Vol. B19 (1988) ACTA PHYSICA POLONICA No 12

ESTIMATES OF QUARK MASSES, MOMENT RATIO AND PHOTOPRODUCTION CROSS-SECTIONS IN THE STATISTICAL MODEL

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(Received November 18, 1987; final version received May 12, 1988)

The wave function suggested from the statistical model has been used as an input to estimate the charm and bottom quark masses m_c and m_b in the context of the sum rules. The moment ratio, R_n, from SVZ moments and the photo-production cross-sections for S-state mesons in $\psi(c\bar{c})$ and $\Upsilon\,(b\bar{b})$ families have been computed with interesting predictions.

PACS numbers: 12.40.Ee, 12.35.Ht

1. Introduction

Over the past few years, a successful description of the $\psi\,(c\bar{c})$ and $\Upsilon\,(b\bar{b})$ families has been achieved and many predictions of the charmonium as well as bottomium levels have been confirmed experimentally. In non-relativistic charmonium models of ψ interpreted as a 1S state and ψ' interpreted as a 2S state, a crucial quantity is the wave function at the origin, $\phi(0)$, since, in particular, the leptonic width is controlled by the square modulus of the wave function at the origin, $|\phi(0)|^2$. However, in the absence of a satisfactory solution to the confinement problem, the wave function of a meson in terms of its constituents cannot be determined from first principle. In the framework of a static model, such as the MIT bag model, the wave function, however, can be calculated and expressed in terms of a few phenomenological parameters related to a hadron. Unlike the conventional quark model for hadron, in the statistical model [1–5], we arrive at the square modulus of the S-state wave function of a meson, $|\phi(r)|^2$ corresponding to an average background linear or harmonic oscillator potential in which the valence quark, q (or antiquark, $\bar{q}$), moves in the virtual cloud of quarks (or antiquarks). Interestingly, in this model, one arrives at $|\phi(r)|^2$ without any reference to the context of the conventional wave equation or field theoretical formalism; further, this $|\phi(r)|^2$ depends only on one parameter r_0, corresponding to the size of a meson i.e. our model wave function is independent of all other uncertain parameters like the quark mass, m_q, as well as other interaction parameters. With such an unconventional wave function, it would be interesting to know how much spectroscopic and other relevant information can be extracted. In the present work, we have used our model wave function as an input in the well known sum rules corresponding to leptonic decay widths as well as moment sum rules to estimate the constituent quark masses. We have further computed the moment ratio, R_n, from SVZ moments which is, surprisingly, found to possess the maximum value of the inverse square of the corresponding (1S) ground state masses of ψ and Υ families. Moreover, we have also made calculations of the photo-production cross-sections of ψ and Υ particles and our predicted values agree closely with those computed by several other workers.

2. *The statistical model*

The statistical model for a hadron has so far successfully described some of the interesting properties of mesons [1–4] and baryons [5] at least in the domain of S-state levels in the past few years. As discussed at length in these works, our statistical model of a S-state $q\bar{q}$ meson, suggests that hadrons are composite particles consisting of large numbers of virtual quarks and antiquarks in addition to real valence quark (q) and antiquark ($\bar{q}$) such that only the valence pair ($q\bar{q}$) determines the quantum numbers of the meson. The valence quark in our model is assumed to be acting independently of and without any correlation with the valence antiquark and we assume that the valence quark (and also the valence antiquark) moves in an average linear or harmonic oscillator background potential due to interaction with its own virtual quarks (or antiquarks) in the sea or cloud. We have assumed that the valence quark (or antiquark) is identical and indistinguishable with the virtual quarks (or antiquarks) in the sea. The indistinguishability of the valence quark with the virtual quarks in the sea (and similarly for the antiquarks) calls for the existence of the well-known quantum mechanical uncertainty or indeterminism in the location of the valence quarks (and similarly for antiquark). Consequently, we came across a situation of a continuous distribution of quarks (and also for antiquarks) through the virtual cloud, and hence there is also a continuous distribution of colour (and flavour). Therefore, we can imbibe the concept of the number density of quarks i.e. $n_q(r)$ (and similarly for antiquarks $n_{\bar{q}}(r)$ for its distribution in the respective phase space or configuration space of a hadron. Further, for a continuous distribution of colour, we came across local colour symmetry at each point of colour neutral meson so that the number density of quarks, $n_q(r)$, and that of antiquarks, $n_{\bar{b}}(r)$, at each point r must be the same. In other words, due to colour neutrality of a hadron $n_q(r) = n_{\bar{q}}(r)$. As is well known, the number of particles in some volume δv is proportional to the probability of finding a particle in this volume. Hence, with this approach we may assign this number density, $n_q(r) = n_{\bar{q}}(r) = |\phi(r)|^2$. Without going into further detailed derivation of $|\phi(r)|^2$ we may recall an expression for the exact normalized wave function corresponding to average harmonic oscillator background potential of the type $V(r) = ar^2+b$ experienced by the quark inside the hadron as [2]

