MULTIPLICITY DISTRIBUTIONS IN THE LUND SHOWER MODEL OF e^+e^- ANNIHILATION

BY L. VAN HOVE AND A. GIOVANNINI*

CERN — 1211 Geneva 23, Switzerland

(*Received May 14, 1988*)

We report on a theoretical investigation of parton and hadron multiplicity distributions produced by quark-antiquark and gluon-gluon systems at centre-of-mass energies from 22 GeV to 2 TeV, on the basis of the Lund Shower Model, which is the Monte Carlo parton-shower and hadronization model accounting best for the available e^+e^- annihilation data. Both classes of distributions are studied for full phase space and central rapidity windows. They are found to have negative binomial properties analogous to those observed in many experiments. We also find a simple relation between the partonic and hadronic distributions; it can be linked with the concepts of preconfinement and local parton-hadron duality. At partonic level, the results suggest a simple interpretation in terms of independent emission of "bremsstrahlung gluon jets" having geometric multiplicity distributions.

PACS numbers: 12.38.Qk

1. *Introduction*

Following the unexpected findings of the UA5 Collaboration at the CERN $p\bar{p}$ collider in 1985 [1a], many experimental groups analyzed multiplicity distributions in e^+e^- annihilation [1b], hadron-proton collisions [1c], proton-nucleus collisions [1d] and deep inelastic muon-proton scattering [1e]. They confirmed the very wide occurrence of the negative binomial (NB) shape for the charged particle distributions in central rapidity intervals $|y| < y_0$ (y = longitudinal rapidity in the c.m. frame; in the e^+e^- annihilation experiment of Ref. [1b] the longitudinal direction is defined by the thrust axis of the final state particles). For total multiplicities, the NB shape holds at high energies whereas at lower energies one finds narrower distributions with Poisson or positive binomial shape. A recent survey of the available experimental evidence is given in Ref. [2].

Continuing our search for a dynamical understanding of the widespread occurrence

* On leave of absence from Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino. Supported in part by Ministero della Pubblica Istruzione (Italy) under grant 1986.

of NB multiplicity distributions [3, 4] and encouraged by a recent study of Kittel [5], we report on a theoretical investigation of multiparticle production by quark-antiquark ($q\bar{q}$) and gluon-gluon (gg) systems up to c.m. energy $\sqrt{s} = 2$ TeV, based on the model which has achieved greatest overall success in accounting for the available e^+e^- annihilation data. Our reason to concentrate on $q\bar{q}$ and gg systems is that they offer the cleanest cases to study QCD-based particle production mechanisms, avoiding the complications linked with the partonic structure of initial-state hadrons and with final-state diffraction dissociation effects.

The extensive data on e^+e^- annihilation obtained at PETRA ($\sqrt{s} \lesssim 45$ GeV) and PEP ($\sqrt{s} \lesssim 30$ GeV) have been analyzed by means of various QCD-based models (most of these data do not concern multiplicity distributions). A thorough comparison between the various models, carried out recently by the MARK II Collaboration [6, 7], shows that the best overall description is given by the Lund Shower Model with coherent evolution (we adopt the terminology of Ref. [6]), the optimal values of the shower evolution parameters being

$$\Lambda = 0.4 \text{ GeV} \qquad \text{(QCD scale in LLA)},$$

$$Q_0 = m_{\min} = 1.0 \text{ GeV} \qquad \text{(cut-off for parton evolution)}. \tag{1}$$

The model is described in Ref. [8] and is implemented in the program called JETSET version 6.3 [9]. Basically it contains the Monte Carlo generation of a parton shower from the initial $q\bar{q}$ or gg system, followed by a hadronization prescription of Lund string type. The parton shower follows the QCD Leading Log Approximation (LLA), except for the imposition of an angular ordering condition which simulates the destructive interference between soft gluons. The simulation of this "coherent evolution" by angular ordering was first incorporated in a Monte Carlo model by Marchesini and Webber [10] and the JETSET 6.3 shower algorithm is of similar character. JETSET 6.3 is completely different with regard to the hadronization of the final partons of the shower for which the Lund string fragmentation algorithm [11] is used.

The work reported below uses JETSET 6.3 with default values, i.e., coherent evolution with angular ordering, p_T^2 as scale parameter in α_s, and global definition of the splitting variable z (these choices correspond to the curves marked "d" in Figs. 5 and 7 of Ref. [8], where the necessary definitions are also given).

