TEMPERATURE EFFECTS IN THE EVOLUTION OF A SCALAR FIELD IN THE INFLATIONARY UNIVERSE*

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We investigate influence of temperature effects on evolution of a scalar field in the new inflationary model of the Universe. We study a simple model of a potential and show that temperature effects only slightly influence the duration of the inflationary period and the amplitude of density perturbations.

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1. *Introduction*

The inflationary model of the early evolution of the Universe introduced by Alan Guth [1], and later modified by Albrecht and Steinhardt, and independently by Linde [2] provides a solution of several problems inherent in the standard Big Bang cosmology, for example the flatness, horizon and isotropy problems. In the inflationary scenario it is assumed that at very early stages of evolution the Universe was very hot (temperature was higher than the critical temperature of transition from symmetric to nonsymmetric state of the grand unified theory) and expanding. As the Universe cools down, due to the special form of the potential (Fig. 1) it could be trapped in the symmetric phase (false vacuum state) and supercools. Inflation occurs during the slow transition from the false

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vacuum state to the true vacuum state. This transition is usually referred as slow roll over. As long as the energy density can be approximated by the energy density of the false vacuum the Universe is expanding exponentially and its geometry can be described by the de Sitter metric

$$ds^2 = dt^2 - e^{2Ht}(dr^2 + r^2 d\Omega^2), \tag{1}$$

where $H = \dot{R}/R$ is the Hubble constant and $d\Omega^2$ is the line element on the unit sphere.

In Grand Unified Theories the Higgs field usually has a very complicated group structure [3]. To describe the evolution of the field it is useful to consider only the direction

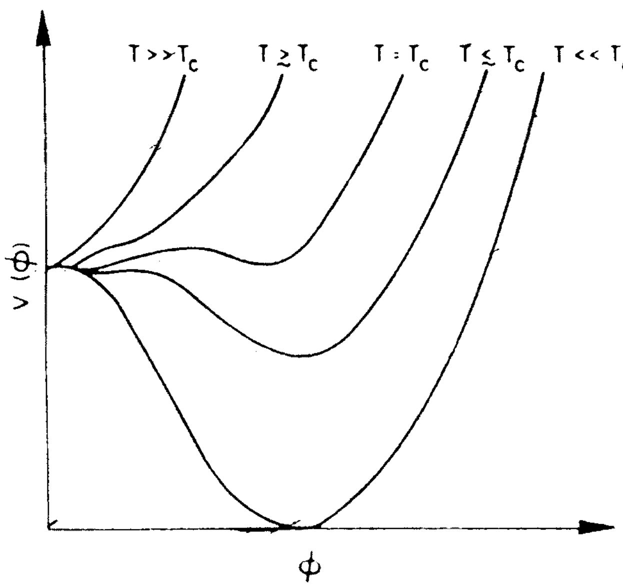

Fig. 1. The Coleman-Weinberg type effective potential for the Higgs field

of spontaneous symmetry breaking and use a one dimensional potential [4, 5]. Recently several different potentials have been studied and it was shown that they provide enough inflation to overcome some of the problems of the standard Big Bang cosmological model.

In this note we would like to consider temperature effects on the evolution of a scalar field and to study their impact on the inflationary scenario. Following the analysis of Guth and Pi [6] we consider a simple potential and study the zero temperature evolution of the scalar field. We also estimate the amplitude of energy density perturbations and duration of the inflationary epoch.

In the next chapter we assume that the system is immersed in a thermal bath and kept at constant temperature T. We investigate the finite temperature evolution of the scalar field. It turns out that temperature effects only slightly influence the duration of the inflationary epoch and the amplitude of energy density perturbations.

2. *The zero temperature model*

Following Guth and Pi [6] let us consider a scalar field with a potential

$$V = V_0 - \tfrac{1}{2}\omega_0^2 \Phi^2, \tag{2}$$

where V_0 and ω_0 are constants. We keep only the quadratic term because the scalar field responsible for inflation is very weakly coupled [4, 5]. The evolution of the scalar field in an expanding Universe is described by the equation

