)\phi.$$

Inserting these solutions to Eqs. (8) and (7) we get $\mathscr{H} = 0$ and $I = 0$. So, there is no physical degree of freedom.

2.3. The action (3)

This action is (up to convention)[1] the Townsend action [8]. Aragone (see Ref. [11] of the paper [5]), performing the canonical analysis, has shown that the Townsend action describes a particle with the helicity $\pm\frac{1}{2}$ and a dipole ghost with the helicity $\pm\frac{3}{2}$. The result has been confirmed by Deser, Siegel and Townsend [5]. The canonical analysis of the action (3), performed by us, gives the same result. As far as we know, the analysis of Aragone is not published. Therefore we give a short résumé of our calculations.

It is convenient to use the following decomposition

$$\psi^{\mu\nu} = (\phi^k, \phi, \psi^k, \psi) \quad (k = 1, 2, 3)$$

where $\psi^{0k} = \phi^k + \gamma^k\phi$ with $\gamma_k\phi^k = 0$, $\psi_{ij} = \varepsilon_{0ijk}(\psi^k + \gamma^k\psi)$ with $\gamma^k\psi_k = 0$. With the help of the formulae of Appendix we can rewrite the action (3) as the sum of the $\pm\frac{1}{2}$ and $\pm\frac{3}{2}$ helicity pieces:

$$I = I(\pm\tfrac{1}{2}) + I(\pm\tfrac{3}{2}).$$

After solving the constraints, resulting from varying the action with respect to the Lagrange multipliers ψ and $\bar{\psi}$, we get

(*i*)
$$I(\pm\tfrac{1}{2}) = \int dx(\Omega_k\partial^0\Psi^k - \mathscr{H}) \tag{9}$$

where

$$\mathscr{H} = -3\Omega_k\gamma^0\gamma_m\partial^m\Psi^k,$$

$$\Omega_k \equiv \tfrac{1}{4}(\Pi_k(\pm\tfrac{1}{2}) + 3i\mathscr{P}_k(\pm\tfrac{1}{2})\gamma_5),$$

$$\Psi_k \equiv \phi_k(\pm\tfrac{1}{2}) + i\gamma_5\psi_k(\pm\tfrac{1}{2}).$$

The momenta conjugated to ϕ_k and ψ_k are

$$\Pi_k = \frac{i}{2}\bar{\Phi}_k\gamma^0 - \tfrac{1}{4}\bar{\psi}_k\gamma_5\gamma_0 \quad \text{and} \quad \mathscr{P}_k = \tfrac{1}{4}\bar{\Phi}_k\gamma_5\gamma_0$$

respectively.

From the action (9) we obtain the equation

$$i(\gamma\partial)\partial_k\Psi^k = 0.$$

Introducing the bispinor $\chi \equiv \partial_k\Psi^k$, we get

$$I(\pm\tfrac{1}{2})_{\text{effective}} = \int dx i\bar{\chi}(\gamma\partial)\chi.$$

(*ii*)

$$I(\pm\tfrac{3}{2}) = \int dx(\Xi_k\partial^0\Phi^k + \Theta_k\partial^0\Lambda^k + \Theta_k\gamma_0\gamma_m\partial^m\Lambda^k + \Xi_k\gamma_0\gamma_m\partial^m\Phi^k - 2\Xi_k\gamma_0\gamma_m\partial^m\Lambda^k),$$

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[1] We use $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

where

$$\Phi^k \equiv \phi^k(\pm\tfrac{3}{2}), \quad \Lambda^k \equiv \phi^k(\pm\tfrac{3}{2})+i\gamma_5\psi^k(\pm\tfrac{3}{2}),$$

$$\Xi^k \equiv \Pi^k(\pm\tfrac{3}{2})+i\mathscr{P}^k(\pm\tfrac{3}{2})\gamma_5, \quad \Theta^k \equiv -i\mathscr{P}^k(\pm\tfrac{3}{2})\gamma_5.$$

The field equations are

$$(\gamma\cdot\partial)\Lambda^k = 0, \quad (\gamma\cdot\partial)\Phi^k = 2\gamma_m\partial^m\Lambda^k,$$

$$\partial^\mu(\Xi^k\gamma^0)\gamma_\mu = 0, \quad \partial^\mu(\Theta^k\gamma_0)\gamma_\mu = 2\partial^m(\Xi^k\gamma_0)\gamma_m.$$

We see that Φ and Θ are the independent variables. Eliminating Λ and Ξ we get

$$I(\pm\tfrac{3}{2}) = \int dx\,\frac{i}{8\Delta}\,\bar{\Phi}(\gamma\partial)\gamma_m\partial^m(\gamma\partial)\Phi.$$

It is the $\pm\frac{3}{2}$ dipole ghost action [5]. Remembering that $(\gamma\partial)\gamma_m\partial^m = -\gamma_m\partial^m(\gamma^\dagger\partial)$, we have [5]

$$I(\pm\tfrac{3}{2})_{\text{effective}} = \int dx\bar{\Phi}(\gamma^\dagger\partial)\,(\gamma\partial)\Phi.$$

2.4. The action (4)

Putting $\psi_{\mu\nu} = \varepsilon_{\alpha\mu\nu\beta}\gamma_5\gamma^\beta\psi^\alpha$, we get the $m = 0$ Rarita–Schwinger theory.

3. *Final remarks*

Let us briefly summarize our result. We have shown that (*i*) the theories of spin $\frac{3}{2}$ equivalent in the massive case, are no longer equivalent in the $m = 0$ limit; (*ii*) the Townsend description with the help of the antisymmetric tensor-bispinor can be obtained as the $m = 0$ limit of the nonminimal massive theory.

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APPENDIX

We use the following decomposition of the vector-bispinor ($k = 1, 2, 3$)

$$\phi^k = \hat{\phi}^k+\gamma^k\phi,$$

where ϕ is the bispinor and $\gamma_k\hat{\phi}^k = 0$. $\hat{\phi}^k$ can be decomposed into the orthogonal parts

$$\hat{\phi}^k = \hat{\phi}^k(\pm\tfrac{3}{2})+\hat{\phi}^k(\pm\tfrac{1}{2}),$$

where

$$\hat{\phi}^k(\pm\tfrac{1}{2}) = -\frac{1}{2\Delta}\,[3\partial^k-\gamma^k(\gamma^i\partial_i)]\partial_m\hat{\phi}^m, \quad \Delta = -\partial_k\partial^k.$$

We see that

$$\gamma_k \hat{\phi}^k(\pm\tfrac{3}{2}) = \gamma_k \hat{\phi}^k(\pm\tfrac{1}{2}) = 0$$

and

$$\partial_k \hat{\phi}^k(\pm\tfrac{3}{2}) = 0, \qquad \partial_k \hat{\phi}^k(\pm\tfrac{1}{2}) = \partial_k \hat{\phi}^k.$$

The analogous decomposition can be performed for $\bar{\phi}^k$.

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