The Monte Carlo results reported below (Section 2) were obtained in collaboration with T. Sjöstrand. They concern the multiplicity distributions of the final partons and of the (meta)stable charged hadrons. Confirming and extending Kittel's result [5], we find that both the partonic and the hadronic distributions have the same NB properties as observed experimentally in many reactions. At sufficiently high energy ($\sqrt{s} \gtrsim 200$ GeV) we find in addition a close relationship between the partonic and hadronic distributions which we relate in Section 3 to the concept of local parton-hadron duality [12]. In Sections 4 and 5 the clan structure of NB distributions, already used in our earlier work [3, 4], is applied to the analysis of the partonic distributions. It leads to an approximate but physically intuitive description of the partonic distributions in terms of bremsstrahlung gluon jets with geometric (i.e., self-similar) multiplicity distributions. Section 6 summarizes the results and presents concluding remarks. A few relevant mathematical facts concerning NB distributions are grouped in the Appendix.

2. *Main Monte Carlo results*

As explained in the Introduction, in collaboration with Sjöstrand we used the Lund Parton Shower Model to generate the partonic and hadronic systems produced by $q\bar{q}$ and gg pairs at c.m. energies $\sqrt{s} = 22, 29, 200, 630$ and 2000 GeV. Extending recent work of Kittel (who had used two options of the earlier version 6.2 of JETSET with other values of Λ and Q_0 [5]), we investigated the multiplicity distributions and found the following overall results:

A) Good NB fits for final-parton and charged hadron multiplicities, in full phase space and in symmetric intervals $|y| < y_0$ of rapidity[1].

B) Increase of the NB parameter k (see Appendix) for growing interval $|y| < y_0$ at fixed energy $\sqrt{s}$, and increase of k^{-1} with $\sqrt{s}$ for total phase space and for fixed interval $|y| < y_0$.

C) Approximate energy independence of the quantity

$$\bar{N} = k \ln [1+(\bar{n}/k)] = \bar{n}/\bar{n}_c \tag{2}$$

in fixed intervals $|y| < y_0$, and its approximate linearity in y_0 at fixed energy. $\bar{N}$ is the average number of "clans" in the interpretation of NB distributions in terms of independent emission of clans (see Section 4 below). The increase of $\bar{n}$ is therefore mostly due to the increase of the average clan multiplicity $\bar{n}_c = \bar{n}/\bar{N}$.

The only exceptions to NB behaviour are for charged hadrons in intervals $|y| < y_0$ with $y_0 > 3$ (for which the familiar even-odd effect due to charge conservation appears, good NB fits holding for even multiplicities) and with $y_0 \lesssim 1$ (where the hadronic distribution has a small peak at low multiplicity above a smooth NB-shaped background; surprisingly this effect is only pronounced for $q\bar{q}$ at $\sqrt{s} = 2000$ GeV). At $\sqrt{s} = 22$ and 29 GeV, while A, B and C hold for the hadronic distributions, the partonic ones for $y_0 \geqslant 2$ are narrower than Poisson and give poor fits. All Monte Carlo runs generated 2000 events. The goodness of fit parameters (χ^2/number of degrees of freedom) are listed in Tables I and II for a representative selection of cases.

Figures 1–4 give our Monte Carlo results for $\bar{N}$ and $\bar{n}_c$ at three energies. The values for total multiplicities are approximately equal to those for the largest y_0 shown. The errors are mostly $\lesssim 4\%$ (the error on k is larger when the distribution is close to Poisson or narrower). Figures 1 and 2 for $\bar{N}$ strikingly illustrate property C, especially Fig. 1 for partons. In addition, Figures 3 and 4 reveal the interesting property that the $\bar{n}_c$ values

---

[1] For programming convenience, the rapidity was defined with respect to the linear sphericity axis for the final partons, which corresponds to the largest eigenvalue of the tensor $S_{\alpha\beta} = (\sum_i p_{i\alpha} p_{i\beta} |p_i|^{-1}) (\sum_i |p_i|)^{-1}$ with $p_{i\alpha}(\alpha = x, y, z)$ the momentum of parton i. The results are expected to be similar if sphericity or thrust had been used.

TABLE I

χ^2/number of degrees of freedom for negative binomial fits to the multiplicity distributions of final partons at $\sqrt{s} = 29$. 200, 2000 GeV in rapidity windows $|y| < y_0$ and in full phase space

y_0	0.5	2	4	5	6	Total
gg						
29 GeV	8.95/8	16.9/14	24.7/12	—	—	17.5/12
200 GeV	28.3/17	51.9/36	25.2/35	47.4/34	—	40.5/34
2000 GeV	69.5/36	92.2/84	109/83	61.6/79	77.8/78	79.2/78
$q\bar{q}$						
29 GeV	8.16/7	59.5/13*	45.2/14*	—	—	47.8/12*
200 GeV	28.9/13	49.5/29	46.3/29	45.2/29	—	37.0/29
2000 GeV	23.3/26	76.5/63	103/68	88.7/69	93.9/66	75.0/64

* bad fit.