$$|\phi(r)|^2 = A(r_0^2-r^2)^{3/2}\theta(r_0-r), \tag{1}$$

where, $A = 8/\pi^2 r_0^6$ is the normalization constant, and r_0 corresponds to the size of a meson. Therefore, we get the square modulus of the wave function at $r = 0$ as

$$|\phi_{ns}(0)|^2 = \frac{8}{\pi^2 r_{0(ns)}^3}. \tag{2}$$

In our subsequent investigation, we shall use our model wave function as an input, since our $|\phi(0)|^2$ is simpler than those suggested by other theoretical works. Moreover, it is neither dependent on the quark mass parameter, m_q, nor on the interaction parameter a and b of the average background potential. i.e. independent of all other uncertain parameters except r_0. Since the experimental data of the corresponding radii of vector meson such as ψ and Υ meson are not available, we may get an order of $r_0(ns)$ from the theoretically predicted values [6–8]. In this context it is relevant to note that different authors have theoretically computed the radius parameter, r_0, of $\psi(c\bar{c})$, $\Upsilon(b\bar{b})$, and $\xi(t\bar{t})$ families in different phenomenological approaches [6–8]. An analysis of their predicted values of $r_0(ns)$ for the aforesaid family shows the corresponding values of $r_0(ns)$ are almost the same, which in turn suggests that the corresponding radii are independent of the mathematical formalism used and also independent of the corresponding interaction between quark (q) and antiquark ($\bar{q}$). Therefore, we may assume r_0 for each member of $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$ families as almost constant and as good as experimentally observed size parameters. However, for our numerical calculations, we shall use subsequently the theoretically estimated values of $r_0(ns)$ from Kaburagi et al. [6].

3. Estimate of quark masses from sum rules

A. Sum rules in annihilation

Although direct evidence of quarks has so far eluded experimental test, few experiments that have been carried out to probe hadronic structure by means of electromagnetic and weak interactions suggest that hadrons have constituents known as quarks. Hence, final hadrons formed through an intermediate quark antiquark state may offer some clues about the hadronic constituents. The newly discovered mesons with flavours charm and bottom possess the important features that heavy quark antiquark bound states annihilate into lepton pairs and hadrons. The expression of the decay rate of a vector meson into a lepton pair, $V \to e^+e^-$ is known by Van Royon-Weisskopf formula [9]

$$\Gamma(V_{ns} \to e^+e^-) = \frac{16\pi\alpha^2 e_q^2}{M_{ns}^2} |\phi_{ns}(0)|^2 \tag{3}$$

and may be recast below flavour threshold as

$$\sum \frac{\Gamma(V_{ns} \to e^+e^-)}{M_{ns}^{\nu}} \simeq \int \frac{\Gamma(V_{ns} \to e^+e^-)}{M_{ns}^{\nu}}\, dn$$

$$= \frac{128\alpha^2 e_q^2}{\pi} \int_{r_0(1s)}^{r_0(2s)} \frac{dn}{M_{ns}^{\nu+2} r_0(ns)}, \tag{4}$$

where $|\phi_{ns}(0)|^2 = 8/\pi^2 r_3^0(ns)$ and ν is any integer.

