LETTERS TO THE EDITOR

INTERMITTENCY IN THE ISING SYSTEMS*

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It is shown that the Ising model in two dimensions reveals the intermittent behaviour at the critical temperature. We conjecture that intermittency exists generally at the second order phase transition points.

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Until recently intermittency was studied in a variety of phenomena described by the classical physics [1]. Some time ago it was suggested that the spectra of particles produced in the high energy collisions may also show intermittent behaviour [2–4]. Several models of the multiparticle production with the intermittency were proposed and the specific experimental tests suggested [5, 3, 6]. Very interestingly, the Kraków-Louisiana-Minnesota group has found an evidence for the intermittency in the high energy proton-nucleus and nucleus-nucleus collisions [7].

In general intermittency is defined as the occurrence of the large fluctuations which may dominate the average characteristics of the system [1]. In the case of the multiparticle production the good averages to look at are the factorial moments [2]

$$F^{(i)} = \left\langle M^{i-1} \sum_{m=1}^{M} \frac{k_m(k_m-1)\dots(k_m-i+1)}{n(n-1)\dots(n-i+1)} \right\rangle_{\text{ev}} \tag{1}$$

where k_m is the number of particles emitted in the m-th bin of rapidity in a given event, and $\langle\rangle_{\text{ev}}$ denotes averaging over all events in the sample; n is the multiplicity of the event. M determines the size of the rapidity bin $\delta Y = \frac{\Delta Y}{M}$ into which the relevant phase space

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ΔY has been divided. The working definition of the intermittency employed in Refs [2–4] is that the factorial moments, Eq. (1), grow with M, i.e. with the decreasing bin size δY. It was argued that this can happen only if the system under consideration reveals the strong fluctuations appearing over *all* rapidity ranges. If below certain scale δY^* say, the system is "smooth", then the moments, Eq. (1), would flatten for $\delta Y < \delta Y^*$.

In this letter we point out that the intermittent behaviour is naturally expected in a variety of statistical systems at the phase transition point of the second order type. The reason for the requirement of the second order phase transition is simple. Only in this case the fluctuations of arbitrary sizes appear in the system [9] (the correlation length diverges), and consequently the factorial moments, Eq. (1), would grow with the decreasing scale. We illustrate this conjecture by the well known example of the Ising system in two dimensions. The model is defined by the action

$$Y = -\beta \sum_{\langle ij \rangle} s_i s_j \tag{2}$$

with β being the inverse temperature, and the summation extends over all pairs of nearest neighbouring spins.

We shall work in the framework of the Lattice Gauge Theories. It is well known that in the Ising 2 system, close to the critical temperature, the domains of the ordered spins of arbitrary sizes appear [10]. Therefore, the natural definition of the factorial moments remains as in Eq. (1) with k_m denoting now the number of spins oriented in the positive direction in the m-th "cell", and the whole lattice is divided into the $M(M \equiv 2^{2(l-1)})$ cells ($m = (m_1, m_2)$; $m_i = 1, 2, \ldots, 2^{l-1}$; $i = 1, 2$). The average over the events becomes the usual LGT average over the configurations. Monte Carlo simulations were performed in a standard fashion [11, 12]. The configurations were generated with the aid of the Metropolis algorithm [13]. After the first 2000 thermalizing sweeps we have measured the moments, Eq. (1) at every 30-th sweep in order to reduce the sweep-to-sweep correlations. Altogether we have run 10000 sweeps for each β value. All computations were done on the IBM Personal Computer.

Fig. 1 summarizes our results for the factorial moments as a function of the cell size (the whole 32×32 lattice has been divided into $2^{2(l-1)}$ cells). The smallest ($l = 5$) cell contains 4 spins while the largest one ($l = 1$) constitutes the whole lattice. By definition all moments are normalized to unity at $l = 1$. The highest computed moment ($i = 5$) is necessarily equal to zero for $l = 5$. Statistical errors are of the order of the size of the displayed symbols. The solid lines are hand drawn just to guide the eye.