TABLE II

χ^2/number of degrees of freedom for negative binomial fits to the multiplicity distributions of charged hadrons at $\sqrt{s} = 29$, 200, 2000 GeV in rapidity windows $|y| < y_0$ and in full phase space

y_0	0.5	2	4+	5+	6+	Total
gg						
29 GeV	14.0/15	40.8/26	10.7/14	—	—	10.8/14
200 GeV	63.0/33	71.6/63	57.0/29	23.5/29	—	27.0/29
2000 GeV	127/71	144/153	127/80	93.1/74	66.0/73	70.6/74
$q\bar{q}$						
29 GeV	13.0/12	22.3/23	20.0/11	—	—	16.8/10
200 GeV	65.1/20*	114/54	65.6/28	51.3/27	—	64.5/27
2000 GeV	194/54*	152/119	142/68	86.3/67	100/63	86.6/62

+ even multiplicities only, * bad fit.

for the $q\bar{q}$ and gg systems are approximately equal. At $\sqrt{s} = 29$ GeV, the Monte Carlo values of $\bar{N}$, $\bar{n}_c$ for the hadronic distributions of the $q\bar{q}$ system are within at most 10% of the HRS Collaboration data [1b], although the analysis of Refs. [6, 7] did not involve the multiplicity distributions of [1b]. One will have to wait for the new e^+e^- colliders SLC at SLAC and LEP at CERN to have data at higher energies.

For $\sqrt{s} \geqslant 200$ GeV we also found the following approximate relations between the NB parameters $\bar{n}_{p,h}$ and $k_{p,h}$ of the partonic and hadronic distributions at the same $\sqrt{s}$ and in the same rapidity domains

$$\bar{n}_h \simeq 2\bar{n}_p \tag{3}$$

$$k_h \simeq k_p. \tag{4}$$

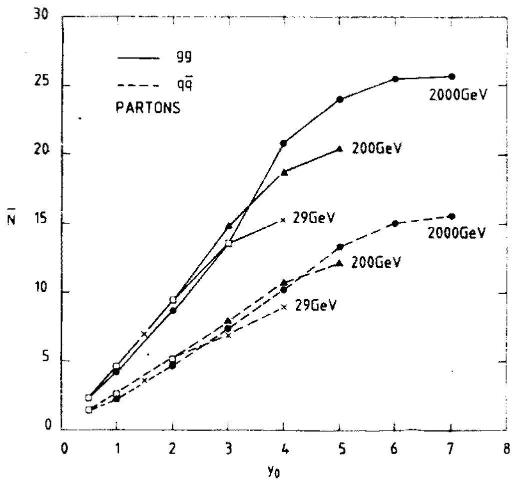

Fig. 1. Average number of clans $\bar{N}$ of final parton multiplicity distributions in rapidity windows $|y| < y_0$ at $\sqrt{s} = 29, 200, 2000$ GeV. The lines are drawn to guide the eye. When calculated points coincide within the errors ($\lesssim 4\%$), they are replaced by an open square

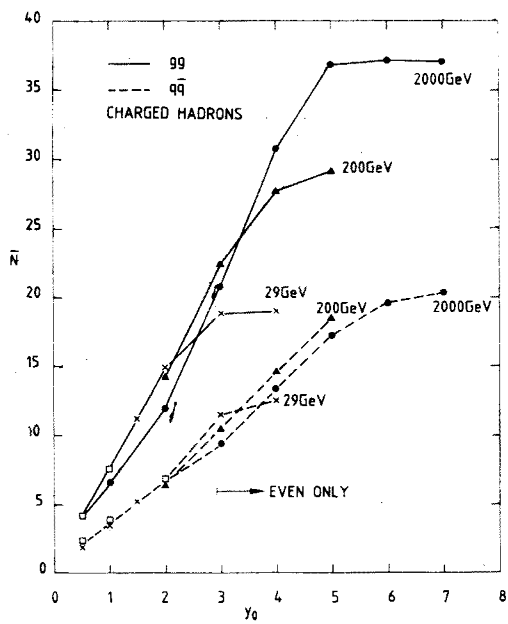

Fig. 2

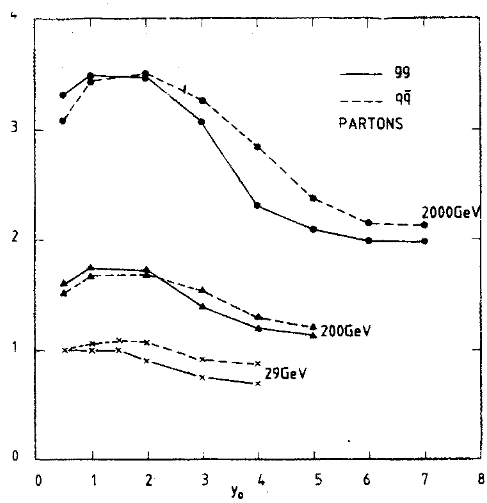

Fig. 3

Fig. 2. Same as Fig. 1 for charged hadron distributions

Fig. 3. Average clan multiplicity $\bar{n}_c$ of final parton distributions in rapidity windows $|y| < y_0$ at $\sqrt{s} = 29$ 200, 2000 GeV