In our previous work [2] we have derived a mass formula of mesons, M_{ns}, which assumes the form:

$$M_{ns} = 2m_q + \frac{4}{3m_q r_0^2(ns)} + \frac{3\pi a r_0^2(ns)}{4} + \frac{b}{2}. \tag{5}$$

In the present work, we assume the empirical relation

$$r_0(ns) = \varepsilon n^\beta \tag{6}$$

where ε and β are constants. It is surprising to note that, assuming the above relation, the value of the radius parameter, r_0, for different levels can be well computed. To obtain the values of ε and β, we have used $r_0(1S) = 1.91\ \text{GeV}^{-1}$ for $\psi(1S)$ as computed by Kaburagi et al. [6] and we get $\varepsilon = 1.91\ \text{GeV}^{-1}$ for $n = 1$. For $n = 2$, $\varepsilon = 1.91\ \text{GeV}^{-1}$ and $r_0(2S) = 4.184\ \text{GeV}^{-1}$ as inputs in (6), we found $\beta = 1.13$. Now, using the obtained value of $\varepsilon = 1.91\ \text{GeV}^{-1}$ and $\beta = 1.13$ one gets $r_0(3S) = 6.39\ \text{GeV}^{-1}$ and $r_0(4S) = 8.77\ \text{GeV}^{-1}$ from (6), where the values compare favourably with $r_0(3S) = 6.403\ \text{GeV}^{-1}$ and $r_0(4S) = 8.586\ \text{GeV}^{-1}$ computed by Kaburagi et al. [6], which in turn favours our conjecture for the empirical relation (6). Similarly, for the $\Upsilon(b\bar{b})$ we get $\varepsilon = 1.090\ \text{GeV}^{-1}$ for $r_0(1S) = 1.090\ \text{GeV}^{-1}$ and $n = 1$ as inputs in (6). Retaining the same value of $\beta = 1.13$ as obtained in the ψ family, it is possible to reproduce the radii of the remaining S-states of Υ family. Therefore, it is meaningful to use our empirical relation (6) so that we come across the following relation from (4), (5) and (6):

$$\frac{\beta\varepsilon^{1/\beta}}{128\pi\alpha e_q^2} \sum \frac{\Gamma(V_{ns} \to e^+e^-)}{M_{ns}^\nu} = \int_{r_0(1s)}^{r_0(2s)} r_0^{(1-4\beta)/\beta} \left[2m_q + \frac{4}{3m_q r_0^2} + \frac{3\pi a r_0^2}{4} + \frac{b}{2}\right]^{-(\nu+2)} dr_0. \tag{7}$$

Approximately evaluating the integral on the right hand side of (7) for ψ and Υ families we get respectively,

$$\frac{\beta\varepsilon^{1/\beta}}{128\pi\alpha e_q^2} \sum \frac{\Gamma(V_{ns} \to e^+e^-)}{M_{ns}^\nu} \simeq \frac{0.1}{(2m_c)^{\nu+2}} - \frac{\nu+2}{(2m_c)^{\nu+3}} \left[\frac{0.02}{m_c} + 1.64a + 0.05b\right] \tag{8}$$

$$\frac{\beta\varepsilon^{1/\beta}}{128\pi\alpha e_q^2} \sum \frac{\Gamma(V_{ns} \to e^+e^-)}{M_{ns}^\nu} \simeq \frac{0.3}{(2m_b)^{\nu+2}} - \frac{\nu+2}{(2m_b)^{\nu+3}} \left[\frac{0.2}{m_b} + 1.7a + 0.15b\right]. \tag{9}$$

Adjusting the numerical values of m_c and m_b, we have performed our calculations for different sets of values of ν in the range $0 \leqslant \nu \leqslant 35$ so as to be consistent with the sum rule (8) and (9). For our numerical calculations we have used the experimentally observed