It is seen from Fig. 1 that close to the Curie temperature ($\beta_c \approx 0.44$ for the infinite system) the moments grow rapidly with l. That is the system reveals its intermittent behaviour. This is in agreement with the physical picture of the phase transition being accompanied by the creation of the scale invariant domains of ordered spins [9]. The fluctuations, though weaker, persist also in the high temperature region while in the ordered phase they vanish rather abruptly.

It is unclear, at present, if the rapid change in the behaviour of F's between $\beta = 0.45$ and $\beta = 0.46$ is the genuine property of the factorial moments, or if it results from some

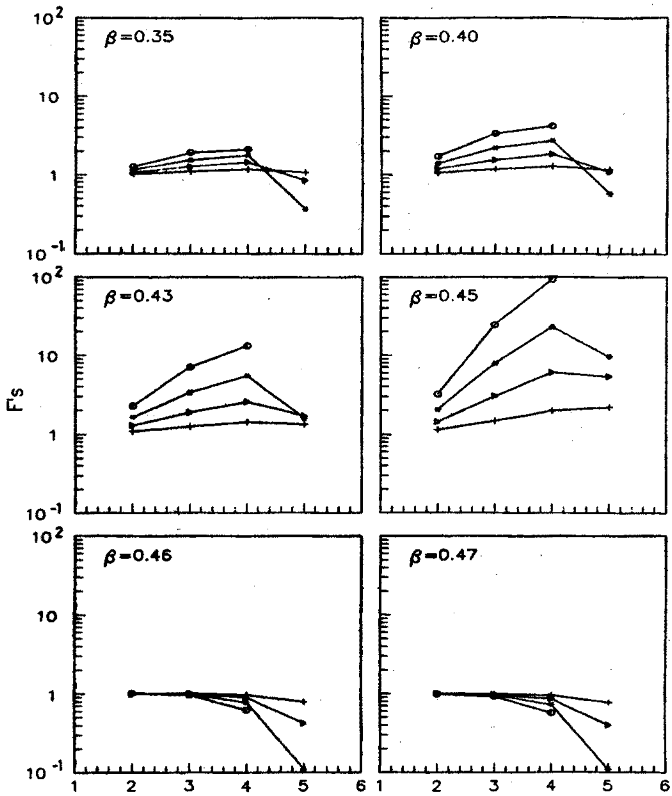

Fig. 1. Monte Carlo results for the factorial moments: $F^{(i)}$, $i = 2(+)$, $3(\Delta)$, $4(*)$, $5(\bigcirc)$; as the function of the cell size, and for various temperatures $T = \frac{1}{3}$

kind of the numerical instability. The former case would be very intriguing, it implies that we have found a new family of the order parameters which is very effective in locating the critical points of the system. More extensive calculations on bigger machines must be done in order to verify/reject the latter possibility.

The lower value of all moments for $l = 5$ results from the finite multiplicity effect [2]. At $l = 5$ we have $k_m \leqslant 4$, and consequently all moments, especially the higher ones, are "kinematically suppressed" for large l. Due to the finite size of our lattice the available l range is rather small, hence the asymptotic form of the dependence of the moments on the cell size did not develop yet. However, Fig. 1 suggests that, in the vicinity of the phase transition, the dependence is power like — typical to the intermittent phenomena.

Our calculations may be improved in many details. One should go for larger lattice, sweep-to-sweep correlations should be treated more carefully etc. However, the relation between the growth of the factorial moments and the existence of the fluctuations occurring over all scales is undoubtedly demonstrated.

Many interesting questions arise. For example, we predict that the growth of $F^{(i)}$'s is closely related to the *order* of the phase transition. In the systems with the first order phase transition, factorial moments should not rise indefinitely since the correlation length there is finite, and hence the size of the fluctuations is also limited by this scale.

The Ising model is the prototype of the Quantum Field Theory, hence it is interesting to extend our study also to the Yang-Mills Lattice Gauge Theories. Since the factorial moments appear to be rather efficient probe of the fluctuations occurring in the system, they may offer a good approach to study the structure of the Yang-Mills vacuum in both: the high temperature and confining phases.

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