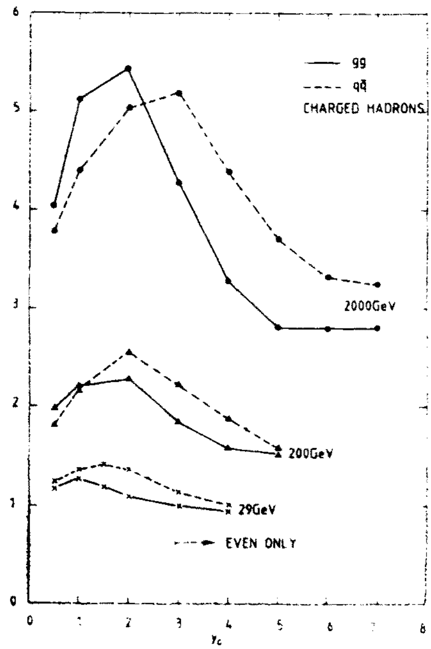

Fig. 4. Same as Fig. 3 for charged hadron distributions

Except for small rapidity intervals ($y_0 \lesssim 1$) where k_h is appreciably larger than k_p, the relations (3), (4) hold to 10–15%. In view of Eqs. (2)–(4), the energy independence of $\bar{N}$ (property C) cannot hold exactly for both the partonic and the hadronic distributions; comparison of Figs. 1 and 2 shows that it holds better at the partonic level. We have also considered intervals $|y - y_1| < y_0$ with $y_0 = 0.5$ and y_1 varying by steps of one over all phase space. While the multiplicity distributions rapidly differ from NB shapes for growing y_1 (an effect presumably due to energy momentum conservation at least for the larger y_1 values), Eq. (3) is found to hold far into the fragmentation regions.

Relation (3) for total multiplicities was already obtained for $\sqrt{s} = 10$ GeV – 10TeV in Ref. [8], see curve "d" of Fig. 7. The other curves of this figure which are all obtained with "conventional" evolution models (no angular ordering) show important variations of the total multiplicity ratio $\bar{n}_h/\bar{n}_p$ with $\sqrt{s}$. For one of these conventional models taken at $\sqrt{s} = 200$ GeV, Sjöstrand has also shown that $\bar{n}_h/\bar{n}_p$ for $|y| < y_0$ increases from 0.7 to 1.2 for y_0 increasing from 0.5 to 4.0 (private communication).

3. Local parton-hadron quality

The validity of Eq. (3) for a wide class of y-intervals suggests that in the coherent evolution model there is a simple local relation between partons and hadrons in most regions of phase space. The existence of such a relation can be linked with the concepts of preconfinement and local parton-hadron duality [12]. We tentatively express this duality in terms of the n-particle inclusive rapidity distributions of partons (p) and hadrons (h) by the following equation

$$Q_{n,\mathrm{h}}(y_1, \dots y_n) = \varrho^n Q_{n,\mathrm{p}}(y_1, \dots y_n) \tag{5}$$

ϱ being a constant. For $n = 1$ and $\varrho \approx 2$, this gives Eq. (3) for any y-interval. Equation (5) can only be approximate because the $Q_{n,\mathrm{h}}$ contain short range correlations due to hadronic resonances which are of course absent in the $Q_{n,\mathrm{p}}$. The interest of (5) lies in the fact that it links the NB properties of the two distributions and reproduces not only (3) but also (4). The proof is as follows. For any rapidity domain D, the generating functions of the multiplicity distributions in D

$$F_{\mathrm{a},D}(z) = \sum_n P_\mathrm{a}(n) z^n; \quad \mathrm{a} = \mathrm{h}, \mathrm{p} \tag{6}$$

are obtained from $Q_{n,\mathrm{a}}$ by constructing the functionals

$$\Phi_\mathrm{a}([u(y)]) = 1 + \sum_1^\infty (n!)^{-1} \int Q_{n,\mathrm{a}}(y_1, \dots y_n) \prod_1^n u(y_i) dy_i \tag{7}$$

and substituting

$$u(y) = z-1 \text{ in } D, \quad u(y) = 0 \text{ outside } D. \tag{8}$$

Hence Eq. (5) implies

$$F_{\mathrm{h},D}(z) = F_{\mathrm{p},D}(z') \text{ with } z'-1 = \varrho(z-1). \tag{9}$$