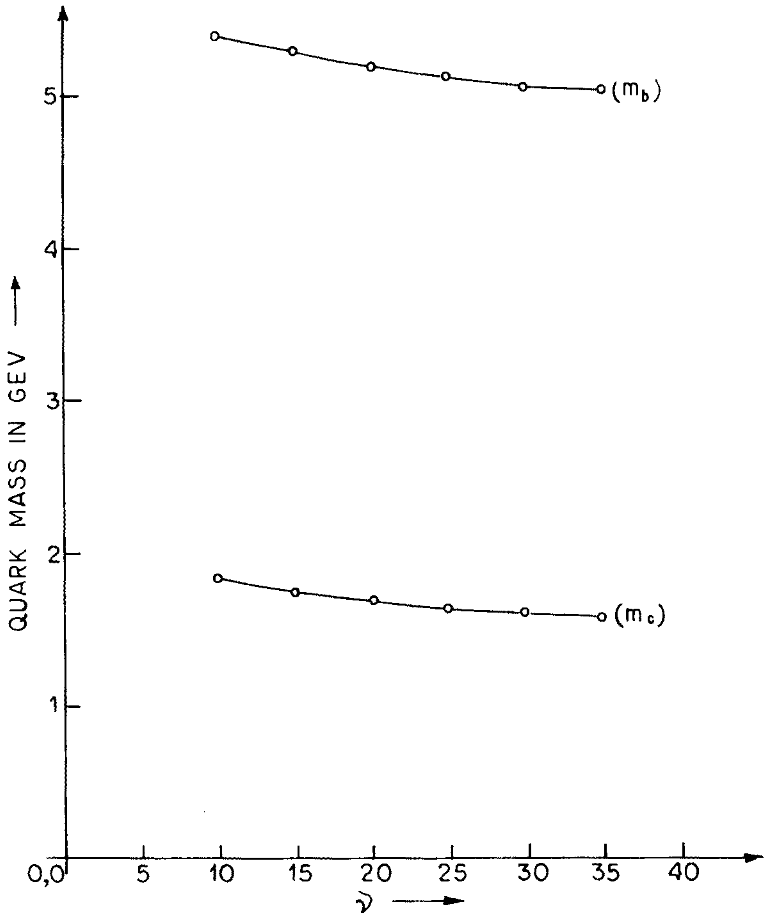

Fig. 1. Charm as well as bottom quark mass values predicted from sum rule in (8) and (9) plotted against the integral values of ν in the range of $10 \leqslant \nu \leqslant 35$

values of $\Gamma_\psi = 4.8$ KeV for ψ (3095), $\Gamma_{\psi'} = 2.1$ KeV for ψ' (3686) and $\Gamma_\Upsilon = 1.2$ KeV for Υ (9459), $\Gamma_{\Upsilon'} = 0.51$ KeV for Υ' (10023). For the strength parameters a and b of the potential $V(r) = ar^2+b$ we have used $a = 0.0228$ GeV^3, $b = -0.8698$ GeV for ψ family and $a = 0.0688$ GeV^3, $b = -0.7858$ GeV for Υ family from our previous work [2]. For radius parameter, r_0, of ψ and Υ families we have considered the theoretically computed values estimated by Kaburagi et al. [6]. Our calculated values of charm and bottom quark masses, m_c and m_b, are plotted against ν in the Fig. 1. We note from Fig. 1 that the quark mass value, m_q, does not change appreciably and approaches $m_c \simeq 1.6$ GeV and $m_b = 5$ GeV, so that m_q scales with respect to ν.

B. Moment sum rule

In the context of testing quantum chromodynamics with sum rules, Miller and Olsson [10] have investigated the consistency of e^+e^- annihilation data with QCD. Moment sum rule through semilocal duality in which only sharp states appear, may be defined [10] as

$$M_\nu^{\exp} = \frac{9\pi}{\alpha^2} \sum_{\substack{\text{sharp} \\ \text{states}}} \frac{1}{s_0^{\nu+1}} \Gamma_{ns} M_{ns}^{2\nu+1} \tag{10}$$

where S_0 is a dimensional factor chosen arbitrarily and all other symbols have their usual meanings. As before, the moment sum rule in the context of our statistical model (SM) becomes:

$$M_\nu^{\mathrm{SM}} \simeq \frac{9\pi}{\alpha^2} \frac{1}{s_0^{\nu+1}} \frac{128\alpha^2 e_q^2}{\pi\beta\varepsilon^{1/\beta}} \int_{r_0(1\mathrm{S})}^{r_0(2\mathrm{S})} r_0^{(1-4\beta)/\beta} \left[2m_q + \frac{4}{3m_q r_0^2} + \frac{3\pi a r_0}{4} + \frac{b}{2}\right]^{2\nu-1} dr_0 \tag{11}$$