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0. \tag{3}$$

In an expanding Universe the scalar field is dynamically damped. The equation of motion for the scalar field can be derived from a time dependent Lagrangian

$$L = (\tfrac{1}{2}\dot{\Phi}^2 - V(\Phi))e^{\Gamma t}, \tag{4}$$

where $\Gamma \approx 3H$.
The scalar field evolves slowly [7] when

$$\omega_0^2 \leqslant 9H^2. \tag{5}$$

We assume that initially the scalar field is concentrated around the false vacuum state ($\Phi = 0$) and the probability $\Psi(\Phi)$ that the field has value Φ is given by the Gaussian distribution

$$\Psi(\Phi) = N \exp(-a(\Phi - \Delta\Phi)^2) \tag{6}$$

$1/\sqrt{2a} \leqslant l$, where $l \sim H^{-1}$ is the typical region where the scalar field is homogeneous [8]. N is the normalization constant $N = \sqrt{\frac{2a}{\pi}}$, and $\Delta\Phi$ — describe initial quantum fluctuations around the false vacuum state. We assume that quantum fluctuations $\Delta\Phi$ have the Gaussian distribution, so

$$P(\Delta\Phi) = \frac{1}{\sqrt{\pi\sigma}} \exp\left(-\frac{1}{\sigma}\Delta\Phi^2\right) \tag{7}$$

and $\sqrt{\sigma} \ll l$.

Instead of solving the equation describing evolution of the scalar field we calculate the Green function. Using the path integral method [9] we obtain

$$G_0(\Phi, \Phi', t) = \left[\frac{\omega e^{\Gamma t/2}}{2\pi i\hbar\, \mathrm{sh}\, \omega t}\right]^{1/2} \exp\left\{-\frac{i}{\hbar}\frac{V_0(e^{\Gamma t}-1)}{\Gamma}\right\}$$

$$\times \exp\left\{\frac{i}{\hbar}\frac{1}{2\,\mathrm{sh}\,\omega t}\left[(\Phi^2 e^{\Gamma t} + \Phi'^2)\left(\omega\, \mathrm{ch}\, \omega t - \frac{\Gamma}{2}\mathrm{sh}\,\omega t\right) - 2\Phi\Phi' e^{\Gamma t/2}\right]\right\} \tag{8}$$

It is now easy to calculate the average value $\langle\Phi^2\rangle(t)$ and in the limit of late times, taking into account that $\mathrm{sh}\,\omega t \sim \mathrm{ch}\,\omega t \sim \frac{1}{2}e^{\omega t}$, $\omega = \sqrt{\omega_0^2+\frac{1}{4}\Gamma^2}$, we obtain

$$\langle\Phi^2\rangle_0(t) = \frac{a^2+\alpha'^2}{16\alpha^2 a}\,e^{2\omega' t}\left[1+4a\Delta\Phi^2\,\frac{\alpha'^2}{a^2+\alpha'^2}\right], \tag{9}$$

where $\alpha = \dfrac{\omega}{2\hbar}$, $\quad \alpha' = \alpha\left(1-\dfrac{\Gamma}{2\omega}\right)$, $\quad \omega' = \omega-\frac{1}{2}\Gamma$.

To estimate the amplitude of density perturbations we use the formula derived by Bardeen et al.

$$\frac{\delta\varrho}{\varrho} = 4\,\frac{\delta\dot{\Phi}}{\dot{\Phi}_{\mathrm{cl}}}\,. \tag{10}$$

In our case $\delta\Phi = \sqrt{\langle\Phi^2\rangle_{\Delta\Phi}}$ and Φ_{cl} is classically evolving field so for large time we have $\Phi_{\mathrm{cl}}(t) = \left[\dfrac{a^2+\alpha'^2}{16\alpha^2 a}\right]^{1/2} e^{\omega' t}$. In the zero temperature case energy density perturbations are

$$\frac{\delta\varrho}{\varrho} = 4\left[1+2a\sigma\,\frac{\alpha'^2}{a^2+\alpha'^2}\right]^{1/2}. \tag{11}$$

The value of $\dfrac{\delta\varrho}{\varrho}$ which we obtained is unrealistically large but this is a problem encountered also by other more realistic models of inflation [10].