Assume one of the distributions, e.g. the partonic one, to be NB in D. This is expressed by

$$F_{\mathrm{p},D}(z') = [1-(\bar{n}_\mathrm{p}/k_\mathrm{p})(z'-1)]^{-k_\mathrm{p}}. \tag{10}$$

The substitution (9) then gives for the hadronic distribution in D

$$F_{\mathrm{h},D}(z) = [1-(\bar{n}_\mathrm{h}/k_\mathrm{h})(z-1)]^{-k_\mathrm{h}} \tag{11}$$

with the parameters

$$\bar{n}_\mathrm{h} = \varrho \bar{n}_\mathrm{p}, \quad k_\mathrm{h} = k_\mathrm{p} \tag{12}$$

(11) has NB structure and (12) agrees with (3) and (4).

4. Clan structure of NB distributions

In previous papers [3, 4], we have argued that cascading is the most likely dynamical reason for the NB behaviour observed in so many multiparticle production processes. The results reported above for the parton distributions are based on a parton cascade model and therefore support this view. In Refs. [3] and [4], we have also exploited what we have called the "clan structure" of NB distributions, i.e., the mathematical fact that any NB distribution can be generated by independent emission of groups of particles which we call clans [4], these clans having on the average a logarithmic multiplicity distribution. In terms of generating functions (g.f.), the clan structure is expressed by the relation

$$F_{\mathrm{NB}}(z) = [1-(\bar{n}/k)(z-1)]^{-k} = \exp[\bar{N}(f_l(b, z)-1)] \tag{13}$$

with $\bar{N}$ given by (2) and f_l by

$$f_l(b, z) = \ln(1-bz)/\ln(1-b), \qquad b = \bar{n}/(\bar{n}+k) \tag{14}$$

f_l (l for logarithmic) is the g.f. of an average clan; it corresponds to the logarithmic multiplicity distribution

$$P_{n_c} = -b^{\bar{n}_c}/[\bar{n}_c \ln(1-b)] \tag{15}$$

with average $\bar{n}_c$ as given in (2). Equation (13) shows that $\bar{N}$ is the mean number of clans.

The clan structure can also be formulated in terms of the n-particle distribution functions, either the inclusive ones $Q_n(y_1, \ldots, y_n)$ considered in Section 3 (here we drop the index a = p, h), or the exclusive ones obtained by expanding the functional (7) in powers of $z(y) = 1+u(y)$. In terms of this functional, it is expressed by

$$\Phi([u(y)]) = \exp[\bar{N}(\varphi([u(y)])-1)] \tag{16}$$

with

$$\varphi([u(y)]) = 1+\sum_1^\infty (n!)^{-1} \int q_n(y_1, \ldots y_n) \prod_1^n u(y_i)dy_i. \tag{17}$$

The q_n are the inclusive distributions of particles in an average clan, and the $\bar{N}q_n$ can be shown to be the correlation functions of the original distribution as defined by Mueller [13]. Clans can therefore be regarded as groupings of particles involving all the correlations of the original distribution. In this sense, they are somewhat analogous to the connected clusters of the Ursell-Mayer and Kahn-Uhlenbeck cluster expansions in statistical mechanics [14]. Just as for these clusters, the concept of clan is of statistical nature. In general, it is not possible to say which particles belong to the same clans on an event-by-event basis.

In the case of shower processes, however, one can try to give to the clan structure a more concrete interpretation. At least approximately, clans are then expected to be groups of particles of common ancestry. A very simple example is reported in Appendix III of Ref. [3], where the word "cluster" was used instead of "clan". In this example, individual clans have a geometric distribution with g.f.

$$f_g(v, z) = z(v+z-vz)^{-1} \tag{18}$$

and average multiplicity v. The average clan with its logarithmic distribution of g.f. (14) is then obtained by averaging (18) over v in an interval $(1, v_{\max})$ with a weight function $\propto 1/v$ [see Eqs. (III. 3) of [3] and Eq. (22) below].

As can be deduced from earlier work by one of us [15], exactly the same clan structure occurs in a Markov-process version of the Konishi-Ukawa-Veneziano model of parton shower [16] when $g \to q\bar{q}$ branching is neglected. In this case the clans are *bremsstrahlung gluon jets*. We shall presently argue that, in the central rapidity region, this very simple clan structure can be used for an approximate but physically intuitive description of the partonic NB distributions reported in Section 2.