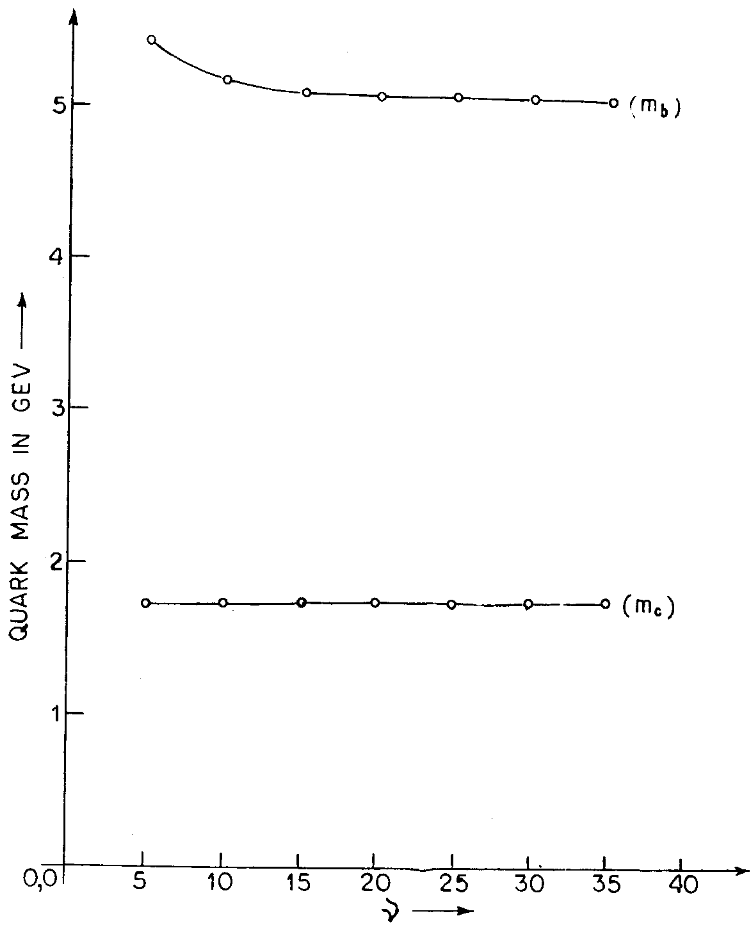

Fig. 2. Charm as well as bottom quark mass values predicted from moment sum rule in (13) plotted against the integral values of ν in the range of $5 \leqslant \nu \leqslant 40$

or

$$M_{\nu}^{\rm SM} \simeq \frac{9\pi}{\alpha^2}\,\frac{1}{s_0^{\nu+1}}\,\frac{128\alpha^2 e_q^2}{\pi\beta\varepsilon^{1/\beta}}\left[2m_q+\frac{4}{3m_q r_0^2}+\frac{3\pi a r_0^2}{4}+\frac{b}{2}\right]^{2\nu-1}\Bigg|_{r_0\cong r_0(an)} \times \int_{r_0(1S)}^{r_0(2S)} r_0^{(1-4\beta)/\beta}\,dr, \tag{12}$$

where we have approximated the mass formula in the neighbourhood of $r_0(av) \simeq [r_0(1S) + r_0(2S)]/2$ regarding the mass formula to be approximately constant in the accepted range of integration. To estimate the quark mass values of m_c and m_b we have

$$\sum \Gamma_{ns} M_{ns}^{2\nu+1} \simeq \frac{128\alpha^2 e_q^2}{\pi\beta\varepsilon^{1/\beta}}\left[2m_q+\frac{4}{3m_q r_0^2}+\frac{3\pi a r_0^2}{4}+\frac{b}{2}\right]^{2\nu-1} \times\left[r_0(2S)^{-2.115}-r_0(1S)^{-2.115}\right]. \tag{13}$$

Now, with m_q as a variable, we have performed our calculation for different sets of values of ν in the range $0 \leqslant \nu \leqslant 40$. We have plotted our computed values of m_c and m_b in the Fig. 2 against the different sets of integral values of ν and we find in this case also that m_q scales with respect to ν i.e. m_q is independent of ν and attains the constant value $m_c \simeq 1.7$ GeV and $m_b \simeq 5$ GeV.