3. *The evolution of the scalar field at finite temperature*

To study the influence of temperature on the evolution of a scalar field in an expanding Universe we use the formalism of temperature Green function [11], which is defined by

$$iG(\Phi, t, \Phi', t') = Z_G^{-1}\,\mathrm{Tr}\,[e^{-\beta\hat{K}}e^{it\hat{K}/\hbar}\hat{\phi}(\Phi)e^{-i\hat{K}(t-t')/\hbar}\hat{\phi}^+(\Phi')e^{-i\hat{K}t'/\hbar}], \tag{12}$$

where $\hat{\phi}(\Phi, t)$ is the field operator, $\hat{K} = \hat{H}-\mu\hat{N}$, $\hat{H}$ is the hamiltonian and $\hat{N}$ is the number of particles operator, μ is the chemical potential, $Z_G = \mathrm{Tr}\,e^{-\beta\hat{K}}$ and $\beta = \dfrac{1}{kT}$.

At first we disregard the damping term. In this case the temperature Green function can be expressed in terms of eigenfunctions of the Hamiltonian. Let u_k be such that $\hat{H}u_k = E_k u_k$, then

$$iG_\beta(\Phi, t, \Phi', t') = \sum\int u_k(\Phi)u_k^*(\Phi')\,(1+n_k)e^{-ie_k(t-t')/\hbar}, \tag{13}$$

where $e_k = E_k-\mu$, and n_k is the thermal average of the number of particles in the k-state. It is reasonable to assume that particles obey the Boltzmann statistics, so we have

$$n_k \sim e^{-e_k\beta}. \tag{14}$$

The finite temperature Green function can be expressed in terms of G_0, we have

$$iG_\beta(\Phi, \Phi', t) = e^{i(\mu - E_0)t/\hbar}[G_0(\Phi, \Phi', t) + e^{\mu\beta} G_0(\Phi, \Phi', t - i\beta\hbar)], \tag{15}$$

where E_0 is the ground state energy at $T = 0$.

When damping is included we construct analogues of creation and annihilation operators. For the Hamiltonian of damped harmonic oscillator

$$\hat{H} = -e^{-\Gamma t} \frac{\hbar^2 \partial^2}{2\partial\Phi^2} + \tfrac{1}{2}\omega_0^2 \Phi^2 e^{\Gamma t}, \tag{16}$$

following the standard procedure [12], we can express the creation and annihilation operators a^+ and a in terms of the creation and annihilation operators of the time independent ($\Gamma = 0$) Hamiltonian

$$\begin{pmatrix} a^+(t) \\ a(t) \end{pmatrix} = \begin{pmatrix} \mathrm{ch}\,(\Gamma t/2) & -\mathrm{sh}\,(\Gamma t/2) \\ -\mathrm{sh}(\Gamma t/2) & \mathrm{ch}\,(\Gamma t/2) \end{pmatrix} \begin{pmatrix} a_0^+ \\ a_0 \end{pmatrix}. \tag{17}$$

The time independent Hamiltonian is represented by the diagonal matrix

$$H_0 = :\tfrac{1}{2}\,\hbar\omega_0 A_0^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A_0:, \tag{18}$$

where $A_0 = \begin{pmatrix} a_0^+ \\ a_0 \end{pmatrix}$, and :: denotes normal ordering of operators a_0^+, a_0.

It is easy to generalize this description for infinite number of oscillators. Using (17) we can express the Hamiltonian of an infinite set of oscillators in terms of annihilation and creation

$$H(t) = \sum_n \hbar\omega_n[\mathrm{ch}\,\Gamma t\, a_n^+ a_n - \tfrac{1}{2}\,\mathrm{sh}\,\Gamma t\,(a_n^+ a_n^+ + a_n a_n)]. \tag{19}$$

The same relation holds for the upside down harmonic oscillator.

To calculate the finite temperature Green function we use the vector description of the creation and annihilation operators. We assume that at the initial moment $t = 0$ the field operators in the damped and undamped case coincide, so $\hat{\phi}(\Phi, t = 0) = \hat{\phi}_\Gamma(\Phi, t = 0)$. The $\hat{K}$ operator can be expressed in terms of the creation and annihilation operators and it can be written in the form $\hat{K} = \Sigma \int (E_k - \mu) a_k^+ a_k$. Using the transformation (17) we obtain the Green function, in the high temperature limit ($\Gamma\beta \to 0$), in the form

$$iG_\beta(\Phi, \Phi', t) = e^{i(\mu - E_0)t\,\mathrm{ch}t/\hbar}[G_0(\Phi, \Phi', t) + e^{\mu\beta\,\mathrm{ch}\Gamma t} G_0(\Phi, \Phi', t - i\beta\hbar)]. \tag{20}$$