5. The evolution of partonic clans

Consider first the parton distribution in a fixed interval $|y| < y_0$, for $y_0 \simeq 1.5$–2.0 where the average clan multiplicities $\bar{n}_c$ are close to their maximum (Fig. 3). Since the mean number of clans $\bar{N}$ is practically constant for $\sqrt{s} = 29$-2000 GeV (Fig. 1), the evolution of the multiplicity distribution with $\sqrt{s}$ is almost entirely due to the evolution of the average clan, which is controlled by the single parameter b, see Eqs. (14) and (15). From (14) we get

$$-(1-b)\ln(1-b)\frac{df_1(b,z)}{db} = -f_1(b,z)+\frac{(1-b)z}{1-bz} = -f_1(b,z)+f_g(b,z), \tag{19}$$

where f_g is the g.f. (18) of the geometric distribution of mean multiplicity

$$v = (1-b)^{-1}. \tag{20}$$

With (20) we can also write (19) as

$$\ln v \, df_1(b,z)/d\ln v = -f_1(b,z)+f_g(v,z). \tag{21}$$

This equation has a simple interpretation. As $\sqrt{s}$ increases at fixed y_0, b and $v = (1-b)^{-1}$ increase. For an infinitesimal change, the multiplicity distribution of an average clan evolves by addition of a geometric distribution given by the last term of (21), the previous term ensuring that the normalization condition $f_1(b,1) = 1$ remains satisfied. The logarithmic clan f_1 can therefore be taken as an average over geometric clans f_g with weight $d\ln v$. This is best seen by integrating (21):

$$f_1(b,z) = \int_1^v f_g(v',z)d\ln v'/\ln v, \qquad v = (1-b)^{-1} \tag{22}$$

(both f_1 and f_g reduce to z for $b = 0$, $v = 1$).

An attractive feature of this interpretation is that the occurrence of geometric distributions is very natural in shower models (see the examples mentioned at the end of Section 4). This is linked with their evolution equation, which is

$$df_g(v,z)/d\ln v = -f_g(v,z)+[f_g(v,z)]^2. \tag{23}$$

The first term in the right-hand side is again the normalization correction as in Eq. (21), but the second term is now the square of f_g itself. It corresponds to a self-similar cascade process, as is expected for gluon jets when $g \to q\bar{q}$ splitting is neglected [15].

We now discuss the variations of the partonic clan structure with the size of the rapidity window $|y| < y_0$ at constant $\sqrt{s}$. For $y_0 \lesssim 1.5$–2.0, $\bar{n}_c$ and b increase with y_0 (Fig. 3). The obvious explanation is that for two windows of sizes y_0 and $y_0' > y_0$, some of the clans belonging to the window y_0' only fall partially inside the window y_0, so that their multiplicity inside the latter window is smaller. This growth of clan size with y_0 can again be interpreted in terms of geometric clans, similarly to the growth with $\sqrt{s}$ at fixed y_0.

The situation is more complicated when y_0 grows above ~ 2 because then $\bar{n}_c$ and b decrease, i.e., the average clan size decreases, despite the growth of the rapidity window. Since $\bar{N}$ and therefore the number of clans grows even somewhat faster than for $y_0 \lesssim 2$ (see Fig. 1), the most straightforward explanation is that the additional clans appearing at larger rapidities are small in rapidity spread and in multiplicity. However, if the additional clans are still assumed to be geometric and produced independently, the overall clan distribution can no longer be uniform in $\ln \nu$ as was the case in Eq. (22). This would imply departures from the logarithmic form for the average clan and therefore from the NB form for the overall multiplicity distribution in large symmetric windows $|y| < y_0$, but this effect could well be very small (a recent paper by Cugnon and Harouna [17] illustrates the insensitivity of the NB form for changes in the multiplicity distribution of clans). Stronger deviations would of course occur for asymmetric windows $|y - y_1| < y_0$, as observed in the Monte Carlo results (Section 2). For increasing y_1, these windows get also more strongly affected by energy-momentum conservation.

6. *Conclusions*

Our main conclusion of the above analysis is that for the central region $|y| \lesssim 2$ we can propose the following *approximate but physically intuitive picture of the coherent shower process in* $q\bar{q}$ *and* gg *systems at high* $\sqrt{s}$:

I. In the parton cascade there is an initial "skeleton" part, to be called the *source*, which emits in bremsstrahlung fashion (independent emission) smaller jets mostly composed of gluons and having a geometric multiplicity distribution [see Eq. (18) for the g.f.]. We call them *bremsstrahlung gluon jets* (BGJ).

II. For fixed window $|y| < y_0$, the mean number $\bar{N}_p$ of BGJ emitted by the source is approximately constant in the energy range $\sqrt{s} = 29$–2000 GeV. It grows linearly with y_0 and is about twice as large for gg as for $q\bar{q}$ (see Fig. 1).