4. *Moment ratio, R_n, from SVZ moments*

Following the well-known Shifman-Vainshtein-Zakharov (SVZ) work [11] we may define the moments M_n' as

$$M_n' = \frac{1}{\pi}\int \frac{{\rm Im}\,\pi(s)}{s^{n+1}}\,ds, \tag{14}$$

where Im $\pi(s)$ is proportional to the total cross-section which in the narrow width approximation is

$${\rm Im}\,\pi \sim \sigma = 12\pi^2 \sum_j \delta(s-M_j)^2 \Gamma_j(V \to e^+e^-)/M_j, \tag{15}$$

where M_j is the mass of the corresponding vector meson and Γ_j is the leptonic decay width of the jth level. Within the framework of the statistical model, the moments may assume the form:

$$M_n' = \sum_j \left(\frac{32}{\pi}\right)\Big/ r_{0j}^3 M_j^N, \tag{16}$$

where, r_{0j} corresponds to the sizes of the mesons of jth s-state levels, and $N = 2n+1$ with $n = 1, 2, 3 \ldots$. As in SVZ work, the moment ratio, R_n, in terms of power moments can be expressed as

$$R_n = \frac{M'_N}{M'_{N-2}}. \tag{17}$$

Using Eqs (16) and (17) we have made numerical estimate of the moment ratio, R_n, both for ψ and Υ families for different values of n in the range $1 \leqslant n \leqslant 40$. Our computed values are displayed in Table I. It may be observed from the table that for any value of n,

TABLE I

Predicted values of moment ratios R_n from (18) for ψ and Υ families listed for the corresponding different integral values of n in the range $1 \leqslant n \leqslant 40$

n	$R_n(c\bar{c})$ (GeV^{-2})	$R_n(b\bar{b})$ (GeV^{-2})
1	0.0976	0.0098
2	0.0992	0.0098
3	0.0989	0.0099
4	0.0998	0.00993
5	0.1000	0.00994
10	0.1006	0.00997
15	0.1007	0.00999
20	0.1007	0.00999
30	0.10076	0.0100
40	0.10076	0.0100

both for ψ and Υ families, R_n is almost constant and attains values $R_n \sim 0.10076\ \text{GeV}^{-2}$ for ψ family and $R_n \sim 0.01\ \text{GeV}^{-2}$ for Υ family. In other words, within the framework of our model the moment ratio is completely dominated by a single resonance. In this context it may be noted that SVZ have assumed the maximum value of R_n to be $(E_1+2m_q)^{-2}$ (where E_1 is the ground state energy) i.e. the maximum value of R_n should have value $(3.095)^{-2} = 0.1044\ \text{GeV}^{-2}$. Our predicted values of R_n are found to compare favourably with the assumed value of R_n of SVZ for charmonium family. Hence, the behaviour of R_n in the statistical model differs from the conventional quark model [12] and enables us to explain the physical meaning of the expected moment ratio, R_n.

5. *Estimate of* $\sigma_c(\gamma p)$ *and* $\sigma_b(\gamma p)$

The mechanism of charm hadron production has been extensively discussed earlier by the authors [13] since the discovery of $\psi(c\bar{c})$ particles. Within the conventional models of strong interactions (e.g. VDM, Regge poles, etc.) the ψ-meson photo production has been considered with the basic aim of calculating the charm particles photo production i.e. $\sigma_c(\gamma p)$. However, without delving further into the mechanism of charm hadron production, we may recall an expression for the quantity $\sigma_q(\gamma p)$ (where q represents c or b) which may be represented as a sum of the contributions of vector mesons, $\psi(c\bar{c})$ and $\Upsilon(b\bar{b})$ [14] as

$$\sigma_q(\mathrm{vp}) \simeq \alpha\sigma(\mathrm{vp}) \sum_{n=1}^{\infty} \frac{4\pi}{g_n^2}, \tag{18}$$

where the symbols have their usual meanings. The vector meson photon coupling constant is determined from the relation