It is now easy to calculate the average value of the scalar field, we have

$$\langle \Phi^2 \rangle_\beta(t) = \frac{\sqrt{2a}}{8} \frac{a^2 + \alpha'^2}{\alpha^2} e^{2\omega' t} \left[(2a)^{-3/2} \left[1 + 4a\Delta\Phi^2 \frac{\alpha'^2}{a^2 + \alpha'^2} \right] \right.$$

$$+ 2\,\mathrm{Re} \left\{ e^{\mu\beta\,\mathrm{ch}\,\Gamma t} \exp\left(\frac{-V_0 e^{\Gamma t} e^{-\Gamma\beta/2} 2 \sin(\Gamma\beta h/2)}{\Gamma\hbar} \right) e^{i\omega'\beta} (A_{1/2})^{-3/2} \right.$$

$$\times\left[1+8a\Delta\Phi^2\frac{a^2}{(a^2+\alpha'^2)}\frac{B_{1/2}^2}{A_{1/2}}\right]\exp\left[-\Delta\Phi^2\left(\frac{2a\alpha\alpha'}{a^2+\alpha'^2}-\frac{4a^2}{(a^2+\alpha'^2)}\frac{B_{1/2}^2}{A_{1/2}}\right)\right]\Bigg\}$$

$$+\ e^{2\mu\beta\,\mathrm{ch}\,\Gamma t}\exp\left(\frac{-2V_0e^{\Gamma t}e^{-\Gamma\beta}2\sin\Gamma\beta h}{\Gamma h}\right)(A)^{-3/2}$$

$$\times\left[1+8a\Delta\Phi^2\frac{a^2}{(a^2+\alpha'^2)}\frac{B^2}{A}\right]\exp\left[-\Delta\Phi^2\left(\frac{2a\alpha\alpha'}{a^2+\alpha'^2}-\frac{4a^2}{(a^2+\alpha'^2)}\frac{B^2}{A}\right)\right]\Bigg]\text{,} \qquad (21)$$

where

$$A = 2(a\cos 2\omega'\beta\hbar-\alpha'\sin 2\omega'\beta\hbar)$$

$$+\frac{\alpha'^2+a^2}{\alpha'^2}(\cos\Gamma\beta\hbar\sin 2\omega\beta\hbar-2e^{2\omega t}\sin\Gamma\beta\hbar)$$

$$A_{1/2} = A(\beta/2) \qquad (22a)$$

$$B = a\sin\omega'\beta\hbar+\alpha'\cos\omega'\beta\hbar$$

$$B_{1/2} = B(\beta/2). \qquad (22b)$$

In the high temperature limit we have the simple formula

$$\langle\Phi^2\rangle_\beta(t) = (1+e^{\mu\beta\,\mathrm{ch}\,\Gamma t})^2\langle\Phi^2\rangle_0(t), \qquad (23)$$

and the amplitude of energy density perturbations in this limit is given by

$$\left(\frac{\delta\varrho}{\varrho}\right)_\beta=(1+e^{\mu\beta\,\mathrm{ch}\,\Gamma t})\left(\frac{\delta\varrho}{\varrho}\right)_0. \qquad (24)$$

4. *Conclusions*

Now we can estimate the duration of the inflationary epoch. One can define the duration of the inflationary epoch as a time necessary for the scalar field to evolve from the value $\Phi(t=0)=0$ to some value $\Phi_1=\Phi(t_1)$ at which the energy density is still dominated by $V(\Phi_1)\sim V(0)$. Using equations (9), (23) we see that the duration of the inflationary epoch in the high temperature limit denoted by $t_{1\beta}$ is related to the duration of the inflationary epoch at zero temperature denoted by t_1 by

$$t_1-t_{1\beta} = \frac{1}{\omega'}\ln(1+e^{\mu\beta\,\mathrm{ch}\Gamma t_1\beta}). \qquad (25)$$

When the scalar field is in thermal equilibrium with radiation the chemical potential μ is equal to zero [13], therefore, we have

$$\omega'(t_1-t_{1\beta}) = \ln 2 \qquad (26)$$

and also

$$\left(\frac{\delta\varrho}{\varrho}\right)_\beta = 2\left(\frac{\delta\varrho}{\varrho}\right)_0. \tag{27}$$

We see that temperature effects only slightly alter the duration of the inflationary period and the amplitude of energy density perturbations.

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