III. The geometric multiplicity distribution of a BGJ depends only on its mean multiplicity ν. The latter is distributed with a weight $\propto d \ln \nu = d\nu/\nu$ over an interval $1 < \nu < \nu_{max}$. For fixed window $|y| < y_0$, ν_{max} increases with $\sqrt{s}$. At equal y_0 and $\sqrt{s}$ the values of ν_{max} are about the same for $q\bar{q}$ and gg systems.

IV. The parameters $\bar{N}_p$, ν_{max} are related by

$$\bar{N}_p = k_p \ln [1 + (\bar{n}_p/k_p)], \qquad \nu_{max} = 1 + (\bar{n}_p/k_p) \tag{24}$$

to the NB parameters $\bar{n}_p$, k_p of the parton multiplicity distribution.

V. For $\sqrt{s} > 200$ GeV the latter are related by

$$\bar{n}_p \simeq \bar{n}_h/2, \quad k_p \simeq k_h \tag{25}$$

to the NB parameters $\bar{n}_h$ and k_h of the charged hadron multiplicity distribution. This can be interpreted as a manifestation of local parton-hadron duality.

It would be of course be of great interest to understand mathematically why such simple approximate properties hold for the Lund Shower Model. While this problem is not yet solved (the asymptotic treatment of Malaza and Webber [18] does not apply in the $\sqrt{s}$ range we have considered), it is likely that the mathematical structure of the model can be sufficiently simplified to localize the elements which control the main behaviour.

On the other hand, the abundant experimental evidence on NB properties of multiplicities in high-energy hadronic and semi-leptonic reactions [1, 2] and their striking similarity with the Monte Carlo results discussed above suggest that the properties I–V could be tentatively extended to those reactions. This could lead to a unified picture for hard and soft hadron production in central rapidity windows in terms of process-dependent sources emitting bremsstrahlung gluon jets of a common type, with an appropriate form of local parton-hadron duality for the final hadronization phase [19].

We are greatly indebted to T. Sjöstrand for his invaluable collaboration on the Monte Carlo calculations and for many remarks and suggestions. We also acknowledge the help of W. Kittel and F. Meyers who made the Nijmegen negative binomial fitting program available for this work. We profited from discussions with G. Altarelli, T. T. Wu and G. Veneziano.

APPENDIX

The NB distribution has the form $(0 < b < 1,\ k > 0)$

$$P_0 = (1-b)^k, \qquad P_n = P_0 k(k+1)\dots(k+n-1)b^n/n! \quad \text{for} \quad n \geq 1. \tag{A.1}$$

The average $\bar{n}$ and the dispersion D are given by

$$\bar{n} = \frac{kb}{1-b}, \qquad \frac{D^2}{\bar{n}^2} \equiv \frac{\overline{n^2}-\bar{n}^2}{\bar{n}^2} = \frac{1}{\bar{n}} + \frac{1}{k}. \tag{A.2}$$

The generating function (g.f.) is

$$F_{\mathrm{NB}}(z) \equiv \sum_0^\infty P_n z^n = [(1-b)/(1-bz)]^k. \tag{A.3}$$

It leads immediately to the clan structure discussed in the main text, see Eqs. (13)–(15) and (2).

The geometric distribution of g.f. f_g given by Eq. (18) — more precisely it is a truncated geometric distribution — has the probabilities

$$p_0 = 0, \qquad p_n = (v-1)^{n-1}/v^n \quad \text{for} \quad n \geq 1 \tag{A.4}$$

and the mean multiplicity v. Combining Eqs. (2), (13) and (22), one obtains the relation

$$F_{\mathrm{NB}}(z) = \exp\{k \int_1^{1+(\bar{n}/k)} [f_g(v, z)-1]dv/v\} \tag{A.5}$$

which is equivalent to Eqs. (13) and (22). It shows that the NB distribution can be generated by independent emission of geometric clans with mean multiplicity ν ranging from 1 to $1+\bar{n}/k$, the average number of such clans in $(\nu, \nu+d\nu)$ being $kd\nu/\nu$.

One of the present authors [20] studied the class of distributions of points in a continuous domain D_0, characterized by the property that the multiplicity of points in D_0 and in every connected or disconnected subdomain D of D_0 has a negative binomial distribution. A distribution of this class is completely determined by the two functions $Q_1(y) = d\bar{n}/dy$ and $k(y)$, where $d\bar{n}$ and $k(y)$ are the average multiplicity and k-parameter of the NB in the infinitesimal neighbourhood dy of the point y. For a general subdomain D the NB parameters $\bar{n}_D$ and k_D are given by

$$\bar{n}_D = \int_D dyQ_1(y), \qquad \bar{n}_D/k_D = \int_D dyQ_1(y)/k(y). \tag{A.6}$$

For non-overlapping domains D_i and their union D, Eq. (A.6) gives the additivity property

$$\sum_i (\bar{n}_{D_i}/k_{D_i}) = (\sum_i \bar{n}_{D_i})/k_D = \bar{n}_D/k_D. \tag{A.7}$$

We have tested this property on the Monte Carlo data for $q\bar{q}$ at $\sqrt{s} = 2000$ GeV with two choices of domains D_i. The results, listed in Table III for one choice, show clear violations of (A.7). Weaker violations of this type have been found in hadronic and semi-leptonic reactions [1c, 2].