$$\Gamma(\mathrm{V}_n \to \mathrm{e}^+\mathrm{e}^-) = \tfrac{1}{3} M_n\alpha^2/(g_n^2/4\pi), \tag{19}$$

where M_n represents the vector meson mass. Now using (3) in (19) we obtain

$$\frac{4\pi}{g_n^2} = \frac{384}{\pi} [M_n r_0(n)]^{-3} e_q^2. \tag{20}$$

Therefore, within the framework of the statistical model, the cross-sections of the charm as well as bottom particles photo production take the form:

$$\sigma_q(\gamma\mathrm{p}) \simeq 122.23\, e_q^2\sigma(\mathrm{vp}) \sum_{n=1}^{\infty} [M_n r_0(n)]^{-3}. \tag{21}$$

The cross-section, $\sigma_q(\gamma\mathrm{p})$, indicates the dependence on the masses of the vector meson, M_n, the radius parameter, r_0, and $\sigma(\gamma\mathrm{p})$. However, to compute the value of $\sigma_q(\gamma\mathrm{p})$ from (21), each vector meson within the family is taken to have the same total cross-section, $\sigma(\gamma\mathrm{p})$, for scattering on the proton. Using the experimentally observed masses of the $\psi(c\bar{c})$ and $\Upsilon(b\bar{b})$ particles for S-state levels and theoretically obtained [6] values of r_0 for these particles and considering $\sigma(\psi\mathrm{p}) = 1.6$ mb, $\sigma(\Upsilon\mathrm{p}) = 174$ μb we have $\sigma_c(\gamma\mathrm{p}) = 3.25$ μb and $\sigma_b(\gamma\mathrm{p}) = 68.33$ nb. Previously, Quigg and Rosner [15] and Horn [16] have theoretically computed the cross-sections $\sigma_c(\gamma\mathrm{p})$ and $\sigma_b(\gamma\mathrm{p})$ in the different phenomenological approaches. Quigg and Rosner [15] have obtained the value of $\sigma_c(\gamma\mathrm{p}) = 2.8$ μb and $\sigma_b(\gamma\mathrm{p}) = 75$ nb; on the other hand Horn [16] estimated the values of $\sigma_c(\gamma\mathrm{p}) = 2$ μb and $\sigma_b(\gamma\mathrm{p}) = 25$ nb. In comparison, our speculated values of $\sigma_c(\gamma\mathrm{p}) = 3.25$ μb and $\sigma_b(\gamma\mathrm{p}) = 68.33$ nb are in good agreement with those obtained by Quigg and Rosner.

6. *Conclusions*

The statistical model proposed for mesons has been used to investigate some of the properties of $\psi(c\bar{c})$ and $\Upsilon(b\bar{b})$ mesons. Our main contribution becomes a new estimate of $|\phi(0)|^2$ which plays an important role in various meson decays. Further, the close analytical expression for $|\phi(r)|^2$ for a meson is not only simple and elegant but also is easily amenable to application in mesonic decays. Using our model wave function (which is free of several uncertain parameters like the constituent quark masses and the parameters of the average harmonic oscillator potential in which a quark or antiquark moves), we have shown that the sum rule is consistent in the context of our statistical model. Not only a reasonable estimates of the charm quark mass, $m_c \sim 1.6$ GeV, and bottom quark mass, $m_b \sim 5$ GeV, are obtained but also we come across an interesting situation that m_q turns out to be almost independent of the integral values of ν of the sum rules. This appears to be a striking feature of our model unlike other theoretical works. Another interesting feature of our model we came across in the computation of the moment ratio, R_n, is that it is dominated by a single resonance as it possesses the maximum value of the inverse square of the corresponding ground state masses of the $\psi(c\bar{c})$ and $\Upsilon(b\bar{b})$ families as, $\lim_{n \to \text{large}} R_n = \frac{1}{M^2}$.

Considering the simplicity and elegance of the closed analytical form for $|\phi(r)|^2$ of a meson, which has been derived without any reference to the context of the conventional equation of motion or using any field theoretical formalism and the fact that it yields reasonable estimates of the quark masses, $\sigma_q(\gamma p)$ and R_n, which agrees with other theoretical findings, it may be argued that our model for hadron is not far from reality.

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