TABLE III

Test of additivity property (A.7) for $q\bar{q}$ at $\sqrt{s} = 2000$ GeV

	$\bar{n}$	k^{-1}	$\bar{n}/k$
D_1 : $\lvert y\rvert < 1$	8.1	0.91	7.3 ┐ sum
D_2 : $\lvert y-2\rvert < 1$	7.9	0.55	4.3 │
D_3 : $\lvert y+2\rvert < 1$	7.9	0.55	4.3 ┘ 16.0
D : $\lvert y\rvert < 3$	23.9	0.28	6.9

REFERENCES

[1] a) G. J. Alner et al., UA5 Collaboration, *Phys. Lett.* **B160**, 199 (1985); *Phys. Lett.* **B160**, 193 (1985); *Phys. Lett.* **B167**, 47 (1986). b) M. Derrick et al., HRS Collaboration, *Phys. Lett.* **168B**, 299 (1986); *Phys. Rev.* **D34**, 3304 (1986). c) M. Adamus et al., NA22 Collaboration, *Phys. Lett.* **B177**, 239 (1986); *Z. Phys.* **C32**, 475 (1986); M. Adamus et al., EHS/NA22 Collaboration, *Z. Phys.* **C37**, 215 (1988), d) F. Dengler et al., NA5 Experiment, *Z. Phys.* **C33**, 187 (1986). e)M. Arneodo et al., EMC, *Z. Phys.* **C35**, 335 (1987).

[2] A. Giovannini, L. Van Hove, *Acta Phys. Pol.* **B19**, 495 (1988).

[3] A. Giovannini, L. Van Hove, *Z. Phys.* **C30**, 391 (1986).

[4] L. Van Hove, A. Giovannini, in Proc. of XVII Int. Symp. on Multiparticle Dynamics, ed. M. Markytan et al., World Scientific, Singapore 1987, p. 561.

[5] W. Kittel, Proceedings of the Workshop on Physics at Future Accelerators, La Thuile and Geneva, CERN 1987, Report 87–07, vol. II, p. 424.

[6] A. Petersen et al., *Phys. Rev.* **D37**, 1 (1988).

[7] W. Hofmann, in Proc. of the 1987 Int. Symp. on Lepton and Photon Interactions at High Energies, eds. W. Bartel, R. Rückl, *Nucl. Phys.* B. (Proc. Suppl.) **3**, 671 (1988).
[8] M. Bengtsson, T. Sjöstrand, *Nucl. Phys.* **B289**, 810 (1987).
[9] T. Sjöstrand, M. Bengtsson, *Computer Physics Commun.* **43**, 367 (1987), especially sect. (6.3), *The Coherent Evolution Case.*
[10] G. Marchesini, B. R. Webber, *Nucl. Phys.* **B238**, 1 (1984); B. R. Webber, *Nucl. Phys.* **B238**, 492 (1984).
[11] T. Sjöstrand, *Computer Physics Commun.* **39**, 347 (1986), especially sects. 3, 4, 5.
[12] D. Amati, G. Veneziano, *Phys. Lett.* **B83**, 87 (1979); Ya. I. Azimov et al., *Z. Phys.* **C27**, 65 (1985); *Z. Phys.* **C31**, 213 (1986). For experimental support see: H. Aihara et al., TPC/Two Gamma Collaboration, *Phys. Rev. Lett.* **57**, 945 (1986).
[13] A. H. Mueller, *Phys. Rev.* **D4**, 150 (1971).
[14] K. Huang, *Statistical Mechanics*, John Wiley and Sons, New York-London 1963, Ch. 14.
[15] A. Giovannini, *Nucl. Phys.* **B161**, 429 (1979).
[16] K. Konishi, A. Ukawa, G. Veneziano, *Nucl. Phys.* **B157**, 45 (1979).
[17] J. Cugnon, O. Harouna, *Europhys. Lett.* **4**, 1127 (1987).
[18] E. D. Malaza, B. R. Webber, *Nucl. Phys.* **B267**, 702 (1986); E. D. Malaza, *Z. Phys.* **C31**, 31 (1986).
[19] See the following paper, *Acta Phys. Pol.* **B19**, 931 (1988).
[20] L. Van Hove, *Physica* **147A**, 19 (1987).